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Uncertainty, efficiency and incentive compatibility: Ambiguity solves the conflict between efficiency and incentive compatibility ☆

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Abstract

A fundamental result of modern economics is the conflict between efficiency and incentive compatibility, that is, the fact that some Pareto optimal (efficient) allocations are not incentive compatible. This conflict has generated a huge literature, which almost always assumes that individuals are expected utility maximizers. What happens if they have other kind of preferences? Is there any preference where this conflict does not exist? Can we characterize those preferences? We show that in an economy where individuals have complete, transitive, continuous and monotonic preferences, every efficient allocation is incentive compatible if and only if individuals have maximin preferences.

JEL classification: D50; D81; D82

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1. Introduction

One of the fundamental problems in mechanism design and equilibrium theory with asymmetric information is the conflict between efficiency and incentive compatibility. That is, there are allocations that are efficient but not incentive compatible. This important problem was alluded to in early seminal works by Wilson (1978), Myerson (1979), Holmstrom and Myerson (1983), and Prescott and Townsend (1984). Since incentive compatibility and efficiency are some of the most important concepts in economics, this conflict generated a huge literature and became a cornerstone of the theory of information economics, mechanism design and general equilibrium with asymmetric information.

It is a simple but perhaps important observation, that this conflict was predicated on the assumption that the individuals were expected utility (EU) maximizers, that is, they would form Bayesian beliefs about the type (private information) of the other individuals and seek the maximization of the expected utility with respect to those beliefs. Since the Bayesian paradigm has been central to most of economics, this assumption seemed natural.

The Bayesian paradigm is not immune to criticism, however, and many important papers have discussed its problems; e.g. Allais (1953), Ellsberg (1961) and Kahneman and Tversky (1979) among others. The recognition of those problems have led decision theorists to propose many alternative models, beginning with Bewley (1986, 2002), Schmeidler (1989) and Gilboa and Schmeidler (1989), but extending in many different models. For syntheses of these models, see Maccheroni et al. (2006) and Cerreia-Vioglio et al. (2011) among others.

The fact that many different preferences have been considered leads naturally to the following questions: Does the conflict between efficiency and incentive compatibility extend to other preferences? Is there any preference under which there is no such conflict? The purpose of this article is to answer these questions.

Our main result shows that all efficient (Pareto optimal) allocations are also incentive compatible *if and only if* individuals have (a special form of) the maximin expected utility (MEU) preferences introduced by Gilboa and Schmeidler (1989): Wald's maximin preference.[1] Therefore, the conflict between efficiency and incentive compatibility is much broader than previously established.

The implication that all efficient allocations are incentive compatible may suggest that the set of efficient allocations for maximin preferences is small, but we show that this is not the case. At least in the case of one-good economies, the set of efficient allocations under maximin preferences includes all allocations that are incentive compatible and efficient for EU individuals. This result (Theorem 5.1) seems somewhat surprising, since other papers have indicated that ambiguity may actually be bad for efficiency, limiting trading opportunities. See for instance Mukerji (1998) and related comments in section 6.

The paper is organized as follows. In section 2, we describe the setting and introduce definitions and notation. Section 3 presents the main result in the paper: all Pareto optimal allocations

[1] See the formal definition of these preferences in section 2.4.

are incentive compatible if and only if all individuals are expected utility maximizers. We illustrate how our results can be cast in the mechanism design perspective in section 4. Section 5 establishes that the set of efficient and incentive compatible allocations in the EU setting are also MEU efficient. Section 6 reviews the relevant literature and section 7 discusses future directions of research. All proofs are collected in the appendix.

2. Model

Let $I = \{1, ..., n\}$ be the set of individuals in the economy. Each agent $i \in I$ observes privately his own signal t_i in some finite set of possible signals T_i. Write $T = T_1 \times \cdots \times T_n$. A vector $t = (t_1, ..., t_i, ..., t_n) \in T$ represents the vector of all types. As usual, T_{-i} denotes $\Pi_{i \neq j}^n T_j$ and, similarly, t_{-i} denotes $(t_1, ..., t_{i-1}, t_{t+1}, ..., t_n)$. We may write $t = (t_i, t_{-i})$ and, occasionally, we will write t as (t_i, t_j, t_{-i-j}), with the obvious meaning.

Each individual cares about an outcome (e.g. consumption bundle) $b \in \mathcal{B} = \mathbb{R}_+^L$, for some $L \in \mathbb{N}$.[2] An *allocation* is a function $x : T \to \mathcal{B}^n$, with $x(t) = (x_1(t), ..., x_n(t)) \in \mathcal{B}^n$, meaning that individual $i = 1, ..., n$ receives $x_i(t)$ when types are $t = (t_1, ..., t_n)$. In this case, x_i will be called an *individual allocation*, or i's individual allocation. We will also adopt the usual notation $x = (x_i, x_{-i})$ for allocations. Let $\mathcal{A}$ denote the *set of all allocations*.

The set of functions $f : T \to \mathbb{R}^L$ will be denoted $\mathcal{C}$. Thus, we may identify $\mathcal{C}^n$ with the set of functions $f : T \to \mathbb{R}^{Ln}$, where $f(t) = (f_1(t), ..., f_n(t))$ and $f_i(t) \in \mathbb{R}^L$ for each $i \in I$. Note that the set of allocations $\mathcal{A}$ is a subset of $\mathcal{C}^n$.

Given $x = (x_1, ..., x_n) \in \mathcal{C}^n$, $\bar{x} \in \mathcal{C}$ is the function defined by $\bar{x}(t) \equiv \sum_{i=1}^n x_i(t)$. When x is an allocation, we may say that $\bar{x}$ is the corresponding *aggregate allocation*. Given any set $\mathcal{S} \subset \mathcal{C}^n$, we denote by $\bar{\mathcal{S}}$ the set of functions $h : T \to \mathbb{R}^L$ such that $h = \bar{g}$ for some $g \in \mathcal{S}$.

Each individual has an initial *endowment* $e_i : T \to \mathcal{B}$. Given an allocation of initial endowments $e = (e_1, ..., e_n) \in \mathcal{A}$, an allocation $x \in \mathcal{A}$ is *feasible given* e if $\bar{x} = \bar{e}$, that is, $\sum_{i=1}^n x_i(t) = \sum_{i=1}^n e_i(t)$ for all $t \in T$. If e is clear from the context, we may say only that x is feasible. We denote by $\mathcal{A}(e)$ the set of allocations that are feasible given $e \in \mathcal{A}$.

2.1. Private endowments

We assume that individual i's endowment depends only on t_i and not on the types of other individuals, that is, we will maintain the following assumption throughout the paper, even if it is not explicitly repeated:

Assumption 2.1 *(Private endowments).* For every $i \in I$, $t_i \in T_i$ and $t_{-i}, t'_{-i} \in T_{-i}$, the endowments satisfy: $e_i(t_i, t_{-i}) = e_i(t_i, t'_{-i})$.

This assumption is almost always assumed in the literature regarding general equilibrium with asymmetric information, no-trade, auctions and mechanism design. In the latter, endowments are usually assumed to be constant with respect to types (as in Morris (1994)) or not explicitly considered. Note that if endowments are constant, Assumption 2.1 is automatically satisfied. In auctions, the players are assumed to be buyers or sellers with explicit fixed endowments, which

[2] Our results hold if the set of bundles $\mathcal{B}$ is assumed to be just a (convex subset of a) topological vector space. We assume $\mathcal{B} = \mathbb{R}_+^L$ just to simplify notation and arguments.

again implies Assumption 2.1. Even when the endowments may vary with types, as in Jackson and Swinkels (2005), where the private information is given by (e_i, v_i), i.e., endowments and values, Assumption 2.1 is still satisfied, because the endowment depends only on player i's private information. In fact, if Assumption 2.1 were not satisfied, the individuals would not know their own endowments, which may appear as awkward. Thus, Assumption 2.1 may be considered a mild and natural assumption.

Let $\mathcal{E}$ denote the set of endowments $e \in \mathcal{A}$ that satisfy Assumption 2.1 and, for each $i = 1, ..., n$, let $\mathcal{E}_i$ denote the set of functions $e_i : T \to \mathcal{B}$ such that $e_i(t_i, t_{-i}) = e_i(t_i, t'_{-i})$ for all $t_i \in T_i$ and $t_{-i}, t'_{-i} \in T_{-i}$. Thus, we may identify $\mathcal{E}$ with $\mathcal{E}_1 \times \cdots \times \mathcal{E}_n$.

While it is clear that $\mathcal{E} \subsetneq \mathcal{A}$, it is also true, but less obvious, that $\bar{\mathcal{E}} \subsetneq \bar{\mathcal{A}}$,[3] that is, there are aggregate allocations $\bar{x} \in \bar{\mathcal{A}}$ for which there are no endowments $e = (e_1, ..., e_n) \in \mathcal{E}$ satisfying $\bar{x} = \bar{e}$. The following example clarifies the issue.

Example 2.2. Let $n = 2$, $L = 1$, $|T_1| = p = 2$ and $|T_3| = q = 3$, such that $T_1 = \{t_1, t'_1\}$ and $T_2 = \{t_2, t'_2, t''_2\}$. If $e = (e_1, e_2)$ satisfies Assumption 2.1, then e_1, e_2, and $\bar{e} \equiv \sum_{i=1}^{2} e_i$ are of the form:

$$
\begin{array}{c|c|c|c|}
e_1 & t_2 & t'_2 & t''_2 \\ \hline
t_1 & a & a & a \\ \hline
t'_1 & b & b & b \\ \hline
\end{array}
\quad + \quad
\begin{array}{c|c|c|c|}
e_2 & t_2 & t'_2 & t''_2 \\ \hline
t_1 & c & d & e \\ \hline
t'_1 & c & d & e \\ \hline
\end{array}
\quad = \quad
\begin{array}{c|c|c|c|}
\bar{e} & t_2 & t'_2 & t''_2 \\ \hline
t_1 & a+c & a+d & a+e \\ \hline
t'_1 & b+c & b+d & b+e \\ \hline
\end{array}
$$

It is easy to see that the form on the right does not span R^6_+, which is the set of all allocations $x = \big(x(t_1, t_2)_{(t_1,t_2)\in T_1\times T_2}\big)$ since $|T_1 \times T_2| = 6$. For instance, there are no $e = (e_1, e_2)$ satisfying Assumption 2.1 such that $\bar{e} = e_1 + e_2$ is given by

$$
\begin{array}{c|c|c|c|}
\bar{e} & t_2 & t'_2 & t''_2 \\ \hline
t_1 & 1 & 2 & 3 \\ \hline
t'_1 & 1 & 1 & 1 \\ \hline
\end{array}
.
$$

From the above, it is useful to characterize the set $\bar{\mathcal{E}}$ of allowable aggregate allocations.

Lemma 2.3. *We have $x_i \in \bar{\mathcal{E}}$ if and only if for every $j = 1, ..., n$, and $t_j, t'_j \in T_j$, and $t_{-j}, t'_{-j} \in T_{-j}$, we have*

$$x_i(t'_j, t_{-j}) - x_i(t_j, t_{-j}) = x_i(t'_j, t'_{-j}) - x_i(t_j, t'_{-j}). \tag{1}$$

Moreover, $\bar{\mathcal{E}}$ is convex and if $x_i \in \bar{\mathcal{E}}$, for $i = 1, ..., n$, then $x = (x_1, ..., x_n)$ satisfies $\bar{x} = \sum_{i=1}^{n} x_i \in \bar{\mathcal{E}}$.

2.2. Preferences, axioms and efficiency

We have observed in the previous section that Assumption 2.1 restricts the set of aggregate endowments $\bar{\mathcal{E}}$, that is, $\bar{\mathcal{E}} \subsetneq \bar{\mathcal{A}}$. By Lemma 2.3, we know that if $x_i \in \bar{\mathcal{E}}$, for $i = 1, ..., n$, then $x = (x_1, ..., x_n) \in \mathcal{A}$ satisfies $\bar{x} \in \bar{\mathcal{E}}$. For these reasons, our results concern preferences $\succcurlyeq_i^{t_i}$ defined

[3] Recall that $\bar{\mathcal{E}}$ is the set of functions $\bar{e} \equiv \sum_{i=1}^{n} e_i$ for some $e \in \mathcal{E}$, and similarly for $\mathcal{A}$ and $\bar{\mathcal{A}}$.

over $\bar{\mathcal{E}}$, for each given $t_i \in T_i$ and $i = 1, ..., n$.[4] The axioms below are supposed to be satisfied for all such preferences, that is, for all $i = 1, ..., n$ and all $t_i \in T_i$. We begin by assuming that the preference is a total preorder (weak order):

Axiom 1 *(Weak order).* $\succcurlyeq_i^{t_i}$ is complete and transitive.

As usual, we define the strict preference $x_i \succ_i^{t_i} y_i$ by $x_i \succcurlyeq_i^{t_i} y_i$ and $\neg(y_i \succcurlyeq_i^{t_i} x_i)$; and the indifference preference $x_i \sim_i^{t_i} y_i$ by $x_i \succcurlyeq_i^{t_i} y_i$ and $y_i \succcurlyeq_i^{t_i} x_i$.

We assume that $\succcurlyeq_i^{t_i}$ depends only on the values of $x_i \in \bar{\mathcal{E}}$ for t_i and not $t_i' \neq t_i$. Formally, we require the following:

Axiom 2 *(No irrelevant types).* Let i and $t_i \in T_i$ be fixed. For any $x_i, y_i, x_i', y_i' \in \bar{\mathcal{E}}$, where $x_i(t_i, t_{-i}) = y_i(t_i, t_{-i})$ and $x_i'(t_i, t_{-i}) = y_i'(t_i, t_{-i})$ for all $t_{-i} \in T_{-i}$,[5] we have:

$$x_i \succcurlyeq_i^{t_i} x_i' \iff y_i \succcurlyeq_i^{t_i} y_i'. \tag{2}$$

Axiom 2 means that the values of $x_i(t_i', t_{-i})$ and $x_i'(t_i', t_{-i})$ for $t_i' \neq t_i$ are not relevant for the $\succcurlyeq_i^{t_i}$ preference.

We extend $\succcurlyeq_i^{t_i}$ to the set of bundles as follows. If $b, c \in \mathcal{B}$, we say that $b \succcurlyeq_i^{t_i} c$ if $x_i \succcurlyeq_i^{t_i} y_i$, where $x_i(t) = b$ and $y_i(t) = c$ for all $t \in T$. With this notation, the following is the usual monotonicity axiom for preferences over uncertain outcomes and it is found in most decision theoretic papers; see for instance Gilboa and Schmeidler (1989) and Cerreia-Vioglio et al. (2011) among others.

Axiom 3 *(Monotonicity).* $\succcurlyeq_i^{t_i}$ is monotonic, that is, if $x_i(t_i, t_{-i}) \succcurlyeq_i^{t_i} y_i(t_i, t_{-i})$ for all $t_{-i} \in T_{-i}$ then $x_i \succcurlyeq_i^{t_i} y_i$.

We want to rule out the possibility that a commodity is undesirable, that is, we want to impose that a bundle with more of each commodity is better. This is formalized by the following:

Axiom 4 *(Monotonicity w.r.t. Euclidean order).* $\succcurlyeq_i^{t_i}$ is monotonic with respect to the Euclidean order, that is, $x_i(t_i, t_{-i}) \geqslant y_i(t_i, t_{-i})$ for all $t_{-i} \in T_{-i}$ implies $x_i \succcurlyeq_i^{t_i} y_i$, with strict preference ($\succ_i^{t_i}$) if $x_i(t_i, t_{-i}) >> y_i(t_i, t_{-i})$ for all $t_{-i} \in T_{-i}$.

It should be noted that Axioms 3 and 4 are independent. It is easy to see that Axiom 3 does not imply 4, since Axiom 3 allows the possibility that all commodities are "bads" and 0 is a satiation point, that is, we could have a decreasing rather than an increasing preference, as Axiom 4 requires. However, the fact that Axiom 4 may hold while Axiom 3 is violated is less immediate. The following example exhibits a preference that satisfies Axiom 4 but not Axiom 3.

[4] The restriction of the preferences to $\bar{\mathcal{E}}$ is further justified because the "market property" used in Theorem 3.1 below to characterize the preferences as maximin—namely, that every interim efficient allocation is incentive compatible—has no bearing on preferences over allocations that come from endowments not satisfying Assumption 2.1. In other words, if the preference were defined over $\bar{\mathcal{A}}$ and not $\bar{\mathcal{E}}$, our "market property" would have no implications about allocations $x_i \in \bar{\mathcal{A}} \setminus \bar{\mathcal{E}}$, which can never be aggregate endowments under Assumption 2.1. Individuals might have arbitrary preferences over those allocations and still be maximin over $\bar{\mathcal{E}}$.

[5] Note that we allow $x_i(t_i', t_{-i}) \neq y_i(t_i', t_{-i})$ and $x_i'(t_i', t_{-i}) \neq y_i'(t_i', t_{-i})$ if $t_i' \neq t_i$.

Example 2.4. Let $n = 2$, $L = 2$, $T_1 = \{t_1\}$ and $T_2 = \{a, b\}$. Let $\succcurlyeq_1^{t_1}$ be represented by the following utility function: $U(x_1) = x_{11}(t_1, a)x_{12}(t_1, b)$, where $x_{1j}(t)$ is the consumption of good $j = 1, 2$. It is clear that this preference satisfies Axiom 4. Let $x_1 = ((x_{11}(t_1, a), x_{12}(t_1, a)), (x_{11}(t_1, b), x_{12}(t_1, b)) = ((3, 1), (1, 4))$ and $y_1 = ((y_{11}(t_1, a), y_{12}(t_1, a)), (y_{11}(t_1, b), y_{12}(t_1, b)) = ((2, 3), (2, 3))$. Then, $U(x_1) = 3 \cdot 4 = 12 > 6 = 2 \cdot 3 = U(y_1)$, that is, $x_1 \succ_1^{t_1} y_1$. However, $U(x_1(t_1, a)) = 3 \cdot 1 < 2 \cdot 3 = U(y_1(t_1, a))$ and $U(x_1(t_1, b)) = 1 \cdot 4 < 2 \cdot 3 = U(y_1(t_1, b))$. This shows that Axiom 3 is violated.

Next, we assume the standard continuity axiom:

Axiom 5 *(Continuity).* For all $f, g, h \in \bar{\mathcal{E}}$, the sets $\{\alpha \in [0, 1] : \alpha f + (1 - \alpha)g \succcurlyeq_i^{t_i} h\}$ and $\{\alpha \in [0, 1] : h \succcurlyeq_i^{t_i} \alpha f + (1 - \alpha)g\}$ are closed.

The above properties allow us to conclude the following:

Lemma 2.5. *Let Axioms 1–5 hold. Fix i. Then, there exists a continuous function $U_i : T_i \times \bar{\mathcal{E}} \to \mathbb{R}_+$ that represents $\succcurlyeq_i^{t_i}$, that is, for all $x_i, y_i \in \bar{\mathcal{E}}$,*

$$x_i \succcurlyeq_i^{t_i} y_i \iff U_i(t_i, x_i) \geqslant U_i(t_i, y_i), \tag{3}$$

for every $t_i \in T_i$. Moreover, U_i satisfies the following properties:

1. *If x_i and y_i are such that $x_i(t_i, t_{-i}) = y_i(t_i, t_{-i})$ for all $t_{-i} \in T_{-i}$, then $U_i(t_i, x_i) = U_i(t_i, y_i)$.*
2. *If we define $u_i : T_i \times \mathcal{B} \to \mathbb{R}_+$ by*

$$u_i(t_i, b) \equiv U_i(t_i, \tilde{x}_i) \tag{4}$$

where $\tilde{x}_i(t) \equiv b$ for all $t \in T$, we have the following:

$$(u_i(t_i, x_i(t_i, t_{-i})) \geqslant u_i(t_i, y_i(t_i, t_{-i})), \forall t_{-i} \in T_{-i}) \implies x_i \succcurlyeq_i^{t_i} y_i.$$

3. *If x_i and y_i are such that $x_i(t_i, t_{-i}) \geqslant y_i(t_i, t_{-i})$ for all $t_{-i} \in T_{-i}$, then $U_i(t_i, x) \geqslant U_i(t_i, y)$. The inequality is strict if for all $t_{-i} \in T_{-i}$, $x_i(t_i, t_{-i}) >> y_i(t_i, t_{-i})$.*

We conclude this section by formalizing the standard notion of efficiency in our setting.

Definition 2.6 *(Efficiency).* Given endowment $e \in \mathcal{E}$, we say that a feasible allocation $x \in \mathcal{A}(e)$ is (interim) efficient if there is no feasible allocation $y \in \mathcal{A}(e)$ such that $y_i \succcurlyeq_i^{t_i} x_i$ for every i and $t_i \in T_i$, with strict preference for some i and t_i.

2.3. Contracts, reports and incentive compatibility

A *contract* is a function $f = (f_1, ..., f_n) \in \mathcal{C}^n$, such that $\bar{f} = \sum_{i=1}^n f_i = 0$. Contracts will play the following role in our model. Given initial endowment $e \in \mathcal{E}$, if individuals want to consume the feasible allocation $x \in \mathcal{A}(e)$, they need to sign a contract $f^{x,e}$, defined by

$$f^{x,e} \equiv x - e. \tag{5}$$

This contract will specify the transfers that need to take place among the individuals in order to implement x. The condition $\bar{f} = 0$ in the definition of contracts guarantees that no good is destroyed or created. Observe that if individual i with endowment e_i receives transfer $f_i^{x,e}(t)$ when types are t, his consumption will be:

$$e_i(t) + f_i^{x,e}(t) = x_i(t),$$

which is exactly the amount specified in allocation x for individual i.

Note that the execution of a contract requires information: while individual i knows his endowment $e_i(t)$, which depends only on his private information t_i by Assumption 2.1, he may not know what should be the transfer $f_i(t)$ that he is supposed to receive, since this depends on the private information of all individuals (t). Thus, to implement the contract f, it is necessary to pool their private information. Therefore, we will assume that individuals report their types in order that contracts may be executed. By doing so, it is possible that individuals misreport their private information. Thus, we consider that each agent j follows a reporting strategy $s_j : T_j \to T_j$ that specifies, for each t_j a type $s_j(t_j) \in T_j$.[6] The *truthful* reporting strategies $s_j^* : T_j \to T_j$, defined by $s^*(t_j) \equiv t_j$, for each $j = 1, ..., n$, will be central to our analysis.

Thus, if agents $j \neq i$ follow reporting strategies $s_j : T_j \to T_j$, so that $s_{-i}(t_{-i}) = (s_j(t_j))_{j\neq i}$ and agent i reports $r_i \in T_i$ when his type is t_i, agent i will end up consuming

$$f_i(r_i, s_{-i}(t_{-i})) + e_i(t_i, t_{-i})$$

under contract f. Therefore, given a profile of strategies $s : T \to T$, $s = (s_1, ..., s_n)$ and contract f, the final allocation to be consumed by agents will be $x^{e,f,s}$ defined by:

$$x^{e,f,s}(t) \equiv f(s(t)) + e(t), \forall t \in T.$$

Note that by changing his strategy from $s_i : T_i \to T_i$, to $\tilde{s}_i : T_i \to T_i$, individual i can change his consumption from $x_i^{e,f,(s_i,s_{-i})}$ to $x_i^{e,f,(\tilde{s}_i,s_{-i})}$. In particular, if all players $j \neq i$ are using truthful strategies $s_j^* : T_j \to T_j$, by deviating to strategy $s_i : T_i \to T_i$ individual i consumes, when types are t,

$$x^{e,f,(s_i,s_{-i}^*)}(t) = f(s_i(t_i), t_{-i}) + e_i(t_i, t_{-i}). \tag{6}$$

This motivates the standard definition of incentive compatibility:

Definition 2.7. Given an endowment $e \in \mathcal{E}$, an allocation $x = (x_1, ..., x_n) \in \mathcal{A}(e)$ is incentive compatible (IC) if for all i, $t_i \in T_i$, and strategies $s_i : T_i \to T_i$,

$$x_i = x_i^{e,f,s^*} \succcurlyeq_i^{t_i} x_i^{e,f,(s_i,s_{-i}^*)}, \tag{7}$$

where $f = f^{x,e} = x - e$ is the corresponding contract for x and e.

2.4. Maximin preferences

For our purposes, we will need to focus on Wald's maximin preference—which inspired (and is a particular case of) Gilboa–Schmeidler's MEU:

$$x_i \succcurlyeq_i^{t_i} y_i \iff \min_{t_{-i}\in T_{-i}} u_i(t_i, x_i(t_i, t_{-i})) \geqslant \min_{t_{-i}\in T_{-i}} u_i(t_i, y_i(t_i, t_{-i})), \tag{8}$$

[6] We avoid considering mixed strategies, because we are most interested in truthful reports and this extra generality would complicate notation without any extra insight.

where $u_i : T_i \times \mathcal{B} \to \mathbb{R}$ is a continuous utility function, increasing in $\mathcal{B}$. If we define

$$U_i(t_i, x) \equiv \min_{t_{-i} \in T_{-i}} u_i(t_i, x_i(t_i, t_{-i})), \tag{9}$$

then U_i and u_i satisfy (3) and (4), which establishes the consistency between the notation adopted here with the notation introduced in Lemma 2.5 for preferences satisfying Axioms 1–5. Incidentally, it is not difficult to see that maximin preferences satisfy Axioms 1–5.

Notice that this preference is an instance of the Maximin Expected Utility (MEU) preferences defined by Gilboa and Schmeidler (1989). To see this, let Δ_i denote the set of measures π on T_{-i}. Then, the preference defined by (8) is equivalently defined by:

$$\begin{aligned} x_i \succcurlyeq_i^{t_i} y_i \iff & \min_{\pi \in \Delta_i} \int_{T_{-i}} u_i(t_i, x_i(t_i, t'_{-i}))\, \pi(dt'_{-i}|t_i) \\ & \geqslant \min_{\pi \in \Delta_i} \int_{T_{-i}} u_i(t_i, y_i(t_i, t'_{-i}))\, \pi(dt'_{-i}|t_i), \end{aligned}$$

which is easily seen to be a particular case of Gilboa and Schmeidler's MEU.[7]

3. Main result

The central theme of this paper is the interplay of uncertainty, efficiency and incentive compatibility. To motivate our result, let us first discuss the conflict between efficiency and incentive compatibility that was noted in 70's.

Consider the following particular example. There are two individuals, with type sets $T_1 = \{U, D\}$ and $T_2 = \{L, R\}$. Their utilities are $u_i(t, b) = b \in \mathbb{R}_+$, $\forall i \in I, t \in T$ and their endowments are constant and equal to 1: $e_i(t) = 1$, $\forall i \in I, t \in T$. Assume moreover that the individuals are Bayesian with uniform common prior, that is, $\pi(\{(t_1, t_2)\}) = \frac{1}{4}$ for any $(t_1, t_2) \in T = T_1 \times T_2$.

It is easy to see that the following allocation is (interim) efficient:

x_1	L	R
U	2	1
D	1	0

and

x_2	L	R
U	0	1
D	1	2

.

Observe that such allocation induces the trade of one unit from individual 2 to individual 1 in the state (U, L), one unit from individual 1 to individual 2 in the state (D, R) and no transfers in the other states. Although this allocation is Pareto optimal, it is not incentive compatible. To see that, observe that if individual 1's type is D, he can say that it is U and avoiding making a transfer if individual 2's type is R, while receiving a unit if individual 2's type is L. With this lie, individual 1 is strictly better at all states.

Given this conflict, one could ask: does the conflict between efficiency and incentive compatibility still hold for other kind of preferences? More clearly, given an economy composed of individuals with other preferences, is it true that some efficient allocations are not incentive compatible? In principle, this question would be repeat for each set of preferences. This is *not*

[7] In Gilboa and Schmeidler (1989)'s MEU, the set Δ_i can be substituted in the expression above by any compact and convex set $\mathcal{P}_i \subset \Delta_i$.

what we do in this paper. Instead, we ask all these questions at once: given an economy with any kind of "reasonable" preferences, what (if any) is the set of preferences for which all efficient allocations are incentive compatible? Note that this question embraces three of the most important concepts in economics—namely, uncertainty, efficiency and incentive compatibility—putting the question mark exactly on the concept that is still more debated in economics: the attitudes towards uncertainty displayed by individuals, captured by their preferences.

Our main result answers this question.

Theorem 3.1. *Let Assumption 2.1 hold. Let $I = \{1, ..., n\}$ be a set of individuals with (interim) preferences $\succcurlyeq_i^{t_i}$ over $\bar{\mathcal{E}}$. The statements (i) and (ii) below are equivalent:*

(i) *For all $i \in I$ and $t_i \in T_i$, $\succcurlyeq_i^{t_i}$ satisfies Axioms 1–5. Moreover, every interim efficient allocation is incentive compatible.*
(ii) *All individuals have maximin preferences.*

This theorem could be considered a non-standard characterization of the maximin preference. It is a non-standard characterization because the typical paper in the decision theoretic literature would impose axioms referring only to individual behavior. While our Axioms 1–5 are of this kind, the property "every interim efficient allocation is incentive compatible" is meaningful only in a multiple individuals setting, as we consider here.

As an illustration of this theorem, the reader can go back to the example at the beginning of this section. The allocation discussed there is not efficient under maximin preferences, although it was for Bayesian preferences. The no trade allocation where the individuals keep their endowments is both efficient and incentive compatible under maximin preferences. Section 4 offers more illustrations of this theorem. We can give an intuition for why this theorem is true as follows.

Heuristic proof of Theorem 3.1. First, consider the implication that if all preferences are maximin, then every efficient allocation is incentive compatible ($(ii) \Rightarrow (i)$). This implication is proven in the appendix (see Proposition A.1). Note that an individual with maximin preference does not care if he gets something above the worst case scenario in that allocation, that is, he is indifferent between receiving only the worst outcome and receiving something better in some state. Now, if an allocation $x = (x_i)_{i \in I}$ is such that individual j with type t_j can gain something by lying about her type (saying that her type is $t_j' \neq t_j$), this means that x_i is specifying for individual i at the state t_j more than he would get at state t_j'. Indeed, the extra benefit that j gets by lying should come from someone; that someone is our i here. But since i has maximin preferences, i is perfectly happy to get only what is specified under t_j'. This implies that we can find another allocation y, similar to x, in which nobody is worse and j is strictly better. Therefore, we prove that if all individuals have maximin preferences and an allocation is not incentive compatible then it cannot be efficient. The proof of the first implication is just a formalization of this argument.

Now, the implication $(i) \Rightarrow (ii)$ is a little bit more complicated. In this discussion, we restrict to the case $n = 2$, just one good and linear u_i. We first observe that if there is an individual that has not maximin preferences, say individual 1, then there is an allocation x_1 such that

$$x_1 \succ_1^{t_1} m_{x_1}(t_1, t_2) \equiv \min_{t_2 \in T_2} x_1(t_1, t_2).$$

Indeed, if $x_1 \sim_i^{t_i} m_{x_1}$ for all x_1, the preference would be maximin. The key idea is to use x_1 to define an allocation that is efficient but not incentive compatible. Since $x_1 \succ_i^{t_i} m_{x_1}$, this allocation is such that there is a type t_2 of player 2 under which 2 receives more than his worst-case scenario outcome. This is the key feature to establish that the defined allocation is efficient. Next, since under some types of individual 2, 1 is receiving more, this means that 2 could lie and get for herself this extra benefits that 1 is getting. Of course, at this level of generality it is not completely clear that 2 could benefit in this way; the formalization in the actual proof is exactly to show that 2 indeed can be strictly better off by lying. Therefore, we have created an allocation that is efficient but not incentive compatible, thus contradicting statement (i) in the Theorem.

4. A mechanism design perspective

It is natural to ask what is the relevance of the above results from a mechanism design perspective. This section clarifies this issue. We begin by translating the usual mechanism design setting into our framework. The set of individuals and their information is exactly as we described before and there is a mechanism designer who wants to implement an efficient allocation. Instead of initial endowments, the mechanism design literature uses to consider only initial levels of utility, to inform whether it is individually rational or not to participate in the mechanism. Of course, this is made only for simplicity and in many cases, endowments could be explicitly defined. In the sequel, we consider separately the two cases.

4.1. Case with explicit initial endowments

Suppose that the individuals have initial endowments $e = (e_i)_{i \in I}$. The mechanism designer wants to find a mechanism that implements a feasible allocation $x = (x_i)_{i \in I}$. Here, we are concerned with efficient allocations. A mechanism is incentive compatible if no individual has an interest of misreporting his information (see Definition 2.7). A mechanism is *budget balanced* if it can be implemented for any report by the agents, without the need of extra goods. The following result is a corollary to the implication $(ii) \Rightarrow (i)$ in Theorem 3.1.

Corollary 4.1. *Suppose that $x = (x_i)_{i \in I}$ is an (interim) efficient feasible allocation and each agent $i \in I$ has a maximin preference as defined in subsection 2.4. Then, there exists a mechanism that implements x and is incentive compatible and budget balanced.*

4.2. Public outcomes

In some models, as in d'Aspremont and Gérard-Varet (1979), the individuals may care about the whole allocation, that is, the set of bundles is $\mathcal{B} = O \times \mathbb{R}$, where $O \subset \mathbb{R}^{\ell}_{+}$ is the finite set of *possible physical outcomes*, as in the set of all possible public projects. The last component of $\mathcal{B}$, namely $\mathbb{R}$, refers to monetary transfers among the n individuals. The (ex post) utility is given by:

$$u_i(t_i, (a, \tau_i)) = v_i(t_i, a) + \tau_i(t), \tag{10}$$

where $\tau_i(t) \in \mathbb{R}$ is the transfer individual i receives and $a \in O \subset \mathbb{R}^{\ell}_{+}$ is the outcome. Because of the possibility of transfers, in this setting it is natural and common to consider *outcome efficiency* instead of the standard efficiency.[8]

[8] This practice is further justified by Lemma 4.3 below.

Definition 4.2. We say that $a^* : T \to O$ is outcome efficient if for all $t \in T$,

$$\sum_{i \in I} v_i(t_i, a^*(t)) = \max_{o \in O} \sum_{i \in I} v_i(t_i, o).$$

Note that the setting above is slightly more general than in the rest of the paper (at least in one aspect), since the consumers may care not only about their own consumption (a_i) but above the entire a. Indeed, in this setup we do not need even to refer to individual consumptions. In other words, it is possible to consider externalities in this setup. The following simple result establishes the connection between outcome efficiency and Pareto efficiency. Since we are not considering endowments here, we need to substitute the feasibility constraint by a condition of the type $\sum_{i \in I} \tau_i(t) = c$, for some $c \in \mathbb{R}$. Although the actual c is not important, we will focus on the case of budget balance, that is, $c = 0$.

Lemma 4.3. *$a^* : T \to O$ is outcome efficient if and only if there exists $\tau = (\tau_i)_{i \in I}$ such that (a^*, τ) is ex post efficient and $\sum_{i \in I} \tau_i(t) = 0$. Moreover, if a^* is outcome efficient, then there exists $\tau = (\tau_i)_{i \in I}$ such that (a^*, τ) is interim efficient with $\sum_{i \in I} \tau_i(t) = 0$.*

In a sense, this result shows that when we have monetary transfers, we just need to worry about outcome efficiency, instead of interim efficiency. This justifies the focus on this definition of efficiency, usually considered in the mechanism design literature.

Let us introduce some terminology. A mechanism $m = (d, \tau)$ consists of a decision rule $d : T \to O$ and transfers $\tau : T \to \mathbb{R}^n$. A mechanism m is budget balanced if $\sum_{i \in I} \tau(t) = 0$ for all $t \in T$ and it is incentive compatible if there is no individual i and types t_i, t_i' such that

$$\min_{t_{-i} \in T_{-i}} \left[v_i \left(t_i, d(t_i', t_{-i}) \right) + \tau(t_i', t_{-i}) \right] > \min_{t_{-i} \in T_{-i}} \left[v_i \left(t_i, d(t_i, t_{-i}) \right) + \tau(t_i, t_{-i}) \right].$$

Finally, we say that d is incentive compatible if there exist transfers $\tau : T \to \mathbb{R}^n$ such that (d, τ) is incentive compatible.

Theorem 4.4. *Assume that individuals have maximin preferences. If the decision rule $d : T \to O$ is outcome efficient, then it is incentive compatible.*

Remark 4.5. As it is well known, Vickrey–Clark–Groves mechanisms can implement efficient *outcomes* for general preferences, such as expected utility preferences. However, such mechanisms cannot implement all interim efficient allocations. The difference, of course, is that interim efficient allocations must specify the monetary transfers. Efficient outcomes together with some transfers will lead to allocations that are not incentive compatible. Thus, the implication $(i) \Rightarrow (ii)$ in our main result is not valid if instead of all interim efficient allocations, we restrict to only outcome efficient allocations. The relevant issue is that monetary transfers matter for the preferences, as they should. If we ignore them, the preferences cannot be pinned down as we did. On the other hand, if we know that the preferences are maximin, the above result establishes that efficiency of outcomes implies incentive compatibility.

4.3. Myerson–Satterthwaite setup

A seller values the object as $v \in [0, 1]$ and a buyer values it as $t \in [0, 1]$. Both values are private information. An allocation will be efficient in this case if trade happens if and only if

$t \geqslant v$. Under the Bayesian paradigm, that is, the assumption that both seller and buyer are expected utility maximizers (EUM), Myerson and Satterthwaite (1983) have proved that there is no incentive compatible, individual rational mechanism (without subsidies) that would achieve ex post efficiency in this situation.

Consider now the following simple mechanism: the seller places an ask a and the buyer, a bid b. If the bid is above the ask, they trade at $p = \frac{a+b}{2}$; if it is below, there is no trade. Therefore, if they negotiated at price p, the (ex post) profit for the seller will be $p - v$, and for the buyer, $t - p$. If they do not negotiate, both get zero. By Myerson and Satterthwaite (1983)'s result mentioned above, if the individuals are EUM, this mechanism does not always lead to efficient allocations. The problem is that this mechanism would be efficient if and only if both seller and buyer report truthfully, that is, $a = v$ and $b = t$, but these choices are not incentive compatible if the individuals are EUM. Now, we will show that $a = v$ and $b = t$ are incentive compatible choices if both seller and buyer have maximin preferences.

Recall that $a = v$ and $b = t$ are incentive compatible if buyer and seller do not have any incentive to choose a different action. If the buyer chooses $b = t$, the worst-case scenario is to end up with zero (either by buying by $p = t$ or by not trading). Can she do better than this? If she chooses $b > t$, the worst-case scenario is to buy by $p > t$, which leads to a (strict) loss. If she considers $b < t$, the worst-case scenario is to get zero (it always possible that there is no trade). Therefore, neither $b < t$ nor $b > t$ is better (by the maximin criterion) than $b = t$ and she has no incentive to deviate. The argument for the seller is analogous.[9]

5. How do Bayesian and maximin efficiency compare?

From the fact that all maximin efficient allocations are incentive compatible, the reader may wonder whether maximin efficiency is an excessively strong requirement, which could explain (one direction of) our results. Indeed, if there are very few efficient allocations under maximin preferences, then the result that all of them are incentive compatible would be less compelling. In this section, we address this issue in particular cases. For one-good economies, we show that whenever an allocation is Bayesian efficient and incentive compatible, then it is also maximin efficient. That is, the set of maximin efficient allocations is at least as large as the set of Bayesian efficient and incentive compatible allocations. For economies with numéraire (or transferable utility), studied in section 4.2, we show that maximin efficiency is equivalent to Bayesian efficiency.

However, the formal statement of these results require a clarification of the relationship between the maximin and the Bayesian preferences. For this, we consider two economies:

- a maximin economy, exactly as described and studied up to now, which will be denoted (in this section only) by $\mathbf{E}^M$; and
- a Bayesian economy, where the agents have Bayesian preferences $\succeq_i^{t_i}$, defined by the same utility functions $u_i : T \times \mathcal{B} \to \mathbb{R}$ of the maximin economy, and posteriors $\pi_i(\cdot|t_i)$ on T_{-i}, that is,

[9] The reader may be concerned with the multiplicity of equilibria in this example. Indeed, to choose $b < t$ could also be an equilibrium. However, the multiplicity of equilibria is also possible in the standard Bayesian framework and is also a concern there, specially in issues related to implementation. Since this issue is not restricted to our framework and a large part of the mechanism design literature does not discuss it, we will follow the standard practice and leave further discussions to future work.

$$x_i \succeq_i^{t_i} y_i \iff \int\limits_{T_{-i}} u_i(t_i, x_i(t_i, t_{-i}))\, \pi_i(dt_{-i}|t_i) \geqslant \int\limits_{T_{-i}} u_i(t_i, y_i(t_i, t_{-i}))\, \pi_i(dt_{-i}|t_i).$$

This Bayesian economy will be denoted by $\mathbf{E}^B$.

5.1. One-good economies

Our result for one-good economies is the following:

Theorem 5.1. *Consider a one-good economy* ($L = 1$) *and* $n = 2$. *If* x *is a Bayesian efficient allocation which is also incentive compatible in* $\mathbf{E}^B$, *then it is also maximin efficient and incentive compatible in* $\mathbf{E}^M$. *The reverse is not true, that is, there are efficient allocations in* $\mathbf{E}^M$ *(hence incentive compatible by the previous results) that are not efficient in* $\mathbf{E}^B$.

This result shows that the maximin preferences do not destroy efficient and incentive compatible outcomes. To the contrary, any incentive compatible outcome that is efficient under a Bayesian preference will be also efficient under the corresponding maximin preference.

5.2. Economies with numéraire

Now, consider an economy as described in section 4.2, that is, the set of bundles is $\mathcal{B} = O \times \mathbb{R}$, where $O \subset \mathbb{R}_+^\ell$ indicates the set of *possible physical outcomes*. The last part of $\mathcal{B}$ ($\mathbb{R}$) refers to monetary transfers among the n individuals. The (ex post) utility is given by: $u_i(t, (a, \tau_i)) = v_i(t_i, a) + \tau_i(t)$, where $\tau_i \in \mathbb{R}$ and $a \in O \subset \mathbb{R}_+^\ell$. The following result shows that efficiency for agents with maximin preferences coincides with efficiency for agents with Bayesian preferences.

Proposition 5.2. *In the economy with transfers described above, an allocation is Bayesian efficient if and only if it is also maximin efficient.*

Despite the fact that efficiency agrees for the two kind of preferences, incentive compatibility does not. That is, some efficient allocation will be incentive compatible under maximin preferences but will not be incentive compatible under Bayesian preferences.

6. Discussion of the related literature

6.1. General equilibrium with asymmetric information

It is well known that in a finite economy with asymmetric information once people exhibit standard expected utility, then it is not possible in general to find allocations which are Pareto optimal and also incentive compatible; see for an example the appendix. The key issue is the fact that, in a finite economy each agent's private information has an impact and therefore an agent will take advantage of this private informational effect to influence the equilibrium allocation to favor herself. This is what creates the incentive compatibility problem. To get around this problem, Yannelis (1991) imposes the private information measurability condition, that is, he requires that the allocations considered are always measurable with respect to the private information of the agents. In this case, any allocation that is private information measurable and ex ante Pareto optimal allocation is incentive compatible. See Koutsougeras and Yannelis (1993), Krasa and Yannelis (1994), Hahn and Yannelis (1997) and Podczeck and Yannelis (2008) for an extensive discussion of the private information measurability of allocations. In fact, the private information measurability is not only sufficient for proving that ex ante efficient allocations are incentive compatible, but it is also necessary in the one-good case.

It is useful to try to understand why measurability was used to solve the problem of the conflict between efficiency and incentive compatibility. If an agent trades a non-measurable contract, this means that the contract makes promises depending on conditions that she cannot verify. Therefore, other agents may have an incentive to cheat her and do not deliver the correct amount in those states. This possibility is exactly the failure of incentive compatibility. To the contrary, if she insists to trade only measurable contracts (allocations), then she cannot be cheated and incentive compatibility is preserved.

However, the requirement of private information measurability raises two main concerns. First, it is an exogenous, theoretical requirement, which may be difficult to justify in real economies. The second concern, which is more relevant, is that the private information measurability restriction may lead to reduced efficiency and in certain cases even to no-trade. Thus, on the one hand, the private information measurability restriction implies incentive compatibility, but on the other hand, it reduces efficiency. To the contrary, the maximin expected utility allows for trade and results in a Pareto efficient outcome which is also incentive compatible.

Different solutions to the conflict between efficiency and incentive compatibility for the standard (Bayesian) expected utility for replica economies have been proposed by McLean and Postlewaite (2002). Those authors impose an "informational smallness" condition and show the existence of incentive compatible and efficient allocations in an approximate sense for a replica economy. The informational smallness can be viewed as an approximation of the idea of perfect competition and as a consequence only approximate results can be obtained in this replica economy framework. Sun and Yannelis (2007) and Sun and Yannelis (2008) formulate the idea of perfect competition in an asymmetric information economy with a continuum of agents. In this case each individual's private information has negligible influence and as a consequence of the negligibility of the private information, they are able to show that any ex ante Pareto optimal allocation is incentive compatible. The above results are obtained in the set up of standard (Bayesian) expected utilities and they are only approximately true in large but finite economies.

Subsequently to the completion of this paper, de Castro et al. (2011) revisited the Kreps (1977)'s example of the non-existence of the rational expectation equilibrium. They showed that there is nothing wrong with the rational expectation equilibrium notion other than the assumption that agents are expected utility maximizers. Using the maximin preferences studied here, de Castro et al. (2011) recomputed the Kreps' example and showed that the rational expectation equilibrium not only exists, but it is also unique, efficient and incentive compatible. Similarly, Angelopoulos and Koutsougeras (2014) study value allocation under ambiguity. Also subsequently to the writing of this paper, de Castro et al. (2017a), Liu (2016) and de Castro et al. (2017b) adopted the maximin preference analyzed in this paper and showed that any maximin efficient and individually rational allocation (e.g. solution concepts like the rational expectations equilibrium, core and value) are implementable as truth telling maximin equilibrium.

Another related paper is Morris (1994). He departures from the Milgrom and Stokey (1982) no-trade theorem, which requires the common prior assumption, and shows that the incentive compatibility requirement allows for obtaining equivalent no-trade theorems under assumptions weaker than the common prior assumption. In this context, no trade theorems may be interpreted as a loss of efficiency created by the constraint of incentive compatibility.

In a series of papers, Correia-da Silva and Hervés-Beloso (2008, 2009, 2012, 2014) introduced economies with uncertain delivery, where agents negotiate contracts that are not measurable with respect to their information. Although they considered MEU preferences, their focus was different as they did not consider the incentive compatibility studied here.

6.2. Decision theory

The maximin criterion has a long history. It was proposed by Wald (1950) and Rawls (1971), and axiomatized by Milnor (1954), Maskin (1979), Barbera and Jackson (1988), Nehring (2000) and Segal and Sobel (2002). Binmore (2008, Chapter 9) presented an interesting discussion of the principle, making the connection of the large worlds of Savage (1972). Gilboa and Schmeidler (1989) generalized at the same time the maximin criterion (see footnote 7) and Bayesian preferences by allowing for multiple priors. Bewley (2002) introduced a model of decision under incomplete information.

De Castro and Yannelis (2013) offer an interpretation of the Ellsberg's paradox in terms of incompleteness of preferences, which comes from the lack of measurability of certain acts (the ambiguous acts) with respect to a partition that represents the decision maker's information. This approach is somewhat related to Gilboa et al. (2010), who considered decision makers who have two preferences. One of these preferences is incomplete and corresponds to the part of her preference that she can justify for third persons. They call this preference objective and model it as a Bewley incomplete preference. The other preference corresponds to a subjective preference, where the decision maker cannot be proven wrong and this is modeled as a maximin expected utility preference.

Rigotti et al. (2008), de Castro and Chateauneuf (2011), characterized conditions for ex ante efficiency for convex preferences (the first) and MEU preferences (the second). Kajii and Ui (2009) and Martins-da Rocha (2010) characterized interim efficiency for MEU and Bewley preferences, but do not mention incentive compatibility issues.

Mukerji (1998) used a model with ambiguity to analyze the problem of investment holdup and incomplete contracts in a model with *moral hazard*. Interestingly, he obtained results that go in the opposite direction than those obtained here: in the moral hazard model that he considered, ambiguity makes harder to obtain incentive compatibility, not easier as we proved for our general equilibrium with asymmetric information model.[10] The connection between ambiguity and information has been addressed before by Mukerji (1997) and Ghirardato (2001). With respect to efficiency and incentive compatibility, Haller and Mousavi (2007) presented evidence that ambiguity improves the second-best in a simple Rothschild and Stiglitz (1976)'s insurance model.

The analysis of games with ambiguity averse players has also a limited literature. Klibanoff (1996) considered games where players have MEU preferences. Salo and Weber (1995), Lo (1998) and Ozdenoren (2000, Chapter 4) analyzed auctions where players have ambiguity aversion. Subsequently to our paper, Bose et al. (2006) and Bodoh-Creed (2012) studied optimal auction mechanisms when individuals have MEU preferences, while Lopomo et al. (2009) investigated mechanisms for individuals with Bewley's preferences. Bose and Renou (2014) and

[10] We are grateful to Sujoy Mukerji for bringing this paper to our attention.

Di Tillio et al. (2017) studied mechanism design with ambiguity. However, none of these papers have uncovered the property of no conflict between efficiency and incentive compatibility for the maximin preferences considered here.

7. Concluding remarks and open questions

We showed that maximin preferences present no conflict between incentive compatibility and efficiency. We also showed that the maximin preferences are not only sufficient for any efficient allocation to be incentive compatible but they are also necessary. Additionally, this paper provides an axiomatization of the maximin preferences. Applications of our results to mechanism design were given. Finally, we applied our results to the Myerson–Satterthwaite's setup and showed that their negative result does not hold in our framework. We close now by discussing some open questions and directions of future research.

This paper shows that in the case of maximin preferences, the set of efficient allocations is contained in the set of incentive compatible ones. It is of interest to know if these two sets relate for other uncertainty averse preferences (as defined by Cerreia-Vioglio et al. (2011)). In other words, fixing a profile of uncertainty averse preferences, we would like to know how close the sets of efficient and incentive compatible allocations are. Or yet: how close are the set of second-best outcomes (that is, outcomes that are efficient subject to being incentive compatible) and first-best (just efficient) outcomes?

It would also be interesting to know if our results hold for a continuum of agents. Finally, it would be interesting to study an evolutionary model of populations of agents with different preferences. Will a society formed only by maximin agents outperform societies formed by individuals with diverse preferences? What happens if some mutations lead to Bayesian subjects inside this maximin society?

In sum, we hope this paper stimulates new venues of investigation.

Appendix A

Proof of Lemma 2.3. For necessity, suppose that $x_i = \sum_{j=1}^n e_j$, with $e_j \in \mathcal{E}_j$. Then,

$$\begin{aligned}
x_i(t'_j, t_{-j}) - x_i(t_j, t_{-j}) &= \sum_{k=1}^{n} \left[e_k(t'_j, t_{-j}) - e_k(t_j, t_{-j}) \right] \\
&= e_j(t'_j, t_{-j}) - e_j(t_j, t_{-j}) \\
&= e_j(t'_j, t'_{-j}) - e_j(t_j, t'_{-j}) \\
&= \sum_{k=1}^{n} \left[e_k(t'_j, t'_{-j}) - e_k(t_j, t'_{-j}) \right] \\
&= x_i(t'_j, t'_{-j}) - x_i(t_j, t'_{-j}),
\end{aligned}$$

where the first and the last line come from $x_i = \sum_{j=1}^n e_j$ and the second, third and fourth come from the fact that e_k depends on t_j if and only if $k = j$.

For sufficiency, assume that x_i satisfies (1). Our objective is to define $\{e_j\}_{j\in I}$ such that $e_j \in \mathcal{E}_j$ and $x_i = \sum_{j=1}^n e_j$. For this, fix $t^0 = (t^0_j, t^0_{-j}) \in T$. For any $k = 1, ..., n$, define

$$e_k(t^0) \equiv \frac{1}{n} x_i(t^0),$$

which implies $\sum_{k=1}^n e_k(t^0) = x_i(t^0)$. For any $j \in I$ and $t = (t_j, t_{-j}) \in T \setminus \{t^0\}$, define:

$$e_j(t_j, t_{-j}) \equiv e_j(t_j^0, t_{-j}^0) + x_i(t_j, t_{-j}^0) - x_i(t_j^0, t_{-j}^0). \tag{11}$$

Thus, $e_j(t)$ is well defined for all t and j and, by the definition, $e_j(t_j, t_{-j}) = e_j(t_j, t'_{-j})$ for all $t_{-j}, t'_{-j} \in T_{-j}$, that is, $e_j \in \mathcal{E}_j$. It remains to verify that

$$\sum_{k=1}^n e_k(t) = x_i(t) \tag{12}$$

for any $t = (t_j, t_{-j}) \in T \setminus \{t^0\}$. We will do this in two steps. First, fix $j \in I$ and $t_j \in T_j$. Then,

$$\begin{aligned}\sum_{k=1}^n e_k(t_j, t_{-j}^0) &= e_j(t_j, t_{-j}^0) + \sum_{k \neq j} e_k(t_j, t_{-j}^0) \\ &= e_j(t_j^0, t_{-j}^0) + x_i(t_j, t_{-j}^0) - x_i(t_j^0, t_{-j}^0) + \sum_{k \neq j} e_k(t^0) \\ &= x_i(t_j, t_{-j}^0) - x_i(t^0) + \sum_{k=1}^n e_k(t^0) \\ &= x_i(t_j, t_{-j}^0),\end{aligned}$$

that is, (12) holds for all $t \in T \setminus \{t^0\}$ of the form $t = (t_j, t_{-j}^0)$ for some $j \in I$. Using this fact, we are now able to extend the result for every $t \in T \setminus \{t^0\}$:

$$\begin{aligned}\sum_{j=1}^n e_j(t) &= \sum_{j=1}^n e_j(t_j^0, t_{-j}) + n x_i(t) - \sum_{j=1}^n x_i(t_j^0, t_{-j}) \\ &= n x_i(t) + \sum_{j=1}^n \left[e_j(t_j^0, t_{-j}) - x_i(t_j^0, t_{-j}) \right] \\ &= n x_i(t) + \sum_{j=1}^n \left[-\sum_{k \neq j} e_k(t_j^0, t_{-j}) \right] \\ &= n x_i(t) - \sum_{j=1}^n \left[\sum_{k \neq j} e_k(t_j, t_{-j}) \right] \\ &= n x_i(t) - (n-1) \sum_{k=1}^n e_k(t) \\ &= x_i(t),\end{aligned}$$

as we wanted to show.

To see that $\bar{\mathcal{E}}$ is convex, let $x_1 = \sum_{i=1}^n e_{1i}$ and $x_2 = \sum_{i=1}^n e_{2i}$, with $e_{ki} \in \mathcal{E}_i$ for $i = 1, ..., n$ and $k = 1, 2$. Then, for $\alpha \in (0, 1)$, we have $\alpha x_1 + (1-\alpha) x_2 = \sum_{i=1}^n [\alpha e_{1i} + (1-\alpha) e_{2i}]$ and clearly $\alpha e_{1i} + (1-\alpha) e_{2i} \in \mathcal{E}_i$.

Finally, let $x_i = \sum_{j=1}^n e_{ij}$, with $e_{ij} \in \mathcal{E}_j$, for $i = 1, ..., n$. Then, $\bar{x} = \sum_{i=1}^n \sum_{j=1}^n e_{ij} = \sum_{j=1}^n \left(\sum_{i=1}^n e_{ij} \right)$ and, for every $j = 1, ..., n$, $e_{ij} \in \mathcal{E}_j \Rightarrow \sum_{i=1}^n e_{ij} \in \mathcal{E}_j$. Thus, $\bar{x} \in \bar{\mathcal{E}}$. □

Proof of Lemma 2.5. The proof is based on elementary and familiar ideas, but we present it here for completeness. Let us fix i and t_i. Let $\bar{u}$ denote the unitary vector $(1, ..., 1) \in \mathcal{B} = \mathbb{R}^L_+$. Let $\upsilon : T \to \mathbb{R}^L_+$ denote the constant function equal to $\bar{u} = (1, ..., 1) \in \mathbb{R}^L_+ = \mathcal{B}$. Let $x \in \bar{\mathcal{E}}$ be given. (In this proof, we will write only x instead of x_i, as usual, for simplicity. No confusion should arise.) We first claim that there exists $\lambda_x \in \mathbb{R}_+$ such that $x \sim_i^{t_i} \lambda_x \upsilon$.

Indeed, from monotonicity, we know that for a sufficiently high $\bar{\lambda} > 0$, $\bar{\lambda}\upsilon \succcurlyeq_i^{t_i} x$. On other hand, again by monotonicity, $x \succcurlyeq_i^{t_i} 0$. Thus, the sets $\{\lambda \in \mathbb{R}_+ : \lambda\upsilon \succcurlyeq_i^{t_i} x\}$ and $\{\lambda \in \mathbb{R}_+ : x \succcurlyeq_i^{t_i} \lambda\upsilon\}$ are both nonempty. Moreover, by completeness, their union is $\mathbb{R}_+$. We claim that both are closed.

Let $A = \{\alpha \in [0, 1] : \alpha\bar{\lambda}\upsilon + (1-\alpha)0 \succcurlyeq_i^{t_i} x\}$ and $B = \{\alpha \in [0, 1] : x \succcurlyeq_i^{t_i} \alpha\bar{\lambda}\upsilon + (1-\alpha)0\}$. Then, $1 \in A$, $0 \in B$. By continuity both A and B are closed. If we denote by αX the set of all αx for some $x \in X$, then, $\{\lambda \in \mathbb{R}_+ : \lambda\upsilon \succcurlyeq_i^{t_i} x\} = \bar{\lambda}A \cup [\bar{\lambda}, +\infty)$ and $\{\lambda \in \mathbb{R}_+ : x \succcurlyeq_i^{t_i} \lambda\upsilon\} = \bar{\lambda}B$. Therefore, both sets are closed.

Since $\mathbb{R}_+$ is connected and equal to the union of these two nonempty closed sets, there must exist λ_x belonging to both sets. But this implies $\lambda_x\upsilon \sim_i^{t_i} x$. Define $U_i(t_i, x) \equiv \lambda_x$.

Let us show that U_i represents $\succcurlyeq_i^{t_i}$. For any $x, y \in \bar{\mathcal{E}}$, we have $\lambda_x\upsilon \sim_i^{t_i} x$ and $y \sim_i^{t_i} \lambda_y\upsilon$. Thus, by monotonicity,

$$x \succcurlyeq_i^{t_i} y \iff \lambda_x\upsilon \succcurlyeq_i^{t_i} \lambda_y\upsilon \iff \lambda_x \geqslant \lambda_y.$$

This shows that $U_i(t_i, \cdot)$ represents $\succcurlyeq_i^{t_i}$.

Let us show that U_i is continuous. Since T_i is finite, we may focus only on the continuity with respect to $x \in \bar{\mathcal{E}}$. Recall that $\bar{\mathcal{E}}$ is finite dimensional, so that its topology is unambiguous. Let $x^n \to x$. For a contradiction, assume that there exists $\epsilon > 0$ and infinitely many n such that $|\lambda_{x_n} - \lambda_x| > \epsilon$. Then, at least one of following two conditions hold: (i) there exist infinitely many n for which $\lambda_{x_n} > \lambda_x + \epsilon$; or (ii) there exist infinitely many n for which $\lambda_{x_n} < \lambda_x - \epsilon$. Assume that (i) holds.

By passing to a subsequence, we may assume $\lambda_{x_n} > \lambda_x + \epsilon$ for all n. Since the set $\left\{\alpha \in [0, 1] : \alpha\upsilon + x = \alpha(\upsilon + x) + (1-\alpha)x \succcurlyeq_i^{t_i} (\lambda_x + \epsilon)\,\upsilon\right\}$ does not contain zero and is closed, there exists $\alpha > 0$ such that $(\lambda_x + \epsilon)\,\upsilon \succ_i^{t_i} \alpha\upsilon + x$. Since $\alpha\upsilon + x >> x$, there exists n_α such that for all $n \geqslant n_\alpha$, $\alpha\upsilon + x \geqslant x_n$, which implies, by monotonicity, $\alpha\upsilon + x \succcurlyeq_i^{t_i} x_n$. Thus, if $n \geqslant n_\alpha$ we have $(\lambda_x + \epsilon)\,\upsilon \succcurlyeq_i^{t_i} \alpha\upsilon + x \succcurlyeq_i^{t_i} x_n \sim_i^{t_i} \lambda_{x_n}\upsilon$, that is, $\lambda_x + \epsilon > \lambda_{x_n}$, which contradicts $\lambda_{x_n} > \lambda_x + \epsilon$. The case (ii) can be dealt with in an analogous way. This shows continuity.

The other claims in the Lemma follow respectively from Axiom 2 (no irrelevant types), Axiom 3 (monotonicity) and Axiom 4 (monotonicity with respect to the Euclidean order). □

A.1. Proof of $(ii) \Rightarrow (i)$ in Theorem 3.1

The implication $(ii) \Rightarrow (i)$ in Theorem 3.1 comes from the following[11]:

Proposition A.1. *If the preferences of the individuals are maximin and $x = (x_i)_{i \in I}$ is an interim efficient allocation, then x is incentive compatible.*

[11] As observed before, it is immediate that maximin preferences satisfy Axioms 1–5.

Proof. Suppose that x is not incentive compatible. This means that there exists an individual i, types $t_i', t_i'', t_i' \neq t_i''$ and strategy $s_i : T_i \to T_i$ such that $s_i(t_i') = t_i''$ and $x_i^{e,f,(s_i,s_{-i}^*)} \succ_i^{t_i} x_i^{e,f,s^*} = x_i$. From (6), for individual i with type t_i', we have

$$\begin{aligned} x_i^{e,f,(s_i,s_{-i}^*)}(t_i', t_{-i}) &= f(t_i'', t_{-i}) + e_i(t_i', t_{-i}) \\ &= x_j\left(t_i'', t_{-i}\right) - e_j\left(t_i'', t_{-i}\right) + e_j\left(t_i', t_{-i}\right). \end{aligned}$$

Since the preference is maximin, $x_i^{e,f,(s_i,s_{-i}^*)} \succ_i^{t_i} x_i^{e,f,s^*} = x_i$ means:

$$\min_{t_{-i}\in T_{-i}} u_i\left(t_i', x_j\left(t_i'', t_{-i}\right) - e_j\left(t_i'', t_{-i}\right) + e_j\left(t_i', t_{-i}\right)\right) > \min_{t_{-i}\in T_{-i}} u_i\left(t_i', x_i\left(t_i', t_{-i}\right)\right). \tag{13}$$

We will prove that x cannot be maximin Pareto optimal by constructing another feasible allocation $y = (y_i)_{i\in I}$ that Pareto improves upon x. For this, define

$$y_j\left(t_i, t_{-i}\right) = \begin{cases} x_j\left(t_i, t_{-i}\right), & \text{if } t_i \neq t_i' \\ e_j\left(t_i', t_{-i}\right) + x_j\left(t_i'', t_{-i}\right) - e_j\left(t_i'', t_{-i}\right), & \text{if } t_i = t_i' \end{cases} \tag{14}$$

To see that $\left(y_j\right)_{j\in I}$ is feasible, it is sufficient to consider what happens when $t_i = t_i'$:

$$\begin{aligned} \sum_{j\in I} y_j\left(t_i', t_{-i}\right) &= \sum_{j\in I} e_j\left(t_i', t_{-i}\right) + \sum_{j\in I} x_j\left(t_i'', t_{-i}\right) - \sum_{j\in I} e_j\left(t_i'', t_{-i}\right) \\ &= \sum_{j\in I} e_j\left(t_i', t_{-i}\right), \end{aligned}$$

because $\sum_{j\in I} x_j\left(t_i'', t_{-i}\right) = \sum_{j\in I} e_j\left(t_i'', t_{-i}\right)$, from the feasibility of x_j at (t_i'', t_{-i}).

From (13) and (14), we have $y_i \succ_i^{t_i'} x_i$ and $y_i \sim_i^{t_i} x_i$ for any $t_i \neq t_i'$. It remains to prove that $y_j \succsim_j^{t_j} x_j$ for any $j \neq i$ and $t_j \in T_j$.

For each $t_j \in T_j$, define $X_j(t_j)$ as the set $\{x_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}$ and $Y_j(t_j) \equiv \{y_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}$. Fix a $t = (t_i, t_j, t_{-i-j}) \in T$. If $t_i \neq t_i'$, the definition (14) of y_j implies that $y_j(t) = x_j(t) \in X_j(t_j)$. If $t_i = t_i'$, (14) gives $y_j\left(t_i', t_{-i}\right) = x_j\left(t_i'', t_{-i}\right) \in X_j(t_j)$ since $e_j(t_i', t_{-i}) = e_j(t_i'', t_{-i})$ for $j \neq i$ by Assumption 2.1. Thus, $Y_j(t_j) \subset X_j(t_j)$, for all $t_j \in T_j$. Therefore,

$$\min_{w\in Y_j(t_j)} u_j(t_j, w) \geqslant \min_{w\in X_j(t_j)} u_j(t_j, w). \tag{15}$$

This shows that $y_j \succsim_j^{t_j} x_j$ for all $j \neq i$ and $t_j \in T_j$. Thus, y is a Pareto improvement upon x, that is, x is not maximin efficient. $\square$

The reader can observe that the only place where we used the specific definition of the interim preference as the minimum was to conclude (15). Indeed if we were to use other preferences (in particular the expected utility preferences), this step would not go through.

A.2. *Proof of* $(i) \Rightarrow (ii)$ *in Theorem 3.1*

Before giving the formal proof of the implication $(i) \Rightarrow (ii)$ in Theorem 3.1, we will give an overview of it. We want to show that whenever an individual (say 1) has a preference that is not maximin, there exists an allocation $x = (x_1, ..., x_n)$ that is efficient but not incentive compatible, that is, there will be an individual $j \neq 1$ that has an incentive to misreport his type.

For this, we proceed in steps. First, we characterize the failure of individual 1 of having maximin preferences by the existence of a bundle x_1 and type t_1 such that

$$x_1 \succ_1^{t_1} m_{x_1} \equiv \min_{t_{-1}\in T_{-1}} x_1(t_1, t_{-1}).$$

Of course, the minimization on the right is not well defined if $L > 1$. Thus, we have to reduce the dimension of the problem from L to 1. This allows us to consider minimal values of bundles as above, which will simplify our arguments. This is accomplished in subsection A.2.1.

Next, we obtain from this bundle x_1 another one with a desirable property. Namely, for any other bundle $y_1 \neq x_1$ such that $x_1 \geqslant y_1$, we will have $x_1 \succ_1^{t_1} y_1$. If x_1 has this property, we say it is minimal. This property is useful to establish that the allocation $(x_1, 0)$, where all other agents other than 1 receive zero is efficient. This is accomplished in subsection A.2.2.

Another property is needed: the fact that a single individual $j \neq 1$ has a type t_j'' satisfying

$$x_1(t_1, t_j'', t_{-1-j}') > x_1(t_1, t_{-1}') = \min_{t_{-1}\in T_{-1}} x_1(t_1, t_{-1}) = m_{x_1}(t_1, t_{-1}).$$

This is established in subsection A.2.3. This individual j will be the one who has a profitable deviation: when he is of type t_j'', he will be strictly better off by misreporting t_j'. In this subsection, we also prove efficiency of the defined allocation $(x_1, 0)$. We can then put together these preliminary steps to conclude the proof of sufficiency of Theorem 3.1 in subsection A.2.4.

A.2.1. Characterization of non-maximin preferences and reduction of dimension

Assume that for all $i = 1, ..., n$ and $t_i \in T_i$, the preference $\succcurlyeq_i^{t_i}$ satisfies Axioms 1–5. Then, by Lemma 2.5, there exists U_i representing $\succcurlyeq_i^{t_i}$. Moreover, by the proof of Lemma 2.5, we may assume that

$$x_j(t_j, t_{-j}) \sim_i^{t_i} u_j(t_j, x_j(t_j, t_{-j}))\bar{u}, \tag{16}$$

where $\bar{u}$ denotes the unitary vector $(1, ..., 1) \in \mathcal{B} = \mathbb{R}_+^L$ and u_j is defined from U_j by (4). Now, for each $x_j : T \to \mathcal{B}$, define $m_{x_j} : T \to \mathcal{B}$ by

$$m_{x_j}(t_j, t_{-j}') \equiv \left[\min_{t_{-j}\in T_{-j}} u_j(t_j, x_j(t_j, t_{-j}))\right]\bar{u}. \tag{17}$$

Note that $m_{x_j}(t_j, t_{-j}')$ depends only on t_j and x_j, but not on t_{-j}'. We have the following:

Lemma A.2. $\succcurlyeq_i^{t_i}$ *is maximin over* $\bar{\mathcal{E}}$ *if and only if* $x_i \sim_i^{t_i} m_{x_i}$ *for all* $x_i \in \bar{\mathcal{E}}$.

Proof. *Sufficiency.* Assume that $x_i \sim_i^{t_i} m_{x_i}$ and $y_i \sim_i^{t_i} m_{y_i}$, for all x_i, y_i. We have the following series of equivalences, which are easy applications of the definitions and monotonicity.

$$\begin{aligned} x_i \succcurlyeq_i^{t_i} y_i &\iff m_{x_i} \succcurlyeq_i^{t_i} m_{y_i} \\ &\iff \left[\min_{t_{-j}\in T_{-j}} u_j(t_j, x_j(t_j, t_{-j}))\right]\bar{u} \succcurlyeq_i^{t_i} \left[\min_{t_{-j}\in T_{-j}} u_j(t_j, y_j(t_j, t_{-j}))\right]\bar{u} \\ &\iff \min_{t_{-i}\in T_{-i}} u_i\left(t_i, x_i(t_i, t_{-i})\right) \geqslant \min_{t_{-i}\in T_{-i}} u_i\left(t_i, y_i(t_i, t_{-i})\right). \end{aligned}$$

That is, (8) holds and the preference is maximin.

Necessity. Assume that (8) holds. Thus, from

$$\min_{t_{-i}\in T_{-i}} u_i(t_i, m_{x_i}(t_i, t_{-i})) = \min_{t_{-i}\in T_{-i}} u_i(t_i, x_i(t_i, t_{-i}))$$

we conclude that $x_i \sim_i^{t_i} m_{x_i}$. □

For each $x_i : T \to \mathcal{B}$, define $\hat{x}_i : T \to \{\lambda\bar{u} \in \mathcal{B} : \lambda \in \mathbb{R}_+\}$ by:

$$\hat{x}_i(t) \equiv u_i(t_i, x_i(t_i, t_{-j}))\bar{u}. \tag{18}$$

Then, by Property 3 of Lemma 2.5, we have $\hat{x}_i \sim_i^{t_i} x_i$. Moreover, $u_i(t_i, \hat{x}_i(t_i, t_{-j})) = u_i(t_i, x_i(t_i, t_{-j}))$ for all $t_{-i} \in T_{-i}$. Thus, by monotonicity and the fact that $\hat{x}_i \geqslant m_{x_i}$, we have $x_i \sim_i^{t_i} \hat{x}_i \succcurlyeq_i^{t_i} m_{x_i}$. Therefore, if $\neg(x_i \sim_i^{t_i} m_{x_i})$, we may conclude $x_i \succ_i^{t_i} m_{x_i}$. From this result, we obtain the following:

Corollary A.3. *$\succcurlyeq_i^{t_i}$ is not maximin if and only if there exists $x_i \in \bar{\mathcal{E}}$ such that $x_i \succ_i^{t_i} m_{x_i}$ and $x_i(t) = u_i(t_i, x_i(t))\bar{u}$ for all $t_{-i} \in T_{-i}$, where $\bar{u} = (1, ..., 1) \in \mathbb{R}_+^L$.*

Proof. Sufficiency is immediate from Lemma A.2 above. Let us prove necessity. Assume that $\succcurlyeq_i^{t_i}$ is not maximin. From Lemma A.2, we know that there exists x_i such that $\neg(x_i \sim_i^{t_i} m_{x_i})$. As observed above, this implies that $x_i \succ_i^{t_i} m_{x_i}$. Using the definition (18), we know that $m_{x_i} = m_{\hat{x}_i}$ and $x_i \sim_i^{t_i} \hat{x}_i$. Thus, $\hat{x}_i$ satisfies the conditions of the corollary, since $u_i(t_i, \hat{x}_i(t)) = u_i(t_i, x_i(t))$. □

Assume that 1's preference $\succcurlyeq_1^{t_1}$ is not maximin. Then, there is a $x_1 \in \bar{\mathcal{E}}$ with the properties described in Corollary A.3 above. Note that since $x_i(t) = u_i(t_i, x_i(t))\bar{u}$, we can focus on only one dimension for x_i instead of complicating the notation to deal with the case $L > 1$. Thus, we will assume from now on that $L = 1$. This is without loss of generality since the general case can be treated through the multiplication by $\bar{u} = (1, ..., 1) \in \mathbb{R}_+^L$ as shown above. Besides simplicity of notation, this allows us to write

$$m_{x_1}(t_1, t_{-1}) = \min_{t_{-1}\in T_{-1}} x_1(t_1, t_{-1}). \tag{19}$$

A.2.2. Minimal allocation

Our next step is to obtain additional properties for the $x_1 \succ_1^{t_1} m_{x_1}$ that we will use in the rest of the proof.

Definition A.4 *(Minimal allocation).* An individual allocation $x_1 \in \bar{\mathcal{E}}$ is said to be minimal in t_1 if $x_1 \succ_1^{t_1} m_{x_1}$ and for all $y_1 \in \bar{\mathcal{E}}$,

$$\left(x_1 \geqslant y_1 \geqslant m_{x_1}, y_1 \neq x_1\right) \Longrightarrow x_1 \succ_1^{t_1} y_1. \tag{20}$$

The following result is useful in the proof of sufficiency in Theorem 3.1, but might be of interest in its own.

Proposition A.5. *If $\succcurlyeq_1^{t_1}$ is not maximin, there exists $x_1 \in \bar{\mathcal{E}}$ minimal in t_1.*

Proof. Let $x_1 \in \bar{\mathcal{E}}$ be the allocation given by Corollary A.3 and write $x_1 = \sum_{k=1} e_k$. Since $\succcurlyeq_1^{t_1}$ does not depend on the values of $x_1(t_1', t_{-1})$ for $t_1' \neq t_1$, we may just assume that $x_1(t_1', t_{-1}) = x_1(t_1, t_{-1})$ for all $t_1' \in T_1$. Thus, e_1 should be constant. Since e_k depends only on t_k, we will abuse of notation and write $e_k(t_k)$ instead of $e_k(t_k, t_{-k})$.

Let $t_{-1}^0 = (t_2^0, ..., t_n^0)$ be such that $x_1(t_1, t_{-1}^0) = \min_{t_{-1} \in T_{-1}} x_1(t_1, t_{-1})$. Then, it is also true that for each k, $e_k(t_k^0) = \min_{t_{-1} \in T_{-1}} e_k(t_k)$. Indeed, if for some k, $e_k(t_k^0) > \min_{t_{-1} \in T_{-1}} e_k(t_k) = e_k(t_k')$, then $x_1(t_1, t_k', t_{-1-k}^0) = e_1(t_1) + e_k(t_k') + \sum_{j \neq 1,k} e_j(t_j^0) < e_1(t_1) + \sum_{j \neq 1} e_j(t_j^0) = x_1(t_1, t_{-1}^0)$, contradicting the definition of t_{-1}^0. Note that $m_{x_1}(t) = x_1(t_1, t_{-1}^0)$ for all $t \in T$.

Now, we number the remaining elements of T_i. That is, for each $i = 2, ..., n$, let $T_i = \{t_i^0, t_i^1, ..., t_i^{m_i}\}$ and let $m_1 \equiv 1$. Define $x^{1,1} \equiv x_1$ and $e_k^{1,1} \equiv e_k$. We will define $x^{i,r}$ and $e_k^{i,r}$ recursively as follows, where $i, k \in I = \{1, ..., n\}$ will denote agents and, for each $i \in I$, $r \in \{1, ..., m_i\}$ denotes one of the possible types of i. It will be clear that $e_k^{i,r}$ will be used to guarantee that $x^{i,r} \in \bar{\mathcal{E}}$. Assume that $x^{i,r-1}$ and $e_k^{i,r-1}$ are defined for all $k = 2, ..., n$, with

$$x^{i,r-1} = \sum_{k=1}^{n} e_k^{i,r-1} \text{ and } x^{i,r-1} \sim x_1. \tag{21}$$

If $r \leqslant m_i$, and for $\alpha \in [0, 1]$, define

$$e_\alpha^{i,r}(t_i) \equiv \begin{cases} e_i^{i,r-1}(t_i), & \text{if } t_i \neq t_i^r \\ \alpha e_i^{i,r-1}(t_i^r) + (1-\alpha) e_i(t_i^0), & \text{if } t_i = t_i^r \end{cases}$$

and

$$x_\alpha^{i,r}(t) \equiv \sum_{k \neq i} e_k^{i,r-1}(t_k) + e_\alpha^{i,r}(t_i).$$

It is clear from this definition that $x_\alpha^{i,r} \in \bar{\mathcal{E}}$ and $x_\alpha^{i,r} \leqslant x^{i,r-1}$ for all $\alpha \in [0, 1]$. We stop when $i = n$ and $r = m_i$. If $i < n$, $r - 1 = m_i$ and $\alpha \in [0, 1]$, define

$$e_\alpha^{i+1,1}(t_{i+1}) \equiv \begin{cases} e_{i+1}^{i,m_i}(t_{i+1}), & \text{if } t_{i+1} \neq t_{i+1}^1 \\ \alpha e_{i+1}^{i,m_i}(t_{i+1}^r) + (1-\alpha) e_{i+1}(t_{i+1}^0), & \text{if } t_{i+1} = t_{i+1}^1 \end{cases}$$

and

$$x_\alpha^{i+1,1}(t) \equiv \sum_{k \neq i} e_k^{i,m_i}(t_k) + e_\alpha^{i+1,1}(t_i).$$

It is also clear that $x_\alpha^{i+1,1} \in \bar{\mathcal{E}}$ and $x_\alpha^{i+1,1} \leqslant x^{i,m_i}$.

Consider first the case $r \leqslant m_i$. As already observed, $x_\alpha^{i,r} \leqslant x^{i,r-1}$, with equality if $\alpha = 1$. By monotonicity, $x^{i,r-1} \succcurlyeq_1^{t_1} x_\alpha^{i,r}$. The set $\{\alpha \in [0, 1] : x_\alpha^{i,r} \succcurlyeq_1^{t_1} x_1\}$ is closed by continuity, contains 1 (since $x_1^{i,r} = x^{i,r-1} \sim x_1$ by (21)) and, by monotonicity, it is an interval (perhaps degenerated). Let α^* be the infimum of its points. Since the set is a closed interval, α^* belongs to it. Therefore, $x_{\alpha^*}^{i,r} \sim_1^{t_1} x^{i,r-1}$ and

$$\alpha < \alpha^* \implies x^{i,r-1} \succ_1^{t_1} x_\alpha^{i,r}. \tag{22}$$

Define $x^{i,r} \equiv x_{\alpha^*}^{i,r} \in \bar{\mathcal{E}}$. Then $x^{i,r} \sim_1^{t_1} x^{i,r-1} \sim_1^{t_1} \cdots \sim_1^{t_1} x^{1,1} = x_1 \succ_1^{t_1} m_{x_1}$ and $x^{i,r} \leqslant x^{i,r-1} \leqslant \cdots \leqslant x^{1,1} = x_1$. Notice also that $x^{i,r}(t) = x^{i,r-1}(t)$ for all $t = (t_i, t_{-i})$ such that $t_i \neq t_i^r$.

In the case that $r-1=m_i$, it would be sufficient to substitute i,r by $i+1,1$ in the last paragraph (and t_i^r by t_{i+1}^1 in the last phrase) to obtain the same properties.

In this way, we define $x^{i,r}$ for all $i=2,...,n$ and $r=1,...,m_i$. We claim that $\tilde{x}_1 \equiv x^{n,m_n}$ is minimal in t_1. First, it is clear that $\tilde{x}_1 = x^{n,m_n} \sim_1^{t_1} x_1 \succ_1^{t_1} m_{x_1} = m_{\tilde{x}_1}$. To show that (20) holds, assume otherwise. That is, assume that there exist $y_1 \in \bar{\mathcal{E}}$ such that $\tilde{x}_1 \geqslant y_1 \geqslant m_{\tilde{x}_1}$, $y_1 \neq \tilde{x}_1$ and $y_1 \sim_1^{t_1} \tilde{x}_1 \sim_1^{t_1} x_1$. This means that there exists $t \in T$ such that $y_1(t) < \tilde{x}_1(t)$. Since $\tilde{x}_1 \leqslant x^{i,r}$ for all i,r, then there exists t,i,r such that $y_1(t) < x^{i,r}(t)$. Let i be the smallest number in $\{2,...,n\}$ for which there is some $t \in T$ and some r such that $y_1(t) < x^{i,r}(t)$. Given this i, let r be the smallest number in $\{1,...,m_i\}$ for which there is some t such that $y_1(t) < x^{i,r}(t)$. Then, by the definition of i and r, $y_1(t) = x^{i,r-1}(t)$ for all t if $r>1$ and $y_1(t) = x^{i-1,m_{i-1}}(t)$ for all t if $r=1$. Let us consider the case $r>1$, the other being analogous. Since $x^{i,r}(t) = x^{i,r-1}(t)$ for all $t=(t_i,t_{-i})$ if $t_i \neq t_i^r$, we conclude that $x^{i,r}(t_1,t_{-i}^0) \leqslant y_1(t_i^r,t_{-i}) < x^{i,r}(t_i^r,t_{-i}) = x_{\alpha^*}^{i,r}(t_i^r,t_{-i})$. Therefore, there is a $\alpha \in [0,1]$ such that $y_1(t_i^r,t_{-i}) = x_{\alpha}^{i,r}(t_i^r,t_{-i})$ and $\alpha < \alpha^*$. By (22), we conclude that $x^{i,r-1} \succ_1^{t_1} y_1$, which contradicts $y_1 \sim_1^{t_1} x_1 \sim_1^{t_1} x^{i,r}$. This contradiction concludes the proof. $\square$

A.2.3. Deviation and efficiency

Lemma A.6. *Let $x_1 \in \bar{\mathcal{E}}$ be the individual allocation given by Proposition A.5. Then we can find $t' = (t_1, t'_{-1})$, $j \neq 1$ and $t''_j \neq t'_j$ such that*

$$x_1(t_1, t''_j, t'_{-1-j}) > x_1(t_1, t'_{-1}) = \min_{t_{-1} \in T_{-1}} x_1(t_1, t_{-1}) = m_{x_1}(t_1, t_{-1}). \tag{23}$$

Proof. Let t'_{-1} be such

$$x_1(t_1, t'_{-1}) = \min_{t_{-1} \in T_{-1}} x_1(t_1, t_{-1}) = m_{x_1}(t_1, t_{-1}).$$

Since $x_1 \succ_1^{t_1} m_{x_1}$, there exists $t'' = (t_1, t''_{-1})$ such that

$$x_1(t_1, t''_{-1}) > x_1(t_1, t'_{-1}). \tag{24}$$

Let $x_1 = \sum_{k=1} e_k$. From (24), there exists $j \in I$ such that $e_j(t''_j, t''_{-j}) > e_j(t'_j, t'_{-j})$, where $t'_{-j} = (t_1, t'_{-1-j})$ and $t''_{-j} = (t_1, t''_{-1-j})$. Thus,

$$x_1(t_1, t''_j, t'_{-1-j}) = \sum_{k=1}^{n} e_k(t_1, t''_j, t'_{-1-j}) > \sum_{k=1}^{n} e_k(t_1, t'_j, t'_{-1-j}) = x_1(t_1, t'_{-1}),$$

as we wanted. $\square$

We need another result for the proof of Theorem 3.1.

Definition A.7. An allocation $x = (x_j)_{j \in I}$ is a i-corner allocation if $x_j(t) = 0$ for all $t \in T$ and all $j \neq i$.

Lemma A.8. *Assume that individual 1's preference at t'_1 is not maximin, but it satisfies Axioms 1–5. Let $\bar{x}_1$ be the minimal allocation obtained by Proposition A.5. Then, the 1-corner allocation $(\bar{x}_1, 0)$ is efficient.*

Proof. For a contradiction, assume that $(x_1, x_{-1}) = (\bar{x}_1, 0)$ is not efficient. Then, there exist another feasible allocation (y_j), $i \in I$ and $t_i' \in T_i$ such that: (i) for all j and t_j, $y_j \succcurlyeq_{t_j} x_j$; and (ii) $y_i \succ_i^{t_i'} x_i$. Since $\sum_j y_j = \sum_j x_j = x_1 = \bar{x}_1$, then $x_1 \geqslant y_1$, which implies that $i \neq 1$. Since $y_i \succ_i^{t_i'} x_i = 0$, then $x_1 \geqslant y_1$ and $x_1 \neq y_1$. Since $x_1 = \bar{x}_1$ and $\bar{x}_1$ is minimal in t_1', we have $x_1 \succ_1^{t_1'} y_1$, which contradicts (i). $\square$

A.2.4. Proof of sufficiency in Theorem 3.1

Now we are able to complete the proof of Theorem 3.1.

Proof of Theorem 3.1. Let $x = (x_1, x_{-1}) = (x_1, 0)$, where $x_1 \in \bar{\mathcal{E}}$ is the individual allocation obtained in Proposition A.5. Since $x_1 \in \bar{\mathcal{E}}$, there exists $e = (e_1, e_{-1}) \in \mathcal{E}$ be such that $\sum_{i=1}^n e_i(t) = x_1(t)$. Consider the corresponding contract $f(t) = x(t) - e(t)$, that is,

$$f_1(t_1, t_{-1}) = \sum_{j=2}^n e_j(t_1, t_{-1}); \text{ and for } j \neq 1,\ f_j(t_1, t_{-1}) = -e_j(t_1, t_{-1}). \tag{25}$$

Let $t' = (t_1, t_{-1}')$, $j \neq 1$ and t_j'' be those given by Lemma A.6, that is, they satisfy (23). Now, assume that if individual j is of type t_j'', he reports t_j' instead. That is, assume that j follows strategy $s_j : T_j \to T_j$ such that $s_j(t_j'') = t_j'$, while all other individuals follow the truthful strategies s_i^*. In this case, when individual j is of type t_j'', he will consume:

$$\begin{aligned}
x_j^{e,f,(s_j,s_{-j}^*)}(t_1, t_j'', t_{-1-j}) &= e_j(t_1, t_j'', t_{-1-j}) + f_j(t_1, t_j', t_{-1-j}) \\
&= e_j(t_1, t_j'', t_{-1-j}) - e_j(t_1, t_j', t_{-1-j}) \\
&= \sum_{i=1}^n e_i(t_1, t_j'', t_{-1-j}) - \sum_{i=1}^n e_i(t_1, t_j', t_{-1-j}) \\
&= x_1(t_1, t_j'', t_{-1-j}) - x_1(t_1, t_j', t_{-1-j}) > 0,
\end{aligned}$$

where the first equality comes from (6), the second equality comes from (25), the third equality comes from the fact that for every $i \neq j$, $e_i(t_1, t_j'', t_{-1-j}) = e_i(t_1, t_j', t_{-1-j})$, the last equality comes from the fact that $x_1 = \sum_{k=1}^n e_k$ and the final inequality comes from (23) and (1). Notice that the above is constant with respect to (t_{-1}, t_{-1-j}) because $e_j(t_1, t_j'', t_{-1-j}) - e_j(t_1, t_j', t_{-1-j})$ cannot vary with (t_1, t_{-1-j}). That is, $x_j^{e,f,(s_j,s_{-j}^*)}(t_j'', t_{-j})$ is strictly positive for all t_{-j}. By monotonicity,

$$x_j^{e,f,(s_j,s_{-j}^*)} \succ_j^{t_j''} 0 = x_j^{e,f,s^*}.$$

Thus, $x = (x_1, 0)$ is not incentive compatible, but it is efficient by Lemma A.8. $\square$

A.3. Other proofs

Proof of Corollary 4.1. Let us define a simple mechanism that implements the efficient allocation x. The space of messages for individual i is just the set of types T_i. The mechanism simply implements the transfers that are supposed to occur at the reported types. That is, if agents report the profile of types $t' = (t_1', \dots, t_n')$, while their true types are $t = (t_1, \dots, t_n)$ then individual i's final allocation will be $e_i(t) + x_i(t') - e_i(t')$, since $x_i(t') - e_i(t')$ is transfer supposed to occur if the types are t'. Now the fact that this mechanism is budget balanced comes from the fact that x is feasible, that is, $\sum_{i\in I}\left(x_i(t') - e_i(t')\right) = 0$ for every $t' \in T$. This mechanism is incentive compatible by Proposition A.1. □

Proof of Lemma 4.3. Let (a^*, τ) be ex post efficient, with $\sum_{i\in I}\tau_i(t) = 0$ but $a^* : T \to O$ is not outcome efficient. Then there exists $o \in O$ and t such that

$$r \equiv \sum_{i\in I} v_i(t_i, o) = \max_{a\in O}\sum_{i\in I} v_i(t_i, a) > \sum_{i\in I} v_i(t_i, a^*(t)) \equiv s.$$

Then, for any τ, the allocation (a^*, τ) is dominated by (a', τ'), where $a'(t') = a^*(t')$ and $\tau'(t') = \tau(t')$ for all $t' \neq t$, $a'(t) = o$, and

$$\tau_i'(t) = v_i(t_i, a^*(t)) + \tau_i(t) - v_i(t_i, o) + \frac{r-s}{n}. \tag{26}$$

Indeed, (26) implies $v_i(t_i, o) + \tau_i'(t) = v_i(t_i, a^*) + \tau_i(t) + \frac{r-s}{n} > v_i(t_i, a^*) + \tau_i(t)$ and $\sum_{i\in I}\tau_i'(t) = \sum_{i\in I}\tau_i(t)$. This shows that (a^*, τ) is not ex post efficient, which is a contradiction.

Conversely, assume that a^* is outcome efficient. Let (a, τ) be an ex post efficient allocation with $\sum_{i\in I}\tau_i(t) = 0$. By the first part of the proof, $a : T \to O$ is outcome efficient, that is, $\sum_{i\in I} v_i(t_i, a(t)) = \sum_{i\in I} v_i(t_i, a^*(t))$. Define $\tau_i^*(t) \equiv v_i(t_i, a(t)) + \tau_i(t) - v_i(t_i, a^*)$. Then for each i and $t \in T$, (a^*, τ_i^*) is indifferent to (a, τ_i). Therefore, (a^*, τ^*) is ex post efficient and $\sum_{i\in I}\tau_i^* = 0$.

Finally, let a^* be outcome efficient. Let us assign a number to each $t \in T$ according to the increasing order of $s(t) \equiv \sum_{i\in I} v_i(t_i, a^*(t))$, that is, let $T = \{t^1, t^2, ..., t^K\}$ be such that $s(t^1) \leqslant s(t^2) \leqslant \cdots \leqslant s(t^K)$. For $k = 1, ..., K$, define $\tau_i^*(t^k) \equiv \frac{s(t^k)}{n} - v_i(t_i^k, a^*(t^k))$. It is clear that $\sum_{i\in I}\tau_i^*(t^k) = 0$, for all $k = 1, ..., K$. Moreover, for every $i \in I$,

$$v_i\left(t_i^1, a^*(t_i^1, t_{-i}^1)\right) + \tau_i^*(t_i^1, t_{-i}^1) = \frac{s(t^1)}{n} = \min_{t_{-i}\in T_{-i}}\left[v_i\left(t_i^1, a^*(t_i^1, t_{-i})\right) + \tau_i^*(t_i^1, t_{-i})\right]$$

and

$$\min_{t_{-i}\in T_{-i}}\left[v_i\left(t_i^k, a^*(t_i^k, t_{-i})\right) + \tau_i^*(t_i^k, t_{-i})\right] = \frac{s(t^\ell)}{n},$$

for some $\ell \leqslant k$, with $t_i^k = t_i^\ell$, since $v_i\,(t_i, a^*(t)) + \tau_i^*(t) = \frac{s(t)}{n}$ for all $t \in T = \{t^1, ..., t^K\}$. This implies that if k is the smallest element of $\{1, ..., K\}$ such that $t_i^k = t_i$, for some given $t_i \in T_i$, then

$$\min_{t_{-i}\in T_{-i}}\left[v_i\left(t_i^k, a^*(t_i^k, t_{-i})\right) + \tau_i^*(t_i^k, t_{-i})\right] = \frac{s(t^k)}{n}.$$

We claim that (a^*, τ^*) is interim efficient. For a contradiction, assume that there exists (a, τ) such that

$$\min_{t_{-i}\in T_{-i}}\left[v_i\,(t_i, a(t_i, t_{-i})) + \tau_i(t_i, t_{-i})\right] \geqslant \min_{t_{-i}\in T_{-i}}\left[v_i\left(t_i, a^*(t_i, t_{-i})\right) + \tau_i^*(t_i, t_{-i})\right],$$

for all $i \in I$ and $t_i \in T_i$, with strictly inequality for at least one $i \in I$ and $t_i \in T_i$. Let i be the smallest element of $I = \{1, ..., n\}$ for which the above inequality is strict for some $t_i \in T_i$. Let $T_i' \subset T_i$ be the set of types of i for which the inequality is strict. For each $t_i \in T_i'$, let k_{t_i} be the smallest $k \in \{1, ..., K\}$ for which $t_i^k = t_i$. Let t_i be the element of T_i' with the smallest k_{t_i}. Let $k_i = k_{t_i}$. Then,

$$\begin{aligned} v_i\left(t_i^{k_i}, a(t_i^{k_i}, t_{-i}^{k_i})\right) + \tau_i(t_i^{k_i}, t_{-i}^{k_i}) &\geqslant \min_{t_{-i}\in T_{-i}} \left[v_i\left(t_i^{k_i}, a(t_i^{k_i}, t_{-i})\right) + \tau_i(t_i^{k_i}, t_{-i})\right] \\ &> \min_{t_{-i}\in T_{-i}} \left[v_i\left(t_i^{k_i}, a^*(t_i^{k_i}, t_{-i})\right) + \tau_i^*(t_i^{k_i}, t_{-i})\right] \\ &= \frac{s(t^{k_i})}{n}. \end{aligned}$$

For each $j \neq i$, let k_j be the smallest $\ell \in \{1, ..., K\}$ such that $t_j^\ell = t_j^{k_i}$. Then, for all j,

$$\min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j^{k_j}, a^*(t_j^{k_j}, t_{-j})\right) + \tau_j^*(t_j^{k_j}, t_{-j})\right] = \frac{s(t_1^{k_1}, t_2^{k_2}, ..., t_n^{k_n})}{n}.$$

Let $\hat{t} = (t_1^{k_1}, t_2^{k_2}, ..., t_n^{k_n})$. Therefore, for all $j \in I$

$$\begin{aligned} v_j\left(t_j^{k_j}, a(\hat{t})\right) + \tau_j(\hat{t}) &\geqslant \min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j^{k_j}, a(t_j^{k_j}, t_{-j})\right) + \tau_j(t_j^{k_j}, t_{-j})\right] \\ &\geqslant \min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j^{k_j}, a^*(t_j^{k_j}, t_{-j})\right) + \tau_j^*(t_j^{k_j}, t_{-j})\right] \\ &= \frac{s(\hat{t})}{n}, \end{aligned}$$

where the second inequality is strict if $j = i$. Since a^* is outcome efficient,

$$\begin{aligned} s(\hat{t}) &= \sum_{j=1}^{n} v_j\left(t_j^{k_j}, a^*(\hat{t})\right) \\ &\geqslant \sum_{j=1}^{n} v_j\left(t_j^{k_j}, a(\hat{t})\right) \\ &= \sum_{j=1}^{n} \left[v_j\left(t_j^{k_j}, a(\hat{t})\right) + \tau_j(\hat{t})\right] \\ &\geqslant \sum_{j=1}^{n} \min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j^{k_j}, a(t_j^{k_j}, t_{-j})\right) + \tau_j(t_j^{k_j}, t_{-j})\right] \\ &> \sum_{j=1}^{n} \min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j^{k_j}, a^*(t_j^{k_j}, t_{-j})\right) + \tau_j^*(t_j^{k_j}, t_{-j})\right]. \\ &= s(\hat{t}), \end{aligned}$$

which is a contradiction. This establishes that (a^*, τ^*) is interim efficient. □

Proof of Theorem 4.4. Let $d : T \to O$ be outcome efficient. By Lemma 4.3, we can find $\tau : T \to \mathbb{R}^n$ such that (d, τ) is interim efficient. Suppose that (d, τ) is not incentive compatible. This means that there exists an individual i and types t_i', t_i'' such that:

$$\min_{t_{-i}\in T_{-i}} \left[v_i\left(t_i', d(t_i'', t_{-i})\right) + \tau_i(t_i'', t_{-i})\right] > \min_{t_{-i}\in T_{-i}} \left[v_i\left(t_i', d(t_i', t_{-i})\right) + \tau_i(t_i', t_{-i})\right]. \quad (27)$$

Let T'_{-i} denote the set of those $t'_{-i} \in T_{-i}$ that realize the minimum for t'_i, that is:

$$v_i\left(t'_i, d(t'_i, t'_{-i})\right) + \tau_i(t'_i, t'_{-i}) = \min_{t_{-i}\in T_{-i}} \left[v_i\left(t'_i, d(t'_i, t_{-i})\right) + \tau_i(t'_i, t_{-i})\right].$$

Define $d' : T \to O$ and $\tau' : T \to \mathbb{R}^n$ as follows: if $t_i \neq t'_i$ or $t_{-i} \notin T'_{-i}$, put $d'(t_i, t_{-i}) = d(t_i, t_{-i})$ and $\tau'(t_i, t_{-i}) = \tau(t_i, t_{-i})$; otherwise, define:

$$d'(t'_i, t_{-i}) = d(t''_i, t_{-i}); \text{ and } \tau'(t'_i, t_{-i}) = \tau(t''_i, t_{-i}).$$

Since $\sum_{j\in I} \tau'_j(t_i, t_{-i}) = \sum_{j\in I} \tau_j(t_i, t_{-i}) = 0$ if $t_i \neq t'_i$ or $t_{-i} \notin T'_{-i}$, and $\sum_{j\in I} \tau'_j(t'_i, t_{-i}) = \sum_{j\in I} \tau_j(t''_i, t_{-i}) = 0$ otherwise, then $\sum_{j\in I} \tau'_j(t) = 0$ for every $t \in T$.

For each $j \neq i$, define the set of utilities achieved by individual j with type t_j:

$$\mathcal{U}_j(t_j) \equiv \{v_j(t_j, d(t_j, t_{-j})) + \tau_j(t_j, t_{-j}) : t_{-j} \in T_{-j}\}.$$

Observe that we used (d, τ) in the definition of $\mathcal{U}_j(t_j)$. Thus, unless $t_i = t'_i$ and $t_{-i} \in T'_{-i}$, we have

$$v_j(t_j, d'(t_j, t_{-j})) + \tau'_j(t_j, t_{-j}) = v_j(t_j, d(t_j, t_{-j})) + \tau_j(t_j, t_{-j}) \in \mathcal{U}_j(t_j).$$

Now consider $t' = (t'_i, t'_{-i})$, where $t'_{-i} = (t_j, t'_{-i-j}) \in T'_{-i}$, $t_j \in T_j$. Then,

$$v_j(t_j, d'(t_j, t'_i, t'_{-i-j})) + \tau'_j(t_j, t'_i, t'_{-i-j}) = v_j(t_j, d(t_j, t''_i, t'_{-i-j})) + \tau_j(t_j, t''_i, t'_{-i-j}).$$

Note, however, that $(t''_i, t'_{-i-j}) \in T_{-j}$. Therefore, even if $t_i = t'_i$ and $t'_{-i} = (t_j, t'_{-i-j}) \in T'_{-i}$, $v_j(t_j, d'(t_j, t_{-j})) + \tau'_j(t_j, t_{-j}) \in \mathcal{U}_j(t_j)$. This allows us to obtain the following inequality:

$$\begin{aligned}
&\min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j, d(t_j, t_{-j})\right) + \tau_j(t_j, t_{-j})\right] \\
&= min_{u_j \in U(t_j)} u_j \\
&\leqslant \min_{t_{-j}\in T_{-j}} \left[v_j\left(t_j, d'(t_j, t_{-j})\right) + \tau'_j(t_j, t_{-j})\right].
\end{aligned}$$

This shows that (d', τ') is at least as good as (d, τ) for any $j \neq i$. On the other hand, (27) shows that (d', τ') is strictly (interim) better for i. Therefore, (d, τ) cannot be interim efficient. □

The following result will be useful in the proof of Theorem 5.1.

Lemma A.9. *Assume that $n = 2$. If x is Bayesian Pareto optimal and incentive compatible, then x_j is private information measurable for each $j \in I = \{1, 2\}$.*

Proof. See Lemma 4.1 of Glycopantis et al. (2003). □

Proof of Theorem 5.1. Assume that x is Bayesian efficient and incentive compatible, but not maximin efficient. This means that there exists a feasible allocation y such that $y_j \succcurlyeq_j^{t_j} x_j$ for all $j \in I, t_j \in T_j$ and there is $i \in I, t'_i \in T_i$ such that $y_i \succ_i^{t_i} x_i$, that is,

$$\underline{y}_i(t'_i) = \min_{t_{-i}\in T_{-i}} u_i(t'_i, y_i(t'_i, t_{-i})) > \min_{t_{-i}\in T_{-i}} u_i(t'_i, x_i(t'_i, t_{-i})) = \underline{x}_i(t'_i).$$

Since x_i is private information measurable by Lemma A.9, $x_i(t'_i, t_{-i}) = \underline{x}_i(t'_i)$ for all $t_{-i} \in T_{-i}$. Thus, $u_i(t'_i, y_i(t'_i, t_{-i})) > u_i(t'_i, x_i(t'_i, t_{-i}))$ for every t_{-i}. The monotonicity of u_i now gives

$y_i(t'_i, t_{-i}) > x_i(t'_i, t_{-i})$. Similarly, $y_j \succcurlyeq_j^{t_j} x_j$ and the fact that x_j is private information measurable imply that $y_j(t'_i, t_{-i}) \geqslant x_j(t'_i, t_{-i})$ for all $j \neq i$. But then, $\sum_{i\in I} y_i(t'_i, t'_{-i}) > \sum_{i\in I} x_i(t'_i, t'_{-i}) = \sum_{i\in I} e_i(t'_i, t'_{-i})$ and y is not feasible, which is a contradiction.

Now we give the counterexample for the reverse implication. There are two individuals, $\mathcal{B} = \mathbb{R}_+$, $T_i = \{t'_i, t''_i\}$, $u_i(t, a) = a$, for $i = 1, 2$ and any $t \in T$. The Bayesian beliefs $\mu_i(\{(t_1, t_2)\})$ of individual i for the event $\{(t_1, t_2)\}$ are defined by the following:

$\mu_1(\cdot)$	t'_2	t''_2
t'_1	0.35	0.15
t''_1	0.15	0.35

and

$\mu_2(\cdot)$	t'_2	t''_2
t'_1	0.15	0.35
t''_1	0.35	0.15

Note that the numbers in each table add up to one. Consider the allocations $x = (x_1, x_2)$ and $y = (y_1, y_2)$ defined as follows:

(x_1, x_2)	t'_2	t''_2
t'_1	$(2, 2)$	$(2, 2)$
t''_1	$(2, 2)$	$(2, 2)$

and

(y_1, y_2)	t'_2	t''_2
t'_1	$(3, 1)$	$(1, 3)$
t''_1	$(1, 3)$	$(3, 1)$

Let us first argue that x is maximin efficient. Suppose that there is z such that

$$z_i \succcurlyeq_i^{t_i} x_i, \forall i, t_i \in T_i \text{ and } z_j \succ_j^{t_j} x_j \text{ for some } j \in I. \tag{28}$$

Let us introduce the following notation: $\underline{z}_j(t_j) \equiv \min_{t_i} z_j(t_j, t_i)$. Thus, (28) implies $\underline{z}_1(t'_1)$, $\underline{z}_1(t''_1), \underline{z}_2(t'_2), \underline{z}_2(t''_2) \geqslant 2$ and at least one of these inequalities has to be strict. Observe that this requires $z_1(t_1, t_2) \geqslant 2$ and $z_2(t_1, t_2) \geqslant 2$, for any $(t_1, t_2) \in T_1 \times T_2$. But then feasibility implies $z_1(t_1, t_2) = z_2(t_1, t_2) = 2$, for any $(t_1, t_2) \neq (t''_1, t'_2)$. In turn, this implies that none of the inequalities $\underline{z}_1(t'_1), \underline{z}_1(t''_1), \underline{z}_2(t'_2), \underline{z}_2(t''_2) \geqslant 2$ can be strict. Therefore, (28) cannot hold and x is maximin efficient.

However, x is not Bayesian efficient because y is a Pareto improvement upon x. To see this, observe that

$$\begin{aligned} y_1(t'_1, t'_2)\mu_1(t'_2|t'_1) + y_1(t'_1, t''_2)\mu_1(t''_2|t'_1) &= 3 \cdot \frac{0.35}{0.15+0.35} + 1 \cdot \frac{0.15}{0.15+0.35} \\ &= 3 \cdot 0.7 + 1 \cdot 0.3 \\ &= 2.4 > 2 = x_1(t'_1, t_2), \forall t_2 \in T_2. \end{aligned}$$

This same calculation works for $y_1(t''_1, t'_2)\mu_1(t'_2|t''_1) + y_1(t'_1, t''_2)\mu_1(t''_2|t''_1)$ and the payoffs of x. Thus, y Pareto improves upon x, that is, x is not interim efficient in $\mathbf{E}^B$. □

Proof of Proposition 5.2. In Lemma 4.3, we show that (a^*, τ) is maximin efficient for some $\tau = (\tau_i)_{i\in I} \in \mathbb{R}^n$ satisfying $\sum_{i\in I} \tau_i = 0$ if and only if a^* is outcome efficient. An examination of the proof of that Lemma shows that it establishes the same equivalence for Bayesian efficiency. Therefore, the two concepts are equivalent to outcome efficiency of a^*. □

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