



Paying for the smart grid[☆]

Luciano De Castro^{a,*}, Joisa Dutra^b

^a Department of Managerial Economics and Decision Sciences (MEDS), Kellogg School of Management, Northwestern Univ., Evanston, IL, USA

^b Getulio Vargas Foundation, Rio de Janeiro, RJ, Brazil

ARTICLE INFO

Available online 1 October 2013

JEL classification:

D04
D47
D78
K29
L51

Keywords:

Smart grid
Demand response
Demand response effect
Reliability
Social planner problem in electricity markets
Public good
Regulation of utilities
Investment decisions by utilities

ABSTRACT

Smart grid technologies may bring substantial advantages to society, but the required investments are sizable. This paper analyzes three main issues related to smart grids: reliability, demand response and cost recovery of investments. In particular, we show that generators will lose profits as a direct effect of demand response initiatives, and most of the benefits of smart grids cannot be easily converted into payments. Moreover, there are potential issues in the choices made by utilities for providing smart grids, and the reliability pertinent to smart grids is a kind of public good.

© 2013 Elsevier B.V. All rights reserved.

1. Introduction

The electricity industry is about to experience deep reforms and the adoption of a “smart grid” is a milestone enabler of these changes. The smart grid is an electricity network that is able to integrate in an intelligent manner all the users, namely, power generators, consumers and agents that can act as both improving economic efficiency and reliability of supply while addressing the growing concerns for the environment in the provision of electricity services. Smart grids include an Advanced Metering Infrastructure (AMI), a high level of automation of the grid, distributed generation, storage and an information technology infrastructure (Amin and Wollenberg, 2005; Fox-Penner, 2010). Its main expected benefits are increased reliability of supply, environment protection and a potential lowering in production and capacity costs (EPRI, 2004, 2011). Fig. 1 shows the relative importance of these factors

as estimated by EPRI (2004).¹ More recently, EPRI (2011) updated its estimates to include the benefits of energy efficiency and demand response, among other factors, and projected that the total benefit would be between \$1.3 and \$2 trillion.

Although the expected smart grid benefits are huge, the related costs are also sizable. According to the EPRI (2011), the total costs of implementing the smart grid in the US range between \$338 and \$476 billion, over 20 years. The most important investments must be done in the distribution (\$231 to \$339 bi) and transmission (\$82 to \$90 bi) networks. The high level of required investments may involve public funds. In this spirit the Energy and Independence and Security Act (Congress, 2007) and the American Reinvestment and Recovery Act (Congress, 2009) allow for grants that can be allocated to smart grid related investments.

Considering that the expected benefits exceed the estimated costs by such an expressive amount one may wonder if there are real difficulties in making these technologies available on a broader basis. To answer this question we decompose the smart grid deployment problem in three building blocks that relate to the fundamental economic issues underlying smart grids: reliability, demand response and the level of investments in smart grids.

[☆] A previous version of this paper (under the title “The Economics of the Smart Grid”) was presented in the 49th Annual IEEE Allerton Conference on Communication, Control, and Computing. We are grateful to the participants for their comments. This version is significantly different and contains new results.

* Corresponding author at: Kellogg School of Management, Northwestern University, 2001 Sheridan Rd, Evanston, IL, 60208, USA.

E-mail address: decastro.luciano@gmail.com (L. De Castro).

¹ For a full description of these benefits, see EPRI (2004, 2011), especially p. 4–4 to 4–10.

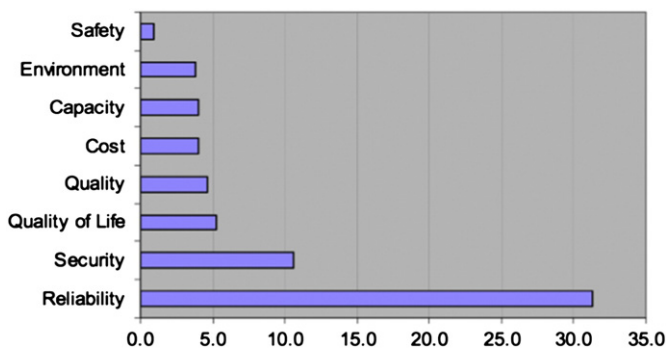


Fig. 1. Estimated annualized value (\$ billions) of each attribute EPRI: 04.

As we can see in Fig. 1, reliability is considered the most valuable benefit associated with smart grids. However it is not easy to translate this benefit into payments to cover costs. The rationale for this is that a given geographic area may gather people whose change of behavior could point to positive net benefits at the same time that some other consumers' habits could be such that would not justify the high required investments. But the economic deployment involves making the technology available to all the people located in a given area. In this context, we are dealing with a public goods dilemma. In such environment consumers have incentives to understate their true willingness to pay for the smart grid in order to lower the expected payments associated to the supply of the smart grid.

The second element of our analysis relates to the increased potential to explore demand response. Consumers may be exposed to pricing mechanisms that, despite being more sophisticated, may improve economic efficiency even under higher transaction costs to respond accordingly.

Nowadays the bulk of residential consumers face flat electricity prices, since most of their meters are not capable of recording time of consumption nor informing real-time prices. The pricing mechanisms that may be implemented in a smart grid environment more closely resemble the ones that have been advocated by economists for decades. There is relevant experimental evidence that time-dependent pricing schemes coupled with technology may grant considerable peak load reductions deferring investments in generation.² This belief is supported by the experimental evidence that has been observed in pilot programs (Faruqui and Sergici, 2010).

However, a widespread adoption of dynamic pricing may require changes at the regulatory level. Standard regulatory practices such as price caps and rate of return embody a direct link between sales on a unit basis and revenues, as well as profitability. Since dynamic pricing may reduce unit sales, this link creates a conflict for utilities. Hence, the extensive use of dynamic pricing depends on the decoupling between sales and revenues, a theme that is sensible from the political point of view.

Lastly, we investigate firms' investment decisions towards the deployment of smart grid technologies. Regulators and policy makers may be reluctant to allow all smart grid costs to be charged directly to consumers. This aspect is even more critical due to a higher technological obsolescence of smart grid assets as compared to physical ones. We present some results derived from the lack of commitment from the regulator to allow cost recovery in smart grid investments.

The abovementioned aspects are only some of the challenges that must be addressed in order to grant a smooth transition to a smart grid framework. This paper is an initial step in understanding smart grid main issues from an economic perspective. We show that these

issues can be decomposed into three separate problems: reliability, demand response and deployment.³ We proceed to analyze these problems and obtain the following findings:

1. Despite being the most important benefit of smart grids, reliability has “public-goods” characteristics that are likely to lead to an insufficient level of deployment. Moreover, this part of the benefit is hardly translated into payments, barring the organization of a reliability market (that would implement Lindahl prices).
2. All generators are worse-off with demand response programs. This also includes generators operating in “shoulder” periods that could potentially benefit from a higher overall consumption due to a shift of consumption from peak to off-peak periods.
3. There are many problems to achieve the optimal level of investments in smart grid technologies under the current regulatory frameworks. First, there are uncertainties with respect to the length of the obsolescence cycle of the new technologies. Second, the uncertainties with respect to the recovery of investments may lead firms to underinvest. Third, the specific choices of technologies that are decided by the firms may be in their interests, but not in the interest of the customers.

Although the above conclusions may seem intuitive—with the important exception of item 2, which surprised us—the main contribution of this paper is to formalize these results in a well defined economic model that could be used by economists in more detailed analyses and studies.

The rest of the paper is organized as follows. After a brief literature review (Section 1.1), Section 2 presents smart grid interested agents' preferences. From this, we define the social planner problem in Section 3. With simplifying assumptions, we are able to decompose this problem in the following way: the reliability problem, studied in Section 4; the demand response problem, discussed in Section 5; and the deployment problem, analyzed in Section 6. Section 7 concludes.

1.1. Literature review

Even though some papers have already addressed the economics of dynamic pricing mechanisms and the incentives faced by end users that are exposed to them, to our knowledge there is no attempt to analyze smart grids' different aspects in a comprehensive economic framework, as this paper does. In this brief review, we will only mention some of the most relevant papers that we were able to identify.

Borenstein and Holland (2005) present a model in which competing load serving entities (LSEs) serve homogeneous retail consumers, who face two different pricing schemes: real-time pricing (RTP) and fixed prices. The authors show that there will be inefficiencies unless all consumers are in RTP. More interestingly, the competitive equilibrium does not achieve even the second best optimal allocation. In their model an increase in the proportion of consumers exposed to RTP lowers consumer surplus to RTP consumers while increasing surplus to consumers remaining on a flat rate as well as to the ones switching to RTP. They also investigate the choice of consumers to adhere to RTP when facing billing and metering costs. If these costs are positive, the second best optimal electricity allocation is not achieved due to an externality imposed by consumers switching to RTP on the consumers who remain in the flat rate. Hence, in the model RTP is likely to improve welfare, but the costs involved in this adoption may not recommend exposing all electricity consumers to RTP. Borenstein and Holland (2005) extend their analysis to heterogeneous consumers using simulations.

Joskow and Tirole (2006) extend Borenstein and Holland (2005)'s model to include the possibility of two-part tariffs, rationing and consumers who have real-time meters but respond only partially to RTP. In their setting, this partial responsiveness is the result of transaction costs of an increased monitoring of the price profile and the

² By “time-dependent pricing schemes” we mean not only real-time pricing, which has been the main focus of attention by economists, but also pricing schemes that incorporate some, but not full, time variability, such as time-of-use tariffs, seasonal rates and critical peak pricing, for example. See Appendix A for a description of some of those pricing schemes.

³ Our analysis does not cover issues related to cyber-security and privacy, which is also arguably important for smart grids.

optimization in the usage of electric appliances. Rational but imperfectly reactive consumers who are exposed to RTP achieve the Ramsey optimum when paying real-time wholesale price for their consumption patterns. According to Joskow and Tirole (2006), the sub-optimality obtained by Borenstein and Holland (2005) is a consequence of two features: (i) some consumers endowed with RT meters are charged uniform prices; and (ii) even if constrained to charge uniform prices, it is suboptimal to charge a linear price.

Joskow and Tirole (2007) investigate the effect of non-market mechanisms such as wholesale market price caps on energy supplies, generation capacity contracting obligations on distribution companies and other LSEs as well as system operating reserve requirements that are often present in competitive wholesale and retail electricity markets. The price caps that are often encountered on spot markets for electricity are reflected on forward prices for energy traded bilaterally or in OTC markets. In the model non-market mechanisms such as capacity obligations cannot be explained by the existence of a group of price-insensitive consumers. However capacity obligations and associated capacity prices are able to restore investment incentives when regulatory opportunism may induce artificial limits set on spot prices.

Other important feature of their analysis is the treatment of reliability as a public good that is investigated through a second product offered by generators as operations reserve. This is distinct from our analysis that tackles the reliability nature in the distribution system as a consequence of the deployment of smart grid technologies. In our setting, smart grid technologies also have a public goods dimension as a result of the higher level of reliability that could be achieved and the inability to exclude electricity users from this higher quality that would be made available.

In a recent paper, Borenstein (2013) observes the low popularity of time-varying electricity pricing among regulators and consumers, especially on a mandatory basis, and proposes an equitable opt-in time-varying residential pricing plan. His plan minimizes cross-subsidies between the groups that adhere to dynamic pricing and the end users that remain on a default flat tariff.

Even though the mentioned papers are able to frame different aspects enabled by smart grids, such as real-time pricing, the novelty of our approach is to put smart grid on the central stage and present an economic analysis of its main issues.

Another difference from the received literature is our set of assumptions. For instance, we consider heterogeneous firms and consumers. While Borenstein and Holland (2005) performed simulations and Joskow and Tirole (2006) included results for studying the effect of heterogeneity, their main focus is on homogenous generators and consumers. Also, they have assumed separability of consumption across time, while we considered preferences depending on functionals, thus allowing any kind of inter-temporal preference.⁴ More importantly, our treatment of reliability is different from all previous papers. We are concerned with reliability relevant for smart grids, which is related to the rate of failure of distribution (and transmission) circuits. In contrast, reliability for the literature is focused on the generation side and refers to the probability that there will be enough capacity to meet demand. This makes our treatment of reliability unique with respect to the previous literature. These model differences have implications for the interpretation and significance of our results, and require specific comments. We make such comments after the results are formalized and presented.

2. A model of smart grid parties' interests

In this section, we introduce a model for the smart grid parties: consumers, distribution companies, generators and society. The last party includes environmental interests not restricted to consumers. In principle, we should also consider transmission companies, which have significant importance for the smart grid. Because their role in our

⁴ Interestingly, this does not create any problem for our separation of the Social Planner problem into three different sub-problems.

analysis is very similar to that of distribution companies, we encapsulate both types of firms in the last term. The reader should keep this in mind to avoid confusion.

We consider a general representation of uncertainty: there is an abstract probability space (Ω, Σ, \Pr) . A random outcome $\omega \in \Omega$ determines prices, allocations, etc., but it is not fully observable. In any case, our discussion and treatment of uncertainty will be limited, since it is not central for the aspects that we discuss.

2.1. Consumers

Let $C = \{1, \dots, N\}$, with $N \gg 2$, denote the set of consumers and T the time set considered: it can be an interval of 24 h, a week or month, or even something as simple as $\{0, 1\}$, denoting off-peak and peak periods. A function $l_i: T \times \Omega \rightarrow \mathbb{R}$ denotes the (random) consumption of power by consumer i , that is, $l_i(t, \omega)$ is the consumption of individual i when the state of nature is $\omega \in \Omega$. For simplicity, we will omit ω in most of the paper and write just $l_i(t)$ for a specific realization of the demand. Note that in principle we allow $l_i(t)$ to be negative; in this case, consumer i would be providing power to the grid instead of receiving it. This could be the case if the consumer has production capacities. The possibility that consumers produce energy and inject it in the grid is actually one of the reasons for allowing smart grid technologies. Let \mathcal{D} denote the set of (measurable) functions $l: T \times \Omega \rightarrow \mathbb{R}$.⁵

The consumer cares about the failures of the electric system, which can occur from two different sources. First, we can have events in which the demand for power is higher than the supply (either because the demand is too high or because some generators have failed). Second, we can have failures in the system for transporting electricity (distribution or transmission). Both kinds of failures impact the consumer in the same way, but they are reduced through different actions (investment in capacity or in the grid). Since this paper discusses the smart grid, we will focus attention only on failures in the grid. Therefore, let $r \in [0, 1]$ be the level of reliability of the service, meaning that the distribution system (transmission plus local distribution) is properly working a fraction r of the time. We assume that r does not depend on the consumption level, contrary to the usual treatments of reliability (Joskow and Tirole, 2007; Wilson, 1993). This difference is justified by our emphasis on failures of the distribution and transmission systems, whose occurrence is not significantly affected by the demand.⁶ If, instead, we were modeling failures caused by insufficient supply, then this assumption would not be reasonable.

Let $v_i: \mathcal{D} \times [0, 1] \rightarrow \mathbb{R}$ represent the utility experienced by the consumer depending on its consumption and the level of reliability. Similarly, let $p: \mathcal{D} \times [0, 1] \rightarrow \mathbb{R}$ denote the total price paid by the consumer.⁷ Note that this pricing functional is defined on functions (power demanded for each hour). Therefore, this functional is more general than what is usually found in the literature.⁸ This generality conveys more clearly the ideas; we are not necessarily advocating for the adoption of more complicated pricing schedules. Besides the

⁵ For some technical applications, it will be convenient to impose more structure in the set \mathcal{D} of functions considered. For instance, we could restrict \mathcal{D} to be equal to the set of L^2 functions in T . This restriction simplifies some technical conditions and does not seem overly restrictive, but it will not play a role in our analysis. In particular, if $T = \{0, 1\}$ as we mentioned above, this restriction is without loss of generality.

⁶ It is true that an extremely high level of demand can increase the probability of failures in the distribution and transmission systems, contrary to our hypothesis. However, if the distribution and transmission systems operate within their specifications, the demand has no significant impact on the probability of failure. Since system operators exert great efforts to avoid operation outside specifications, our assumption seems reasonable.

⁷ The notation allows, in principle, that the price and utility functionals could depend on the random functions $l_i: T \times \Omega \rightarrow \mathbb{R}$. It makes more sense, however, to think that they depend on specific realizations of the load, that is, on the functions $l_i(\cdot; \omega): T \rightarrow \mathbb{R}$, for a fixed ω .

⁸ For instance, Wilson (1993) describes the prices as a function of the hour and the level of power required. This pricing function could not make a distinction between shapes of consumption, for instance, as the one considered here would allow.

shape of consumption, the pricing functional may also depend explicitly on the reliability of the service. This is done just for the sake of generality, since most pricing schemes currently in place do not directly charge for reliability levels. Also, $p(\cdot)$ could be negative to represent a consumer who is a net producer. This is relevant because distributed generation is one of the main components of smart grid.

In Appendix A, we discuss some alternative formats of the pricing function $p(l_i, r)$, especially those that induce a higher demand response.

The consumer utility is given by:

$$u_i(l_i, r) - p(l_i, r). \tag{1}$$

Note that this utility can capture all attitudes towards inter-temporal transfers of electricity consumption, while the received literature has ruled out such transfers.⁹

2.2. Distribution company

The distribution company D needs to decide the level of investment in the smart grid, y .¹⁰ The level y impacts the potential benefits of smart grid, which we capture as an impact in the reliability of the grid r and the possibility of implementing demand response, as we will discuss below. For now, it is convenient to keep the decision y at a high level of generality, assuming that it is just an element of an abstract set Y . This will allow us to model the choice of different technologies. When we want to focus on the dollar amount to be invested, we will specify $Y \subset \mathbb{R}_+$.

The distribution company has total cost $c_d(y, r)$ of providing reliability r at the level of investment y .¹¹ The idea is that with a higher y , the cost of providing r is smaller, that is, the function $c_d(y, r)$ is not necessarily additively separable.

2.3. Generator

For simplicity, we assume there are two types of technology for producing energy: standard and clean, which will be denoted, respectively, by l_s and l_c . The generators have costs $c_s(l_s)$ and $c_c(l_c)$ of producing energy by these technologies. Naturally, we should have

$$l_s + l_c = \sum_{i \in C} l_i. \tag{2}$$

Note that y enters directly into the cost of reliability, but it does not enter into the cost of producing energy. In this case, how can we accommodate the notion, mentioned above, that one of the advantages of the smart grid is the reduction in the production costs? Our point is that this is *not* a direct benefit of the smart grid. This benefit occurs only through a difference in the costs and the technologies associated with the production of $l = l_s + l_c$. There are two ways in which smart grids could affect this production. First, it could impact the shape of l by altering the load profile through shaving the consumption at peak trough demand response. Even if the total energy is not reduced, cheaper technologies could grant lower costs of attending the demand in a higher percentage of the time, reducing the need of high cost technologies.¹² Second, smart grids allow consumers to inject electricity in the grid either through some distributed generation in their premises (in general, this is renewable generation) or through the batteries of plug-in electric

vehicles. Hence the need for power is reduced and this impact can be significant, especially at peak times.

2.4. Society

The society cares for the environment and suffers a disutility from the pollution caused by the standard technologies. We model this by assuming that society suffers a disutility $-v(l_s)$ when the standard technologies produce l_s . Note that we are implicitly assuming that the production technologies that some consumers may have are clean. This comes from the fact that the energy produced by consumers, if it is above their needs, enters as negative sign and, therefore, contributes to reduce $l = \sum_{i \in C} l_i$ and, therefore, l_s .

3. The benevolent social planner problem

Conventional technologies aggregate electricity consumption in large time intervals (e.g. months). Hence, users cannot be exposed to different prices for distinct time periods.

The smart grid would allow this smoothing of the consumption through different pricing schedules, something that is not possible with standard technologies. That is, the pricing function $p: \mathcal{D} \rightarrow \mathbb{R}$ belongs to a set $\mathcal{P}(y)$ of possible pricing functions, which depends on the level y of smart grid investments. For simplicity, we may assume that the functions in $\mathcal{P}(y)$ are differentiable.

Let us consider first the problem of a market designer or a benevolent social planner who wants to choose a pricing schedule $p \in \mathcal{P}(y)$ that covers the costs:

$$\sum_{i \in C} p(l_i, r) \geq c_d(y, r) + c_s(l_s) + c_c(l_c). \tag{3}$$

Given a pricing schedule $p \in \mathcal{P}(y)$, the consumer would choose l_i in order to maximize Eq. (1), that is, the utility of the consumer will be given by:

$$U_i(p, r) \equiv \max_{l_i \in D} u_i(l_i, r) - p(l_i, r). \tag{4}$$

For simplicity, we did not include a (transaction) cost for the consumer to choose l_i in face of more complicated $p \in \mathcal{P}(y)$, which is argued by some consumer advocates as relevant.¹³ Since we will not focus explicitly on the optimal choice of p , such omission is innocuous.

Given y, r and a price $p \in \mathcal{P}(y)$, the social welfare is:

$$W(y, r, p) \equiv \sum_{i \in C} U_i(p, r) - v(l_s) + \left\{ \sum_{i \in C} p(l_i, r) - [c_d(y, r) + c_s(l_s) + c_c(l_c)] \right\},$$

where the terms in the second line refer to the joint profit of distribution and generators. Given that the determination of the prices demands fixed levels of y and r , it is useful to define

$$S(y, r) \equiv \max_{p \in \mathcal{P}(y)} W(y, r, p) \text{ subject to Eqs. (2) and (3)}$$

as the social planner's pricing problem. Finally, the actual social planner problem is to choose y and r in order to maximize $S(y, r)$.

¹³ Joskow and Tirole (2006) allow for a transaction cost of altering consumption in response to real time pricing.

⁹ Joskow and Tirole (2007, footnote 8) states that they "could allow such transfers, at the cost of increased notational complexity."

¹⁰ As we emphasized earlier, in our analysis "distribution company" includes firms specialized in transmission.

¹¹ It may seem more natural to write this cost as $y + c_d(r)$. Our point is that $c_d(r)$ is also a function of y —with a small investment in the smart grid, the distribution company will have to spend more (in personnel, for instance) to achieve the same level of reliability. Therefore, the cost would be $y + c_d^y(r)$, which is just a particular form of $c_d(y, r)$.

¹² This consumption change could also reduce the need of building more generators for reserve, which is also a factor for reducing costs in time.

The description above is useful for establishing a framework that allows us to investigate the questions and problems discussed before. Since the social planner problem is extremely difficult to solve at this level of generality, we impose some simplifying conditions for proceeding with the analysis.

3.1. Assumptions

Our first set of assumptions allows us to break the social planner problem into several parts, making it more tractable. The first assumption refers to the consumer.

Assumption 3.1. Linear separation

The utility function of each consumer is additively separable, that is, $u_i(l_i, r) = v_i(l_i) + \bar{v}_i(r)$.

Although this assumption may seem restrictive, it is satisfied by a monotonic (log) transformation if u_i depends on the product of (a function of) r and l_i ; for example, if $u_i(l_i, r) = \int \tau l_i(t) dt$.¹⁴

The next assumption is much more common in practice.

Assumption 3.2. Decoupling

The pricing functional must specify a distribution and a generation component, that is, $p(l_i, r) = p_d + p_g(l_i)$, so that instead of Eq. (3), we have¹⁵:

$$\sum_{i \in C} p_d = N p_d \geq c_d(y, r); \text{ and} \tag{5}$$

$$\sum_{i \in C} p_g(l_i) \geq c_s(l_s) + c_c(l_c). \tag{6}$$

Moreover, p_d does not depend on r nor on l_i .

This assumption requires that the revenue collected be allocated separately to generation and distribution companies. This amounts to decoupling the energy and the distribution shares of the electric pricing and it is consistent with a vertical unbundling characteristic of the electricity industry in several parts of the world. We should note, however, that the motivation for decoupling in the real world (in short, to align incentives of distribution companies with the aim of energy consumption reduction) is not what motivates us to adopt this assumption. We use it to simplify the social planner problem (see Proposition 3.1 below).

Besides the separation of the energy and distribution parts (remember that we are including transmission on the distribution part), under this assumption the payments for the distribution company do not depend on the reliability level, r . This is consistent with most of the pricing schemes currently in place that do not charge for reliability. This aspect of the assumption will be important in Section 4 below. On the other hand, we also assume that p_d does not depend on l_i . This is not true in many places, where the distribution charges are given by a fixed factor times per total consumption (\$/kWh). In this case, p_d will be, of course, a function of $\int \tau l_i(t) dt$. However, our assumption is not problematic even in this case, because the distribution charges are set to allow recovery of the distribution company costs but not the energy costs. Therefore, the separation assumed in Assumption 3.2 above is a reasonable approximation.

¹⁴ There is a caveat, however. In general, monotonic transformations do not change the optimal allocation, but the consumer problem (Eq. (7)) depends also on the price, in the traditional “partial equilibrium” fashion. Therefore, the optimal choice may not be invariant to monotonic transformations of the utility.

¹⁵ The distribution tariff p_d could be different for different classes of customers, but we abstain from making this distinction.

3.2. Separation of the problem

Let us define the following function:

$$V_i(p) \equiv \max_{l_i \in D} v_i(l_i) - p_g(l_i). \tag{7}$$

It is now easy to obtain our first result.

Proposition 3.1. Under Assumptions 3.1 and 3.2, the social planner problem can be separated in three problems, as follows:

Reliability Problem:

$$S_d(y) \equiv \max_{r, p_d} \sum_{i \in C} \bar{v}_i(r) - c_d(y, r) \text{ subjected to Eq. (5);} \tag{8}$$

Demand Response Problem:

$$S_g(y) \equiv \max_{p \in P(y)} \sum_{i \in C} V_i(p) - v(l_s) + \left\{ \sum_{i \in C} p_g(l_i) - [c_s(l_s) + c_c(l_c)] \right\} \text{ subjected to Eq. (6);} \tag{9}$$

and Deployment Problem:

$$\max_y S_d(y) + S_g(y). \tag{10}$$

The naming of these problems will become clear in the discussion that follows. But before going into this analysis, it is useful to discuss some subtle consequences of Assumption 3.2. Under this assumption, the social planner cannot transfer funds from energy consumption to the payment of the grid. Since one of the benefits of the smart grid is to improve the allocation of energy production (by shifting consumption from peak to off-peak periods, as we discuss in Section 5), this constraint limits recovery of the smart grid costs with its underlying benefits.

In the next sections, we derive some results about each of the above problems. However, obtaining explicit characterizations of the solutions of these problems is beyond the scope of this paper, given the generality of the objects involved (price and utility functionals).

4. Reliability problem

Under the standard assumptions there exists a solution to the reliability problem that can be characterized by the following:

Proposition 4.1. Assume that \bar{v}_i and c_d are differentiable and that $r = 0$ cannot be optimal in problem (8). Then, when it exists, the solution r^* to problem (8) satisfies:

$$\sum_{i \in C} \bar{v}'_i(r^*) = \frac{\partial c_d(y, r^*)}{\partial r}. \tag{11}$$

Proof. By Eq. (5), and considering that the social planner aims to maximize the consumer's utility, $p_d = \frac{1}{N} c_d(y, r)$. Substituting this into Eq. (8), we obtain the claim from the assumptions of differentiability and interiority. ■

Note that the choice of r^* to satisfy Eq. (11) requires unrealistic knowledge from the social planner. It has to know not only the marginal utilities of all consumers in the society, but also the marginal cost of the reliability for each level of the smart grid investment y . It is natural to ask whether Eq. (11) could be implemented through some market mechanism. For instance, the social planner could try to fix a price $p_d(r) \equiv \frac{1}{N} c_d(y, r)$ for the level r of reliability, expecting the individuals to choose how much to pay for their optimal share of reliability.

Unfortunately, this will not work because the level of reliability that individuals would demand in this case would be sub-optimal, as the following proposition shows.

Proposition 4.2. Assume that \bar{v}_i is differentiable, increasing and concave. For any pricing schedule satisfying Eq. (5), the level r demanded by the individuals will be sub-optimal.

Proof. A price scheme satisfying Eq. (5) does not vary with r . Therefore, each customer would choose a level of consumption r_i^* satisfying:

$$\bar{v}'_i(r_i^*) \leq 0,$$

with equality if $r_i^* > 0$. Since \bar{v}_i is increasing and $r \in [0, 1]$, $r_i^* = 1$ for every consumer, which does not satisfy Eq. (11). ■

In some sense, the result in Proposition 4.2 seems artificial, because it comes directly from the fact that the consumers' contributions do not vary with the costs of providing reliability. However, we see this proposition as useful to highlight the importance of charging for reliability. On the other hand, the problems are yet not solved if we introduce a reliability price, as the following proposition considers.

Proposition 4.3. Assume that \bar{v}_i is differentiable, increasing and concave and $\frac{\partial c_d(y, \bar{r})}{\partial r} > 0$. Assume that each electricity user demands a reliability level r_i , but consumes the aggregate reliability $r = \sum_{i \in C} r_i$. The level r demanded by the individuals will be sub-optimal.

Proof. Each consumer's problem will be

$$\max_{r_i} \bar{v}_i \left(r_i + \sum_{j \neq i} \{r_j\} \right) - p_r r_i.$$

Let \bar{r}_i denote the optimal choice of consumer i and $\bar{r} \equiv \sum_{i \in C} \bar{r}_i$. Then, $\bar{v}_i(\bar{r}) \leq p_r$, with equality holding if $\bar{r}_i > 0$. From the distribution company perspective, we must have $p_r \leq \frac{\partial c_d(y, \bar{r})}{\partial r}$. If we set $\delta_i = 1$ if $\bar{r}_i > 0$ and $\delta_i = 0$ if $\bar{r}_i = 0$, then $\sum_i \delta_i \left[\bar{v}'_i(\bar{r}) - \frac{\partial c_d(y, \bar{r})}{\partial r} \right] = 0$. If $\bar{r} > 0$, $\sum_{i \in C} \bar{v}'_i(\bar{r}) > \frac{\partial c_d(y, \bar{r})}{\partial r}$, which shows that Eq. (13) is not satisfied and \bar{r} is sub-optimal. ■

This proposition reveals the public goods problem in the provision of reliability. Essentially, each consumer “free rides” on the reliability level provided by the other electricity users.

In a public goods setting, the market failure is related to the fact that economic agents that make decisions based on their willingness to pay for the good end up achieving a provision level that is lower than the choice that would emerge from a social planner's choice. In turn, the social planner would provide a level corresponding to a social willingness to pay for the public good.

In a decentralized mechanism consumers have incentives to understate their true willingness to pay because, if the good is provided, it is not possible to exclude them from consumption, and their consumption will not lower other consumers' availability for the good. In principle, smart grid technologies could allow exclusion of certain consumers, mitigating this problem. That is, in principle the following solution could be achieved. Utilities could contract different levels of reliability with consumers. Because of gains of scale, the utility would make investments to provide the improved level of reliability to consumers, and this would allow it to provide the same level to the consumers that were not asking for it. However, to make the contracts credible, the utility should be able to voluntarily curtail some consumers just to provide them with the contracted level of reliability. This practice does not seem politically acceptable.

Other economic solutions to the public goods problem include Lindahl pricing and taxes (see Mas-Colell et al., 1995). Under Lindahl

prices, reliability is treated as an exclusive good that could be rejected from the individual. In our situation, this means that if an individual chooses a reliability $r_i < r_i^*$, the distribution company would have to shut off that consumer deliberately, so that he or she would consume exactly r_i . This solution may also lack (political) feasibility.

Alternatively, this problem could be solved by collecting taxes from consumers to fund the provision of reliability. In this setting, every consumer would be taxed by the social planner in terms of his consumption of reliability. The desired amount would be such that the marginal rate of substitution would equal the marginal cost of providing reliability. The optimality condition would demand setting taxes such that $t_i(r_i) = p_i^*(r_i)$ where p_i^* would be the Lindahl prices, but the information needed to implement this solution would be excessively demanding.

Given the difficulty in solving this problem from a theoretical point of view, it would be interesting to investigate how actual regulators are dealing with it. Although we did not perform any formal inquiry into this matter, anecdotal accounts suggest to us that most of the regulators' work is reduced to approve or reject proposals made by utilities. In turn, utilities make their choices based on a set of options made available by equipment suppliers. What option is actually adopted is influenced by a number of arbitrary factors, including the persuasion power of the respective sales people.

5. Demand response problem

We call problem (9) the demand response problem because the decision variable is the price functional, which is the main driver of demand response programs. The social planner has to choose a pricing functional that covers energy production costs and that grants the proper incentives to electricity users. As we argued, the deployment of smart grid must embody functionalities that induce demand response (DR) and allow distributed generation (DG).

The first effect is depicted in Fig. 2. What one hopes with DR is that the peak generators will be less active, while the cycling and base generators will be more active. Since the latter generators produce at a lower cost than the former, the result is a reduction of overall costs of electricity production.

However, it is less clear what is happening with the revenue for each kind of generators. For instance, it seems that a baseload generator will have the opportunity to produce for longer periods. This is indeed likely to occur because some of the energy consumed in the peak, during which the base generator is already at full capacity and could not provide that energy, could now be provided by those generators. Since these generators will work for longer periods, it seems reasonable to infer that they will have higher profits and revenues. Moreover, the peak generators would certainly lose. This is coherent with the standard intuition. As we are going to see, they may be wrong. For clarifying this issue, we need to formalize the generators' costs, the spot price, and the DR consequences on the load function.

5.1. The generators' costs and the spot price

We assume the spot price is determined by the marginal cost of the highest cost technology required to meet the total load (demand) $l(t)$. The cost for attending the load x is $c(x)$, where c is assumed twice differentiable, increasing and convex. Therefore, the total cost introduced before is:

$$c_s(l_s) = \int_T c(l_s(t)) dt.$$

Our results in this section can also be easily adapted to the case of a finite number of different generators. Hence, we could have assumed that there were m generators, and generator $j = 1, \dots, m$ has costs c_j , satisfying $0 < c_1 < c_2 < \dots < c_m$, and capacity k_j , so that whenever the

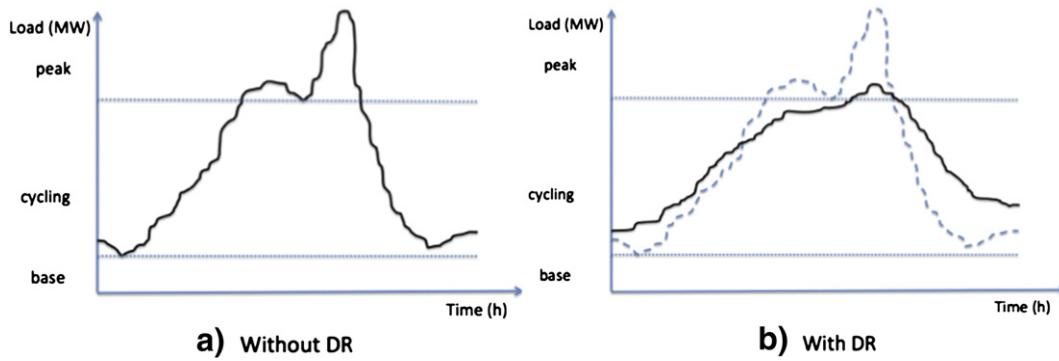


Fig. 2. Change in the load shape with DR.

load is between $L_j \equiv \sum_{i=1}^j k_i$ and L_{j-1} , the price is determined by the marginal generator j , that is, it is equal to c_j .

5.2. DR impact on the load function

Demand Side Mechanisms are expected to shift consumption from high cost (peak) to low cost (off-peak) periods. As a result, one would observe a valley-filling behavior pattern such as diagram (b) in Fig. 2, also known as ironing of the load function. We were not able to find a formal definition of this expected change, so we will propose one.¹⁶ To understand our motivation, it will be useful to depict the loads shown in Fig. 2 in terms of the load–duration curve (Fig. 3). A load–duration curve specifies, for each level of load x , the number of hours $J(x)$ that the customers' load exceeds x .

Now, although the demand in each load level may reduce or increase, DR is expected to reduce the load in the high cost (high demand) periods and perhaps increase it in low cost (low demand) periods. The following definition captures this effect.

Assumption 5.1. With DR, the energy demanded from higher cost generators is lower. That is, for every $L > 0$,¹⁷

$$\int_L^\infty \Pr(I^0(t) > u) du \geq \int_L^\infty \Pr(I^1(t) > u) du. \tag{12}$$

Notice that we have not required that the total energy consumption remains the same. If we add the extra assumption that the total consumption (integral of the load) does not change, this is just Second Order Stochastic Dominance. Although this assumption seems reasonable, it is an empirical question left to future works whether or not it is a good description of the demand response effect in practice.

5.3. Effects of DR on generators' revenues and profits

The following proposition shows that the intuition cited at the beginning of this section is not correct.

Proposition 5.1. Under Assumption 5.1, the revenue and the profit of all generators decrease with DR.

Proof. The revenue of generator x , that operates if $l > x$, is:

$$\int_x^\infty c(u) dF^0(u) = - \left[c(u) (1 - F^0(u)) \right]_x^\infty + \int_x^\infty c'(u) du.$$

¹⁶ After writing the first version of this paper, we learned that Holland and Mansur (2008) characterize the effect of demand response as reducing the variance of the load.

¹⁷ Hereafter, we will use superscripts 1 and 0 to represent the situation with and without DR, respectively.

Since $1 - F^0(\infty) = 0$, the revenue from energy is equal to:

$$c(x) (1 - F^0(x)) + \int_x^\infty c'(u) du,$$

where the first term is the cost of generator x , while the second is its profit. Now, let us define $H^0(x) = \int_x^\infty (1 - F^0(u)) du$. Then, generator x 's profit is:

$$- [c'(u)]_x^\infty + \int_x^\infty c''(u) du.$$

Since $H^0(\infty) = 0$, this simplifies to:

$$c^0(x) + \int_x^\infty c''(u) du.$$

A similar expression holds for the case with DR. Since Eq. (12) is just the assumption that $H^0(u) \geq H^1(u)$ for all u , then we have the conclusion. ■

Proposition 5.1 contradicts the usual intuition that baseload operators, that may have the opportunity to produce more under DR, will gain from consumption shifts. For this, we have assumed that the total demand does not increase with DR (this is implicit in Assumption 5.1). It may be the case that the total demand increases with DR, because consumers have access to cheaper energy. It is not clear if this can indeed be an effect of DR, but in any case, Proposition 5.1 shows that this could be the only way generators could benefit with DR.

It is useful to compare this result with Borenstein and Holland (2005, Theorem 4), which studies the effects of an increase in RTP customers in the profits and overall level of capacity. They say that “the short-run wholesale profits may increase or decrease” if the RTP customers

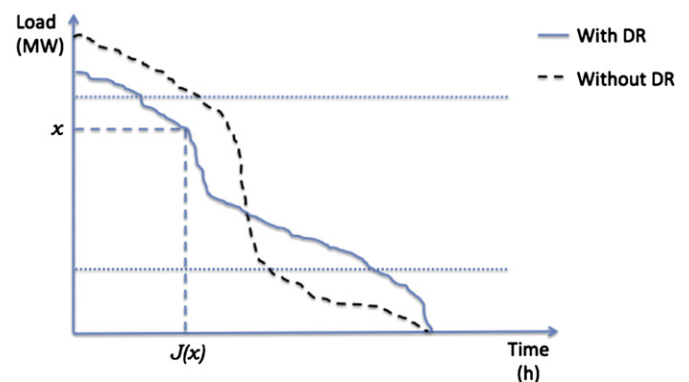


Fig. 3. Load–duration curves with and without DR.

increase (p. 480). In contrast, we obtain an unequivocal prediction: profits (weakly) decrease. As we emphasized before, our assumptions are different; for instance, they consider homogenous firms and consumers, while we allow heterogeneity. However, the main difference is our assumption on the effect of demand response (Assumption 5.1).

In the above analysis, we have not taken into consideration the ancillary service revenues that a generator may provide. For a peak generator, these services can be a substantial part of its revenues and profits. If the introduction of DG and DR increases the need of ancillary services—because of an increase in the volatility of the load, for instance, which is likely to occur (Roozbehani et al., 2011)—then the direct beneficiaries from DR will be exactly peak, not base generators.

It is worth noting that the present analysis does not consider entry of new generators into the industry. If the argument in the previous paragraph is indeed valid, they will probably resist any proposal to bear the deployment costs.

Another implication of the analysis in this section is that it does not seem reasonable to expect that generators will be called to bear some of the costs of smart grid implementation. Since DR is an important part of the effects of the smart grid and it has a negative impact on them, they will probably resist any attempt of sharing its costs.

5.4. The problem of consumers' freedom of tariff choice

A complication that arises for the analysis of DR programs effects is the possibility of consumers choosing between the usual flat rate and some form of time-dependent prices. Borenstein and Holland (2005, Theorem 6) show that freedom of adoption may lead to a level of adoption that is either above or below the optimal level. This depends on the transaction costs for observing real-time prices and adapting consumption accordingly. Therefore, in choosing the optimal pricing functional, the social planner will have to take in account the complications that this freedom brings.

6. Deployment problem

The last problem to consider is the choice of the deployment level of smart grid technologies (Eq. (10)). Assuming that it is unique, let y^* denote the solution to this problem, that is,

$$y^* \equiv \arg \max_y [S_d(y) + S_g(y)], \tag{13}$$

where $S_d(y)$ is the solution of the reliability problem (8) and S_g is the solution of the demand response problem (9).

At this level of abstraction, there is not much that we can say to characterize y^* . Instead, in this section we will discuss hurdles that must be overcome to achieve an optimal implementation. In particular, we will discuss three issues. We begin in Section 6.1 by describing the problems associated with the choice of technology and its obsolescence. In Section 6.2, we study the link between the level of optimal provision y^* and the (lack of) commitment of the regulator to allow cost recovery of the required investments. Even though this problem is not specific to smart grid investments, the high level of required investments makes it more relevant. Finally, in Section 6.3 we discuss the possibility of misalignment of the interests of utilities and consumers.

6.1. Faster obsolescence and technology choice

One critical issue to smart grid investments is that they are subject to a faster technology obsolescence relative to conventional distribution networks. Additionally, there is asymmetric information in the sense that a myriad of options exist, allowing the firm to choose among distinct technologies that differ in the initial cost and the number of periods that the technology lasts.

Consider the choice of the regulated firm that faces two distinct technologies that are equivalent from the consumer's perspective. Denote the investment cost in alternative j by y_j . This initial investment requires additional investments in the amount (again) of y_j , each T_j periods.¹⁸ Let δ be the discount rate (note that this is not necessarily equal to the return level). Therefore, the total cost of option j is:

$$C_j = y_j + \frac{y_j}{(1 + \delta)^{T_j}} + \frac{y_j}{(1 + \delta)^{2T_j}} + \dots = \frac{y_j}{1 - (1 + \delta)^{-T_j}}. \tag{14}$$

From this equation we can illustrate a very simple but relevant finding: the evaluation of the true costs of a technology requires considering not only the initial investment y_j . It is necessary to know the time interval elapsed until new investments are required; otherwise, the firm may end up choosing a technology 1 over 2 due to its lower initial costs ($y_1 < y_2$) despite incurring in a higher total cost ($C_1 > C_2$) because the time interval in 1 is much shorter than in 2 ($T_1 \ll T_2$). This leads us to observe the following:

Result 1. The regulator should consider the plan for all the investments going forward. It is not enough to decide about the first installment of investments.

However, the uncertainty about T_i (that can be significant) may pose an additional burden. Since smart grid technologies are innovative, it is not clear how long they will last. Moreover, after making the investments the utilities may have incentives to press for shortening this period even if it is optimal to stick to a period T_i . This can happen as a result of a rate-of-return type of regulation that rewards expenses or pressure from vendors eager to sell new equipment.

6.2. Cost recovery

One of the main challenges to a level of deployment of smart grid technologies that will grant maximum net benefits to society is the uncertainty related to the ability to recoup investments. We make use of one example to illustrate the consequences of the lack of commitment from the regulator to the cost recovery of investments in these technologies.

Let $\phi(y) \in [0, 1]$ be the distribution company's belief that the regulator will allow cost recovery of its investments in smart grid technologies. This probability decreases with the level of investment y —see Fig. 4.

Denote by R the allowed return on investment. The problem of the regulated firm is to choose the level of investments y that maximizes

$$\pi(y) \equiv \phi(y) \cdot (1 + R)y + [1 - \phi(y)] \cdot 0 - y = \phi(y) \cdot (1 + R)y - y. \tag{15}$$

The optimal level of investments is:

$$y\phi'(y) + \phi(y) = \frac{1}{1 + R}. \tag{16}$$

For the sake of illustration consider a piecewise linear function $\phi(y)$ as shown in Fig. 5, given by:

$$\phi(y) = \frac{\bar{y} - y}{\bar{y} - \underline{y}} = \beta - \alpha y. \tag{17}$$

where \bar{y} and \underline{y} stand for the higher and lower levels of deployment, respectively.

¹⁸ In principle, we could have another level of investment in period T_j , but we assume that this is equal to the initial one for simplicity.

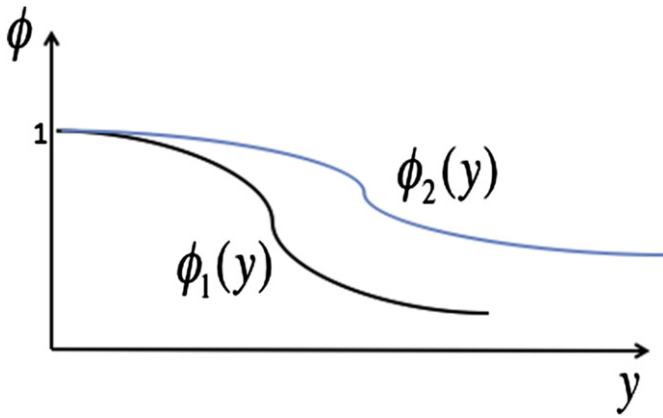


Fig. 4. $\phi(y)$ is probability of recovery of investments y .

The profit function is expressed as:

$$\pi(y) = y \cdot [(1 + R)\beta - 1] - \alpha(1 + R)y^2. \tag{18}$$

Solving for the optimal level of investments in smart grids we have:

$$y^* = \frac{\beta - \frac{1}{1+R}}{2\alpha} = \frac{\bar{y} - \underline{y}}{2(1+R)}. \tag{19}$$

Hence investment increases with β and decreases with α .

If the probability of recovery is too high, there will be over-investment (in relation to the social optimum). The policy of granting the regulated firm enough revenue such that it earns at least a normal rate of return in its invested capital—the $\phi(y) = 1$ case—may lead to the well-known Averch–Johnson problem of overcapitalization; see Newbery (2000). In turn, if the probability of cost recovery is too low there will be under-provision, reinforcing the effect that comes from the public good nature of some aspects of smart grids. Arguably, the second case is more likely. These intuitive facts are summarized by the following lemma, whose proof is omitted.

Result 2. If the probability of recovery is too high, then there is over-investment in the smart grid with respect to the optimal level y^* . If the probability of recovery is significantly small, then there will be under-investment.

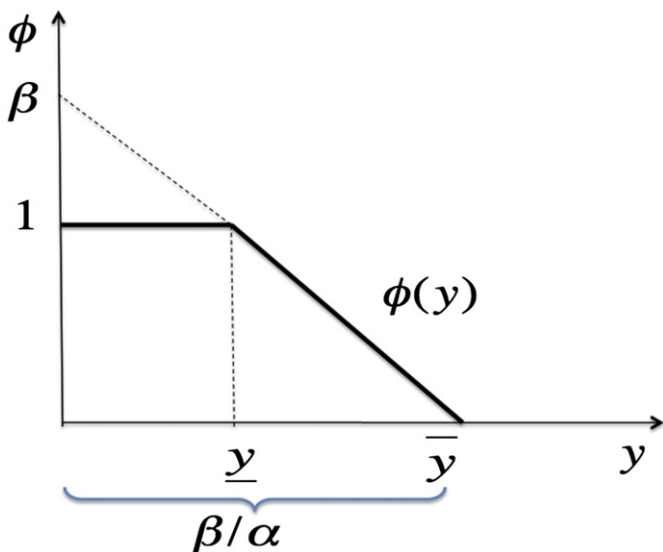


Fig. 5. A piecewise linear ϕ .

The above result can be made more precise and presented in a comparative statics language, as follows:

Lemma 6.1. Comparative statics

Assume that ϕ_1 and ϕ_2 are differentiable and satisfy $y\phi_1'(y) + \phi_1(y) < y\phi_2'(y) + \phi_2(y)$ for every $y > 0$ (see Fig. 4). Let y_1 and y_2 denote respectively the optimal levels of investments in the two cases. Then $y_1 \leq y_2$.

Proof. This can be concluded by a simple inspection of Eq. (16).¹⁹ ■

Recall that $\phi(y)$ represents the probability that the firms attach to the event that the regulator allows recovery of the investment y . This could suggest that the regulator could manipulate ϕ to achieve the optimal level of investment y^* making use of some commitment device. This is theoretically possible, but the regulator would need to know the “proper” level of investments, besides having a sharp control about the commitment device. As we observed before at the end of Section 4, this level of knowledge (not to mention the commitment) from the part of the regulator is utterly unrealistic.

Remark 6.2. In the above analysis, we assumed that $\phi(\cdot)$ is exogenously given. Alternatively, we could have obtained it from an equilibrium notion, at the expense of providing a more complicated model. For the simple point made here, we judged that this would not be necessary.

6.3. Alignment of the interests of firms and consumers

One last aspect to consider emerges when the technology choices available to the firm have equal cost but are not equivalent from the consumer's perspective. Notice that the choice of a smart grid configuration comprises a set of functionalities that differ in their ability to benefit consumers.

For instance, consider that the firm has the possibility of choosing between two smart meters. The first one includes the possibility of receiving price signals in real time and programming the consumption to vary accordingly. It also allows the consumer to sell back to the grid some energy stored in the battery of her car or produced by her solar panels (distributed generation). The second one, in comparison, is much simpler, allowing only the firm to remotely read the consumption and curtailment of the service in case the bills are not paid on time. Since this second option is simpler, it is natural to assume that $y_1 > y_2$. Notice that the two smart meters would lead to different business models to the firm, part of which would not be necessarily covered by the rate-of-return regulation. Thus, it may well happen that the profits in the first case are smaller than the second, that is, $\pi(y_1) < \pi(y_2)$.

On the other hand, the consumer experiences more services with the first option. It may well be the case that the utility u_i derived from investment y_i for $i = 1, 2$ is such that $u_1 - y_1 > u_2 - y_2$. In other words, despite being more expensive, the net benefit of the first decision is higher than the utility of the second one. In this way, the interests of consumers and firms are misaligned.

Arguably, the situation reported in this section is similar to recent experiences in some smart grid deployments in which the firms decided

¹⁹ A more detailed proof would go as follows. Let $\pi_i(y)$ be given by Eq. (15) when $\phi = \phi_i$, for $i = 1, 2$. We want to show that $y_1 \equiv \arg \max_y \pi_1(y) \leq y_2 \equiv \arg \max_y \pi_2(y)$. Suppose that this is not the case, that is, $y_1 > y_2$. By the definition of y_1 and y_2 , we have:

$$(1 + R)[\phi_1(y_1) \cdot y_1 - \phi_1(y_2) \cdot y_2] \geq y_1 - y_2 \geq (1 + R)[\phi_2(y_1) \cdot y_1 - \phi_2(y_2) \cdot y_2],$$

which implies that

$$\phi_1(y_1) \cdot y_1 - \phi_1(y_2) \cdot y_2 \geq [\phi_2(y_1) \cdot y_1 - \phi_2(y_2) \cdot y_2].$$

This can be rewritten as:

$$\int_{y_2}^{y_1} [y\phi_1'(y) + \phi_1(y)] dy \geq \int_{y_2}^{y_1} [y\phi_2'(y) + \phi_2(y)] dy,$$

which contradicts the assumption and concludes the proof.

to install smart meters that were convenient to them, but with limited capabilities for the provision of innovative services for the consumers.

7. Conclusion

This paper describes what we perceive as the main economic problems regarding smart grids: reliability, demand response and cost recovery of investments and its effect on deployment. We observed that reliability enhancement that may be achieved through the adoption of these new technologies has the characteristics of a public good, making it harder to achieve an adequate level of provision through decentralized mechanisms.

Additionally, demand response programs negatively impact generators, which may act to hinder the advancement of such programs. Some consumers will suffer with these programs, which can lower the pace or even lead to a partial adoption.

We also showed how difficult it is to achieve optimal deployment decisions and this is reinforced by uncertainties inherent to smart grids. We do not know the length of the obsolescence cycle of the new equipment. Also, firms may under-invest or choose technological solutions that benefit them, but not the society as a whole. Since the choices of smart grid deployment are complex and cannot be determined exclusively by regulators, who lack the resources and information to decide and enforce these decisions, our observations send a worrying perspective about the future of smart grid deployment.

An important part of the problem related to smart grid is the fact that most decisions should be made by heavily regulated utilities. These companies do not have the experience or the culture of investing in new and unexplored technologies, making risky choices in uncertainty environments, looking after the benefits of consumers and reaping the rewards only when they offer superior products, as seasoned entrepreneurs do. How can we nudge utilities to make the right choices, or rather, how can we design the right regulatory environment to foster these advances?

Smart grids represent a deep technological innovation in the electricity industry that may grant sizable benefits to the parties involved. But these benefits are considerably sensitive to some of the underlying choices in the adoption process. In order to enhance our understanding of the impact of these choices, we need new and innovative research that points to ways in which this potential can be realized.

Appendix A. Examples of pricing schemes for consumers

It is useful to describe some examples of the pricing schedules. The first and second are usual pricing schemes, while the remaining ones are more related to demand response.

1. *Fixed Tariff* is the most frequently adopted pricing mechanism for residential consumers. It involves high levels of cross subsidies and inefficiency. Since consumers face a flat price there is underconsumption in off peak hours (when prices are lower than costs) and overconsumption in peak hours. In this case, there is a fixed tariff $p_e \in \mathbb{R}_+$ that defines the total price of energy. On top of that, the consumer also pays a fee p_d for being connected to the distribution grid, that is,²⁰

$$p(l_i, r) = p_d + p_e \int_T l_i(t) dt. \tag{20}$$

2. With *Inclining Block Rates*, the consumer pays a different tariff for each bracket of its consumption. That is, there are points $0 = x_0 < x_1 < \dots < x_k$, tariffs $p_0, p_1, \dots, p_k \in \mathbb{R}_+$ and a piecewise linear function $f: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ defined recursively by: $f(0) = 0$ and if $x \in [x_j, x_{j+1}]$, $f(x) = p_j(x - x_j) + f(x_j)$, which defines the price functional:

$$p(l_i, r) = p_d + f\left(\int_T l_i(t) dt\right). \tag{21}$$

3. In *Time of Use (TOU)* tariffs, there are two or more time intervals (peak and off-peak) and a correspondent number of tariffs, $p_0, \dots, p_k \in \mathbb{R}_+$ and time periods, T_0, \dots, T_k such that $\cup_k T_k = T$. These tariffs are set ex ante and may significantly vary from real time prices. Hence, TOU rates are not considered dynamic and are considered poorly effective. The price functional (for two time intervals) is:

$$p(l_i, r) = p_d + p_0 \int_{T_0} l_i(t) dt + p_1 \int_{T_1} l_i(t) dt. \tag{22}$$

4. *Seasonal Rates* are pricing schemes that set rates that are different for different times of the year. Consumers face higher charges for usage in peak months and less during non-peak months. It is, therefore, formally identical to TOU mentioned above (sometimes it is considered as an instance of TOU). The only difference is that the time set there corresponds to periods within a day, while in Seasonal Rates the periods are sequences of days.
5. *Critical Peak Pricing (CPP)*: CPP is considered a dynamic pricing mechanism that may induce considerable demand response at a low transaction cost from the consumer's perspective. Under CPP, customers have to pay higher charges for all consumption above a certain threshold during peak hours. Consumers are previously notified of the peak hours, which cannot exceed an agreed limit (for instance, 100 h each year). In general, the notification takes place one day ahead of the peak. To formalize this, let E denote the CPP event (set of hours notified as peak pricing). Let l_i be the allowed consumption for consumer i under CPP events (this can be zero), p_G be the guaranteed price for this allowed consumption and p_P be the peak price. This is usually at least five times greater than the regular price, which we will denote by p_R . The price functional is then given by:

$$p(l_i, r) = p_d + p_P \int_E [l_i(t) - \min\{l_i, l_i(t)\}] dt + p_G \int_E \min\{l_i, l_i(t)\} dt + p_R \int_{T \setminus E} l_i(t) dt. \tag{23}$$

6. *Peak Time Rebate (PTR)*. PTR is formally equivalent to CPP, except for the fact that instead of facing higher rates for consumption in peak hours consumers are granted a discount for consumption in off-peak hours. Contrary to CPP, in PTR pricing schemes consumers who fail to lower their electricity consumption do not face penalties. Therefore, PTR is formally identical to CPP.
7. In a *Real Time Pricing (RTP)* mechanism consumers' charges are related to the underlying spot prices at the time of consumption. It used the real-time price $p(t) \in \mathbb{R}_+$ obtained for each $t \in T$ in the spot market. The total price is thus given by:

$$p(l_i, r) = p_d + \int_T p(t) l_i(t) dt. \tag{24}$$

An important observation to make about the above pricing schemes is that they do not depend on reliability.

²⁰ In this and in the subsequent equations, we assume that T is an interval so that the integrals make sense. It is very easy to adapt these expressions for the case in which $T = \{0, 1\}$. For instance, Eq. (20) would be just: $p(l_i, r) = p(e_i^0 + e_i^1)$ for some $p \in \mathbb{R}_+$.

References

- Amin, S.M., Wollenberg, B., 2005. Toward a smart grid: power delivery for the 21st century. *IEEE Power Energ. Mag.* 3 (5), 34–41.
- Borenstein, S., 2013. Effective and equitable adoption of opt-in residential dynamic electricity pricing. *Rev. Ind. Organ.* 42 (2), 127–160.
- Borenstein, S., Holland, S., 2005. On the efficiency of competitive electricity markets with time-invariant retail prices. *RAND J. Econ.* 36 (3), 469–493.
- Congress, 2007. Energy independence and security act of 2007. Public Law 110–140.
- Congress, 2009. The American Recovery and Reinvestment Act of 2009. Public Law 111–115.
- EPRI, 2004. Power Delivery System of the Future—A Preliminary Estimate of Costs and Benefits. Discussion Paper. Electric Power Research Institute.
- EPRI, 2011. Estimating the Costs and Benefits of the Smart Grid. Discussion Paper. Electric Power Research Institute.
- Faruqui, A., Sergici, S., 2010. Household response to dynamic pricing of electricity: a survey of 15 experiments. *J. Regul. Econ.* 1–33.
- Fox-Penner, P., 2010. *Smart Power: Climate Change, the Smart Grid, and the Future of Electric Utilities* (Island Pr.).
- Holland, S.P., Mansur, E.T., 2008. Is real-time pricing green? The environmental impacts of electricity demand variance. *Rev. Econ. Stat.* 90 (3), 550–561.
- Joskow, P., Tirole, J., 2006. Retail electricity competition. *RAND J. Econ.* 37 (4), 799–815.
- Joskow, P., Tirole, J., 2007. Reliability and competitive electricity markets. *RAND J. Econ.* 38 (1), 60–84.
- Mas-Colell, A., Whinston, M., Green, J., 1995. *Microeconomic Theory*. Oxford University Press, New York.
- Newbery, D., 2000. *Privatization, Restructuring, and Regulation of Network Utilities*, vol. 2. MIT Press.
- Roosbehani, M., Dahleh, M., Mitter, S., 2011. Volatility of Power Grids under Real-time Pricing. Arxiv preprint arXiv:1106.1401.
- Wilson, R., 1993. *Nonlinear Pricing*. Oxford University Press, USA.