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RESEARCH ARTICLE

A new approach to the rational expectations equilibrium: existence, optimality and incentive compatibility

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Abstract
Rational expectations equilibrium seeks a proper treatment of behavior under private information by assuming that the information revealed by prices is taken into account by consumers in their decisions. Typically agents are supposed to maximize a conditional expectation of state-dependent utility function and to consume the same bundles in indistiguishable states [see Allen (Econometrica 49(5):1173–1199, 1981), Radner (Econometrica 47(3):655–678, 1979)]. A problem with this model is that a rational expectations equilibrium may not exist even under very restrictive assumptions, may not be efficient, may not be incentive compatible, and may not be implementable as a perfect Bayesian equilibrium (Glycopantis et al. in Econ Theory 26(4):765–791, 2005). We introduce a notion of rational expectations equilibrium with two main features: agents may consume different bundles in indistinguishable states and ambiguity is allowed in individuals' preferences. We show that such an equilibrium exists *universally* and not only generically without freezing a particular preferences representation. Moreover, if we particularize the preferences to a specific form of the maxmin expected utility model introduced in Gilboa and Schmeidler (J Math Econ 18(2): 141–153, 1989), then we are able to prove efficiency and incentive compatibility. These properties do not hold for the traditional (Bayesian) Rational Expectation Equilibrium.

Keywords Rational expectations · Ambiguity aversion · Efficiency · Incentive compatibility

JEL Classification D50 · D81 · D82

Contents

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1 Introduction

Some of the most important developments in economics were related to modeling of information and the study of its use by agents in certain economic situations. The introduction of the rational expectations paradigm is a good example of such breakthroughs. Rational expectation equilibrium (REE) theory offers a rigorous conceptual framework to modeling the information conveyed by prices into the decision of economic agents. The fact that prices may convey useful information to market participants is well known at least since Hayek (1945). The main feature of REE is the requirement of consistency of the optimal actions of economic agents and the information that those optimal actions reveal through prices.

Agents are typically assumed to maximize a conditional expected utility function and to consume the same bundles in indistinguishable states (see Allen 1981; Radner 1979; Einy et al. 2000a, b and among others). A problem with this model is that a rational expectations equilibrium may not exist even under very restrictive assumptions. This fact was established by Kreps (1977) (see also Green 1977), through an influential and well-known example, which we revisit in Sect. 1.1 (see also Sect. 4). In seminal papers, Radner (1979) and Allen (1981) prove the *generic* existence of REE when individuals are Bayesians. Moreover, a REE may not be efficient, may not be incentive compatible,[1] and may not be implementable as a perfect Bayesian equilibrium (Glycopantis et al. 2005).

[1] Sun et al. (2013) provides a counterexample for a large economy in which REE does not possess the desirable property of incentive compatibility for each agent.

We introduce a notion of rational expectations equilibrium and prove that it exists *universally* (not generically) for a general class of preferences with ambiguity. Specializing the preferences to maxmin expected utility (MEU), we prove that the REE is also efficient and further specializing to a particular kind of MEU, we show that it is incentive compatible. We present an alternative to the traditional rational expectations equilibrium of Radner (1979) and Allen (1981) (see also Einy et al. 2000a, b; Balder and Yannelis 2009) where individuals are ambiguous and allowed to consume different bundles in states they do not distinguish. Specifically, our notion of rational expectations equilibrium differs from the traditional REE in two main perspectives. First, we allow for ambiguity in agents' consumption choices and second, we do not impose optimal allocations to fulfill the private information measurability condition.

We consider a general class of ambiguous preferences which includes as particular cases most of the popular utility representations modeled in the literature such as MEU, α-MEU, invariant biseparable preferences, smooth preferences, ϵ-contamination, variation preferences among others. Clearly, it also includes the Bayesian formulation so that our solution even specializes to the setting with no ambiguity.

We prove the existence of a REE with this general class of preferences. For the existence it is crucial to not impose any private information measurability restriction. Indeed, the second important departure of our model from the literature (Allen 1981; Einy et al. 2000a, b) is that agents are not always able to infer the state from the endowments they receive or from the consumption goods they consume (see Sects. 2.3 and 3.1 for details).

The lack of measurability requirement is particularly motivated in those economies, we will mainly refer to throughout the paper, in which all markets participants have maxmin preferences. Indeed, according to the maxmin formulation, any agent considers the worst possible scenario, that is she expects to receive the bundle which minimizes her welfare. Therefore, she is indifferent among bundles in indistinguishable states, because whatever she will receive ex post, she is sure to obtain something ensuring her the lowest possible bound of happiness. The following reexamination of the financial example introduced by Kreps (1977) clarifies our existence result.

The idea is to consider a family of associated complete information economies (one for each state) and show that any selection from its Walrasian equilibrium correspondence is a REE. Computing the Walrasian equilibrium, two possibilities arise: either prices are fully revealing, i.e., different complete information economies have different market clearing prices, in which case the equilibrium exists and is equivalent to the traditional rational expectations equilibrium a la Allen (1981) and Radner (1979); or prices are partially revealing, i.e., different complete information economies have the same equilibrium prices. In this case, a selection out of the Walrasian equilibrium correspondence is found that guarantees market clearing. The selected allocation may not be private information measurable. From the existence of an *ex post* Walrasian equilibrium we deduce the non emptiness of the set of REE with a general class of preferences. We show that this inclusion is strict but it becomes an equivalence under an additional requirement satisfied by the Bayesian expected utility function and not by the MEU. To study the efficiency and incentive compatibility of a REE we specialize preferences to the maxmin expected utility and prove that in this case the REE is indeed efficient and incentive compatible. We compare the notion of maxmin REE with the traditional one by using the example of Kreps (1977). We show that the two notions are in general different. We illustrate some properties of the maxmin REE and list the assumptions guaranteeing its efficiency and its incentive compatibility. Several examples complete the analysis by underling the role of each assumption.

1.1 Kreps' example

Kreps (1977) provides a simple financial example that allows us to understand the heart of our contribution. He assumes that there are two assets: a riskless asset that costs and pays 1 and a risky asset that is sold at period $t = 1$ by the price $p(\omega) \in \mathbb{R}_+$ and pays $V(\omega)$ in period $t = 2$, where ω denotes the state of the world. There are two individuals, both with utility $U(c) = -e^{-c}$ for the consumption of c units at $t = 2$. Individual 1 knows whether $V(\omega)$ is distributed according to a normal with mean m_1 and variance σ^2 or according to a normal with mean m_2 and variance σ^2. Let s_1 denote the first distribution and s_2, the second. That is, individual 1 knows which distribution s_j $(j = 1, 2)$ governs $V(\omega)$. On the other hand, individual 2 only knows that the distribution governing $V(\omega)$ is in the set $S \equiv \{s_1, s_2\}$, but she can infer s once she observes the prices.[2] To complete the description, assume that individual i is endowed with k_{ij} units of the risky asset if s_j occurs, for $i, j \in \{1, 2\}$. Endowments of the riskless asset are constant and, therefore, ignored.

Now if an individual knows s and buys q units of the risky asset, her consumption will be $x(\omega) = -p(\omega) \cdot q + (q + k_{i\cdot}) \cdot V(\omega)$, leading to the expected utility:

$$u_i(s, x) = E_s \{-\exp[-(-p \cdot q + (q + k_{i\cdot}) \cdot V)]\}, \tag{1}$$

where E_s denotes expectation with respect to $s \in \{s_1, s_2\}$. As natural, we assume that the price $p(\omega)$ depends only on s and write $p(\omega) = p_j$ if $s = s_j$, $j = 1, 2$. Given the normality of the risky asset returns, we have for $j = 1, 2$:

$$u_i(s_j, x) = -\exp\left[pq - m_j\left(q + k_{ij}\right) + \frac{\sigma^2}{2}\left(q + k_{ij}\right)^2\right], \tag{2}$$

which leads to the following optimal quantity if the individual knows which s obtains:

$$q_{ij} = \frac{m_j - p_j}{\sigma^2} - k_{ij}, \quad \text{for } i = 1, 2 \text{ and } s = s_j, \; j = 1, 2. \tag{3}$$

Let us consider the case in which *both individuals are Bayesian*. If individual 2 is uniformed, that is, $p_1 = p_2$, then she considers a mixture of normals (s_1 and s_2). In any case, her optimal choice, although not given by (3), is a *single quantity* $q_{21} = q_{22}$. Kreps first observes that if $m_1 \neq m_2$ and $k_{1j} = 0$, for $j = 1, 2$ then prices cannot be uninformative, that is, we cannot have $p_1 = p_2$. Indeed, in this case $q_{11} \neq$

[2] Nothing changes in the analysis if we assume that individual 2 considers all convex combinations of s_1 and s_2 as possible.

q_{12}, but since $q_{2j} = -q_{1j}$, this would imply $q_{21} \neq q_{22}$, contradicting the previous observation.[3]

Thus, assume that $p_1 \neq p_2$ and individual 2 is informed, that is, all choices are given by (3). Kreps notes that if $m_1 = 4$, $m_2 = 5$, $k_{21} = 2$, $k_{22} = 4$ and $\sigma^2 = 1$, then $p_1 = p_2 = 3$, which contradicts $p_1 \neq p_2$. This contradiction shows that no rational expectations equilibrium exists.

Let us now observe what happens with the MEU formulation. Under full information, there is no ambiguity and the individuals' behaviors are exactly as above. However, in the case that 2 is uniformed ($p_1 = p_2$), then she faces ambiguity and takes the worst-case scenario in her evaluation. She is, therefore, indifferent among a set of different quantities q_{ij}; in particular, she is indifferent among quantities that promises utilities above the minimum between the two states.[4] Which among her equally good quantities will be selected? It is standard to think that a Walrasian auctioneer selects the quantity that clears the market, but the information about the quantity chosen by the Walrasian auctioneer is available to the individual only after all choices are made and, therefore, cannot affect her behavior. This means that the restriction $q_{21} = q_{22}$ used above no longer holds. She could receive different quantities on different states. For example, an equilibrium with the above parameters would be $p_1 = p_2 = 3$ and $q_{21} = -1$ and $q_{22} = -2$.

Remark 1.1 Notice that in the original Kreps' model, private information measurability of the quantities plays a crucial role in the failure of existence. Basically, the problem is that if prices do not reveal information, we may end up requiring that quantities bought in different states be different, but this is possible only if prices do reveal information. The requirement of measurability makes sense if each individual is buying the quantity itself, but it is natural to dispense with this restriction if we see this as a negotiation of contracts in the interim period, whose quantities are finally determined in the *ex post* period. Relaxation of private information measurability is also an important ingredient in our theory (please see discussion in Sect. 3.1).

1.2 Relevant literature

With respect to the traditional REE literature, it is well known by now that a REE as formulated by Radner (1979), Allen (1981) and Grossman (1981) exists only generically.

Condie and Ganguli (2011a) extends the Radner (1979) model to ambiguous agents' preferences which are represented by the Choquet expected utility with a convex capacity. They provide the existence of fully revealing REE for almost all sets of beliefs. Condie and Ganguli (2011b) study the existence problem of partially revealing REE when at least one individual has ambiguity averse preferences. Our model mainly differs from Condie and Ganguli (2011a, b) in the way the information is treated. We describe the information via a partition of the state space as used by Radner

[3] Another way of describing the same problem is to think that the decision on quantities is measurable with respect to the information partition that the individual has after observing prices.

[4] Note that she is indifferent taking in account the information that she has when making decisions. Obviously, she is *not* indifferent *ex post*.

(1968) and Allen (1981). In contrast, Radner (1979) and Condie and Ganguli (2011a) use a model based on signals. In particular, Radner (1979) and Condie and Ganguli (2011a) fix a state-dependent utility and specify various economies by the appropriate notion of conditional beliefs. Radner (1979) describes signals as providing information on the conditional probability distribution over a set of states. All information in Radner (1979) is obtained by knowing everyone's joint signal. As such, the partitions observable by traders are over the space of joint signals as opposed to the state space over which consumption occurs. Radner calls these consumption states the "payoff-relevant part of the environment" (page 659). If an individual receives signal t_i then she knows that the joint signal is in the set of joint signals for which she receives the signal t_i. This imposes additional structure on the types of partitions over the signal space that agents observe.

In Tallon (1998) individuals' conditional belief are only superadditive. Consequently even when the equilibrium price is fully revealing, uninformed agents, perceiving ambiguously the price signal, may not know which state occurred; and therefore it may be worthwhile for them to buy "redundant" information. On the contrary in our model a fully revealing REE and equilibria of the full information economy coincide.

Polemarchakis and Siconolfi (1993) prove the existence of noninformative REE in which prices convey no information. They consider only nominal assets and reduce the existence of noninformative REE to the existence of competitive equilibria in a particular economy with restricted participation in the asset markets. Crucial and also restrictive is the assumption of signal invariance of the utility function, without which the existence of noninformative equilibrium is not guaranteed.

Citanna and Villanacci (2000) do not restrict the analysis of fully revealing or non revealing REE but they consider a partially revealing REE by allowing any degree of information revelation through the prices. Balder and Yannelis (2009) show that if agents correctly predict equilibrium prices (every agent has her own price estimate based on her own private information), then a REE exists universally.

In a series of papers Correia-da Silva and Hervés-Beloso (2008, 2009, 2012, 2014), to whom we refer with CH, introduce economies with uncertain delivery, where:

> "instead of choosing bundles, agents choose lists of bundles out of which the market then selects one bundles for delivery".

We keep their idea of uncertainty delivery in the sense that each individual, given an event of her information partition (informational signal), submits a demand correspondence (not necessarily a function) and accepts to get *ex post* any of the bundles contracted for delivery in indistinguishable states belonging in the given event. The selection must guarantee market clearing.

In spite of this similarity, our paper differs from the works by CH in many aspects. We consider an interim model and we focus on the notion of rational expectations equilibrium according to which agents trade is based on their private information and on the information revealed by prices. This is not captured by CH who instead study models in which trade takes place ex ante. Indeed they write "an agent cannot infer the information of the others because, at the moment of trade, the other agents still have not received their information. From the deliveries made at date 1, agents could be able to infer the true state of nature. But we assume that the information obtained through these inferences cannot be used (in a court of law, for example) to enforce contracts." [see Footnote 4 in Correia-da Silva and Hervés-Beloso (2012)].

This difference is clear in the example illustrated in the Appendix 2 of Correia-da Silva and Hervés-Beloso (2012), in which their equilibrium does not exist, while a rational expectations equilibrium, as defined in our paper, exists. One can easily construct additional examples to show that our REE concept is different that the one of CH. As a matter of fact, the CH does not capture the idea of REE as in their set-up agents do not condition their expectations on the information the equilibrium prices generate, which is the basic feature of a REE notion.

We continue the research project begun by de Castro and Yannelis (2008) who show that only MEU preferences solve the conflict arising between efficiency and incentive compatibility. A similar framework is used in de Castro et al. (2011) in order to prove the existence of core allocations and equilibria in ex ante and interim asymmetric information economies. Efficiency and incentive compatibility properties of equilibria are also analyzed. In de Castro et al. (2011), contrary to this paper, agents do not consider the information revealed by the prices. He and Yannelis (2015) extend de Castro et al. (2011) to economies with countably many states of nature and also prove the Core-Walras equivalence theorem. Zhiwei (2014) and Zhiwei (2016) consider our REE solution concept and study respectively the ex ante efficiency properties and the implementation problem of a maxmin REE as a maxmin equilibrium (see de Castro et al. 2017 for an extension). In this paper we do not consider ex ante models, we do not deal with cooperative solution concepts (i.e., core or value), neither with the implementation problem. It should be pointed out that our work differs from all the above papers as we focus on universal existence of REE and also its efficiency and incentive compatibility, that have not been examined up to now. Also, we do not provide any applications in finance, although in view of recent work (see Faria and Correia-da Silva 2012; Ma et al. 2008; Yi et al. 2015; Munk and Rubtsov 2014) indicates that our modeling is applicable to financial markets.

1.3 Organization of the paper

The paper is organized as follows: in Sect. 2 we describe the economic model and the general class of agents' preferences which may allow for ambiguity. In Sect. 3 we adapt to our framework a very general rational expectations equilibrium notion, called V-REE, and establish its existence. In Sect. 4 we particularize agents' preferences to the maxmin expected utility. We compare the traditional notion of REE with the maxmin REE for which we also provide some properties. Sects. 5 and 6 deal respectively with the efficiency and incentive compatibility of maxmin REE. Some open questions are collected in Sect. 7. The "Appendix" collects longer proofs and useful examples.

2 Model: asymmetric information economy

We use the following notations. For two vectors $x = (x^1, \ldots, x^\ell)$ and $y = (y^1, \ldots, y^\ell)$ in $\mathbb{R}^\ell$, we write $x \geq y$ when $x^k \geq y^k$ for all $k \in \{1, \ldots, \ell\}$; $x > y$ when $x \geq y$ and $x \neq y$; and $x \gg y$ when $x^k > y^k$ for all $k \in \{1, \ldots, \ell\}$. A function $u : \mathbb{R}^\ell_+ \to \mathbb{R}$ is (strictly) monotone if for all $x, y \in \mathbb{R}^\ell_+$, $(x > y)$ $x \gg y$ implies that $u(x) > u(y)$; and it is (strictly) quasi-concave if for all $x, y \in \mathbb{R}^\ell_+$ and all $\alpha \in (0, 1)$ we have that $(u(\alpha x + (1-\alpha)y) > \min\{u(x), u(y)\}$ if $x \neq y)$ $u(\alpha x + (1-\alpha)y) \geq \min\{u(x), u(y)\}$. Given two sets A and B, the notation $A \backslash B$ refers to the set-theoretic difference, i.e., $A \backslash B = \{a : a \in A \text{ and } a \notin B\}$.

2.1 Asymmetric information economy

We consider an exchange economy with uncertainty and asymmetrically informed agents. The uncertainty is represented by a measurable space $(S, \mathcal{F})$, where S is a finite set of possible states of nature and $\mathcal{F}$ is the algebra of all the events, i.e., $\mathcal{F}$ is the power set of S. Let $\mathbb{R}^\ell_+$ be the commodity space and I be a set of n agents, i.e., $I = \{1, \ldots, n\}$. **An asymmetric information exchange economy** $\mathcal{E}$ is the following collection:

$$\mathcal{E} = \{(S, \mathcal{F});\ (\mathcal{F}_i, u_i, e_i)_{i \in I}\},$$

where for all $i \in I$

- $\mathcal{F}_i$ is a partition of S, representing the **private information** of agent i. The interpretation is as usual: if $s \in S$ is the state of nature that is going to be realized, agent i observes $\mathcal{F}_i(s)$, the unique element of $\mathcal{F}_i$ containing s. By an abuse of notation, we still denote by $\mathcal{F}_i$ the algebra generated by the partition $\mathcal{F}_i$.
- a **random utility function** (or state-dependent utility) representing her (*ex post*) preferences $u_i : S \times \mathbb{R}^\ell_+ \to \mathbb{R}$. We assume that for all $s \in S$, $u_i(s, \cdot)$ is continuous and monotone.[5]
- a **random initial endowment** of physical resources represented by a function $e_i : S \to \mathbb{R}^\ell_+$.

Given a vector-valued random variable $f : (S, \mathcal{F}) \to (\mathbb{R}^\ell, \mathcal{B}(\mathbb{R}^\ell))$, where $\mathcal{B}(\mathbb{R}^\ell)$ is the σ-field of Borel subsets of $\mathbb{R}^\ell$, let $\sigma(f)$ denote the smallest sub-algebra of $\mathcal{F}$ for which $f(\cdot)$ is measurable. For some results, we need to assume that $\sigma(u_i) \subseteq \mathcal{F}_i$ or $\sigma(e_i) \subseteq \mathcal{F}_i$ referring respectively to $u_i(\cdot, t)$ is $\mathcal{F}_i$-measurable for all $t \in \mathbb{R}^\ell_+$ and $e_i(\cdot)$ is $\mathcal{F}_i$-measurable. Finally $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ means that $\sigma(u_i) \subseteq \mathcal{F}_i$ and $\sigma(e_i) \subseteq \mathcal{F}_i$.

A **price** p is a function from S to $\mathbb{R}^\ell_+$. In some other papers, a price is defined as a non-zero function from S to $\mathbb{R}^\ell_+$, meaning that for some s (not necessarily for all s) $p(s) > 0$. However, with standard arguments it can be proved that if there is at least one agent i such that $u_i(s, \cdot)$ is monotone for all $s \in S$, then the equilibrium price p is positive in any state (i.e., $p(s) > 0$ for any $s \in S$). Moreover, if $p : S \to \Delta$,

[5] It is known that if $u_i(s, \cdot)$ is continuous and monotone, then it is also monotonically increasing, i.e., $x \geq y$ implies $u_i(s, x) \geq u_i(s, y)$.

where Δ is the $(\ell - 1)$-dimensional unit simplex in $\mathbb{R}^\ell_+$, (as defined for example in Allen 1981) then in particular $p(s) > 0$ for any $s \in S$. Since throughout the paper we assume that $u_i(s, \cdot)$ is monotone for all $s \in S$ and all $i \in I$, the equilibrium price p is positive in each state, i.e., $p : S \to R^\ell_+ \backslash \{0\}$. The algebra $\sigma(p)$ represents the information generated by the price function p. We denote by $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$ the smallest algebra containing both $\mathcal{F}_i$ and $\sigma(p)$.

A function $x : I \times S \to \mathbb{R}^\ell_+$ is said to be a **random consumption vector** or **allocation**. For each i, the function $x_i : S \to \mathbb{R}^\ell_+$ is said to be an allocation[6] of agent i, while for each s, the vector $x_i(s) \in \mathbb{R}^\ell_+$ is a bundle of agent i in state s. An allocation x is said to be **feasible** if $\sum_{i \in I} x_i(s) = \sum_{i \in I} e_i(s)$ for all $s \in S$.

We will describe the agents' preferences below. The above structure, including each agent's preference, is common knowledge for all agents.

2.2 Preferences

In this section, we discuss preferences which may allow for ambiguity. Given, any event $F \in \mathcal{F}$ and any two functions $f, g : S \to \mathbb{R}$, we consider the function $V(\cdot|F)$ such that

$(A1)$ $V(f|\{s\}) = f(s)$ for any $s \in S$;
$(A2)$ $f(s') \geq g(s')$ for some $s' \in F$ implies $V(f|F) \geq V(h|F)$, where

$$h(s) = \begin{cases} g(s') & \text{if } s = s' \\ f(s) & \text{otherwise.} \end{cases}$$

Let $\Pi = (\Pi_i)_{i \in I}$ be an **information structure**, meaning that for any agent i, Π_i is a partition of S. In particular, if $\Pi_i = \mathcal{F}_i$ for any $i \in I$, then the information structure is the initial private information. Let F be an event of Π_i for some agent i (i.e., $F = \Pi_i(s)$ for some state s) and f and g be the utility of i at two different allocations x and y (i.e., $f(s) = u_i(s, x(s))$ and $g(s) = u_i(s, y(s))$), then V represents the interim preferences of agent i while conditions $(A1)$ and $(A2)$ are the consistency requirements between interim and *ex post* preferences. For the rest of this section we avoid to use the subscript i referring to any agent. With an abuse of notation, for any allocation $x : S \to \mathbb{R}^\ell_+$ and any event $F \in \mathcal{F}$, $V(x|F)$ denotes $V(u(\cdot, x(\cdot))|F)$. Notice that $(A2)$ is a monotonicity condition meaning that if $V(y|F) > V(x|F)$ then $u(s', y(s')) > u(s', x(s'))$ for some state $s' \in F$ [see also Axiom 4 in de Castro et al. 2011].

Most of the typical ambiguity preferences representation in finance and in economics obeys the two conditions above. Moreover, V is not restricted to ambiguous preferences as it can be the standard **(Bayesian) interim expected utility** function once there is a unique known probability π on $\mathcal{F}$ with $\pi(s) > 0$ for any s, i.e.,

$$V(x|F) = E_\pi(u(\cdot, x(\cdot))|F) = \sum_{s \in F} u(s, x(s)) \frac{\pi(s)}{\pi(F)}. \tag{4}$$

[6] For simplicity, we will often use the symbol $x_i(s) \in \mathbb{R}^\ell_+$ to denote $x(i, s) \in \mathbb{R}^\ell_+$. Similarly, $x_i(\cdot)$ refers to the function $x(i, \cdot) : S \to \mathbb{R}^\ell_+$. Finally, $x(s)$ refers to the function $x(\cdot, s) : I \to \mathbb{R}^\ell_+$.

Let $\mathcal{C}^F$ be the set of all probabilities with support contained on F and $\mathcal{M}^F$ be a non empty, closed and convex subset of $\mathcal{C}^F$ containing the priors of the agent.[7] If an agent is ambiguous on the set $\mathcal{M}^F$ and she considers the worst possible scenario when evaluating her payoff, then for any two allocations x, y, she prefers x to y if

$$\inf_{\mu \in \mathcal{M}^F} E_\mu \left[u(\cdot, x(\cdot)) \right] \geq \inf_{\mu \in \mathcal{M}^F} E_\mu \left[u(\cdot, y(\cdot)) \right].$$

Thus, for any allocation x the utility V with respect to the information Π in state s is:

$$V(x|F) = \inf_{\mu \in \mathcal{M}^F} E_\mu \left[u(\cdot, x(\cdot)) \right], \text{ where } F = \Pi(s). \tag{5}$$

In the case of a state-independent utility, (5) represents the seminal conditional preferences in the Gilboa-Schmeidler form. For this reason we call it **maxmin expected utility (MEU)** and we denote it by $\underline{u}^\Pi(s, x)$.

Remark 2.1 If $\mathcal{M}^F$ is a singleton set then the maxmin expected utility (MEU) reduces to the standard Bayesian expected utility (4). If $\mathcal{M}^F = \mathcal{C}^F$ then it is the maxmin expected utility considered in de Castro and Yannelis (2008) where it is shown that

$$V(x|F) = \inf_{\mu \in \mathcal{C}^F} E_\mu \left[u(\cdot, x(\cdot)) \right] = \min_{s' \in F} u(s', x(s')). \tag{6}$$

It is proved in de Castro and Yannelis (2008) that efficient allocations are incentive compatible if and only if individuals' preferences are represented by (6).

Whenever for each agent i the partition Π_i is her initial private information $\mathcal{F}_i$, then we do not use the superscript, i.e., $\underline{u}_i(s, x_i) = \min_{s' \in \mathcal{F}_i(s)} u_i(s', x_i(s'))$. When we deal with the notion of rational expectations equilibrium according to which each agent i takes into account also the information that the equilibrium prices generate, F is an event of the information partition $\mathcal{G}_i^p$, where $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$. In this case the utility V is denoted by $V(x_i|F) = \underline{u}_i^{REE}(s, x_i) = \min_{s' \in \mathcal{G}_i^p(s)} u_i(s', x_i(s'))$.

We now observe that conditions $(A2)$ and $(A1)$ above are also satisfied by further extensions of the MEU model that allow for a distinction between ambiguity and ambiguity aversion.

Ghirardato et al. (2004) introduced the **invariant biseparable preferences** represented by the following function adapted to our framework [see Lemma 1 and Theorem 11 in Ghirardato et al. (2004)].

$$V(x|F) = \alpha([x]) \min_{\mu \in \mathcal{M}^F} \sum_{s \in F} u(s, x(s))\mu(s) + (1 - \alpha([x])) \max_{\mu \in \mathcal{M}^F} \sum_{s \in F} u(s, x(s))\mu(s), \tag{7}$$

where $\mathcal{M}^F$ is a non empty, compact and convex set of probabilities on 2^F, $[x]$ is the set of $y : F \to \mathbb{R}_+^\ell$ such that $\sum_{s \in F} u(s, x(s))\mu(s)$ is a positive affine transformation

[7] In particular if $\mu \in \mathcal{C}^F$, then $\mu(s') = 0$ for any $s' \notin F$ and $\sum_{s' \in F} \mu(s') = 1$. For applications of those preferences see Ravanelli and Svindland (2019).

of $\sum_{s\in F} u(s, y(s))\mu(s)$ for all $\mu \in \mathcal{M}^F$; and $\alpha([x]) \in [0, 1]$. Clearly the MEU preference model and more generally the α-MEU preference model, in which $\alpha([\cdot])$ is respectively constantly equal to one or equal to $\alpha \in [0, 1]$, are special cases of the above representation. Moreover, if $\mathcal{M}^F = \{\pi\}$ then the function $\alpha([\cdot])$ disappears and (7) reduces to (4). Notice that (7) satisfies condition $(A2)$; moreover once $F = \{s\}$, the set $\mathcal{M}^F$ contains only the measure μ assigning zero to $\emptyset$ and one to $\{s\}$ and hence (7) also obeys condition $(A1)$ above.

Klibanoff et al. (2005) introduced the **smooth ambiguity model** that, with suitable adaptations to our framework, represents individuals' preferences by the function

$$V(x|F) = \sum_{\mu\in\mathcal{C}^F} \Phi\left(\sum_{s\in F} u(s, x(s))\mu(s)\right)\beta(\mu), \tag{8}$$

where $\mathcal{C}^F$ is the set of all probability measures with support on F, $\Phi : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function reflecting attitude towards ambiguity and β is a probability measure on $\mathcal{C}^F$. Since u and Φ are monotone, (8) satisfies condition $(A2)$ above. Moreover, once F is a singleton $\{s\}$, the set $\mathcal{C}^F$ contains only one prior μ and consequently $\beta(\mu) = 1$. Thus, (8) also satisfies condition $(A1)$ if Φ is the identity function $\Phi(t) = t$.

Variational preferences introduced by Maccheroni et al. (2006) and adapted to our framework have the following representation

$$V(x|F) = \min_{\mu\in\mathcal{C}^F} \left\{\sum_{s\in F} u(s, x(s))\mu(s) + c(\mu)\right\}, \tag{9}$$

where $\mathcal{C}^F$ is the set of all the probability measures with support on F and $c : \mathcal{C}^F \to [0, +\infty)$ is a convex function on $\mathcal{C}^F$. **Multiplier preferences** introduced by Hansen and Sargent (2008) are the special case of (9) where c is the relative entropy. The monotonicity condition $(A2)$ trivially holds, and $(A1)$ is also verified for $c(\mu) = 0$, where μ is the unique measure assigning zero to the event $\emptyset$ and one to the event $F = \{s\}$.

A further ambiguity model consists in taking the convex combination of two quantities: first (6), the minimum expected utility with respect to all possible probability measures with support on F and second (4), the expected utility with respect to a particular probability measure in this set. The following function represents in our framework the so-called ϵ**-contamination preferences**

$$V(x|F) = \epsilon \min_{s\in F} u(s, x(s)) + (1-\epsilon)\sum_{s\in F} u(s, x(s))\pi(s). \tag{10}$$

Remark 2.2 In Sect. 3 we introduce a notion of rational expectations equilibrium and prove its existence with the general function V satisfying only conditions $(A2)$ and $(A1)$ above in economies with strictly positive endowment and quasi-concave (ex post) utility function. Moreover, by strengthening $(A2)$ our notion coincides with the *ex post* Walrasian equilibrium (see Proposition 3.4). We prove the efficiency of the REE for an expected utility represented by (5) and the incentive compatibility for an expected utility represented by (6).

2.3 Timing

We can specify the timing of the economy as follows. There are three periods: ex ante ($t = 0$), interim ($t = 1$) and *ex post* ($t = 2$). Although consumption takes place only at the *ex post* stage, the other events occur as follows:

- At $t = 0$, the state space, the partitions, the structure of the economy and the price function $p : S \to \mathbb{R}^{\ell}_{+}$ are common knowledge. This stage does not play any role in our analysis and it is assumed just for a matter of clarity.
- At $t = 1$, a particular state of nature s has obtained. Each individual learns her private information signal $\mathcal{F}_i(s)$ and the prevailing price $p(s) \in \mathbb{R}^{\ell}_{+}$. Therefore, she learns $\mathcal{G}^p_i(s)$, where $\mathcal{G}^p_i = \mathcal{F}_i \vee \sigma(p)$, meaning that she only knows that one of the states $s' \in \mathcal{G}^p_i(s)$ obtains but not exactly which. With this information, the individual plans how much she wish to consume in any indistinguishable states $s' \in \mathcal{G}^p_i(s)$ and evaluates the utility of this choice by means of the function V (conditional to the event $\mathcal{G}^p_i(s)$). Since her actual consumption, as her endowment, may be contingent to the realized state of nature, she need to be sure that she will be able to pay her consumption plan $x_i(s')$ for all $s' \in \mathcal{G}^p_i(s)$. This leads to the following budget set

$$B_i(s, p) = \left\{ y_i \ : \ p(s') \cdot y_i(s') \le p(s') \cdot e_i(s') \ \text{ for all } s' \in \mathcal{G}^p_i(s) \right\}. \tag{11}$$

  In particular an individual submits a demand correspondence assigning to each $s \in S$ the set $\{x_i(s'), s' \in \mathcal{G}^p_i(s)\}$ of contingent bundles in indistinguishable states. This contract gives to i the right to consume *ex post* one bundle of the demand set which contains indifferent alternatives for i at the time of contracting. Obviously, the individual is not indifferent *ex post*. The interpretation of this model is that the plan that the individual makes at the interim stage ($t = 1$) serves as the channel through which her information is passed to the system, or to the "Walrasian auctioneer," if one prefers. This is necessary for the purpose of aggregation of information among the individuals and to guarantee the feasibility of the final allocations.
- At $t = 2$, whether or not the "Walrasian auctioneer" delivers truthfully, individual i receives and consumes one of the bundles $x_i(s')$ with $s' \in \mathcal{G}^p_i(s)$, not necessarily $x_i(s)$.[8] Thus, agents are allowed to consume different bundles in indistiguishable states.

[8] The information about the bundle chosen by the "Walrasian auctioneer" is available to the individual i only after all choices are made and, therefore, cannot affect her behavior.

3 Rational expectations equilibrium and its existence

This section adopts a variant of a rational expectations equilibrium (REE) notion introduced by Allen (1981), where agents' preferences are represented by the function V and without the private information measurability restrictions on allocations. We show that such a REE equilibrium (universally) exists under mild assumptions.

Definition 3.1 A **V-rational expectations equilibrium (V-REE)** is a pair (p, x), where p is a price function and x is a feasible allocation such that

(i) for all $i \in I$ and all $s \in S$, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$, and
(ii) for all $i \in I$ and for all $s \in S$, $V_i(x_i|\mathcal{G}_i^p(s)) = \max_{y_i \in B_i(s,p)} V_i(y_i|\mathcal{G}_i^p(s))$.

Once agents' preferences are represented by (6), the pair (p, x) is called **maxmin REE**. Notice that contrary to the traditional REE notion the optimal allocation is not required to be measurable with respect to the private information and the information generated by the equilibrium price. This allows agents to consume different bundles in indistinguishable states. However, the private information measurability of allocations requirement is automatically satisfied under strict concavity assumption [see Proposition 4.3 in this paper, Theorem A.6 in Allen (1981) and Section 5 of Kreps (1977)]. Definition 3.1 includes as particular case the traditional notion of REE, when ambiguity does not matter, as the function V is the (Bayesian) expected utility function and $\sigma(x_i) \subseteq \mathcal{G}_i^p$, $\sigma(e_i) \subseteq \mathcal{F}_i$ are imposed for any agent $i \in I$.

Either a traditional (Bayesian)[9] REE or a V-REE are said to be (i) *fully revealing* if the equilibrium price reveals to each agent all states of nature, i.e., $\sigma(p) = \mathcal{F}$; (ii) *non revealing* if the equilibrium price reveals nothing, that is $\mathcal{G}_i^p = \mathcal{F}_i$ for all $i \in I$ or, equivalently, if $\sigma(p) \subseteq \bigwedge_{i \in I} \mathcal{F}_i$; finally (iii) *partially revealing* if the equilibrium price reveals some but not all states of nature, i.e., $\bigwedge_{i \in I} \mathcal{F}_i \subset \sigma(p) \subset \mathcal{F}$. We first notice that whenever the equilibrium price p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$, since $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$, it follows that $\mathcal{G}_i^p = \mathcal{F}$ for all $i \in I$. Thus, for any state $s \in S$ and any agent $i \in I$, $\mathcal{G}_i^p(s) = \{s\}$, and hence $E_\pi(u_i(\cdot, x_i(\cdot))|\mathcal{G}_i^p(s)) = u_i(s, x_i(s))$ as well as $V_i(x_i|\mathcal{G}_i^p(s)) = u_i(s, x_i(s))$ [see $(A1)$]. Moreover, the $\mathcal{G}_i^p$-measurability assumption of the traditional REE allocations plays no role. Therefore, fully revealing traditional REE and fully revealing V-REE coincide, as they both become *ex post* Walrasian equilibrium.

3.1 Private information measurability

In the literature on the Bayesian rational expectations equilibrium, for any agent i the endowment e_i and the optimal allocation x_i are required to be private information measurable, i.e., $\sigma(e_i, x_i) \subseteq \mathcal{G}_i^p$ (see for example Allen 1981; Einy et al. 2000a, b). This means that at time $t = 1$, once a particular state of nature s is realized, agent i, observing also her endowment $e_i(s) \in \mathbb{R}_+^\ell$, can choose the optimal bundle $x_i(s)$ among those $z \in \mathbb{R}_+^\ell$ satisfying the budget inequality $p(s) \cdot z \leq p(s) \cdot e_i(s)$. Moreover, since $\sigma(x_i) \subseteq \mathcal{G}_i^p$ agent i knows perfectly the bundle x_i she is purchasing but she is

[9] By traditional in this paper we mean Bayesian.

uncertain about the welfare she will derive from its consumption, since she only knows that she will derive one of the utilities $u_i(s', x_i)$ with $s' \in \mathcal{G}_i^p(s)$.

An important departure of our model from the literature (Allen 1981; Einy et al. 2000a, b) is that agents are allowed to consume different bundles in indistinguishable states. In other words, the private information measurability conditions are not imposed. Despite this, in our model the selection of a consumption vector from agents' demand correspondence does not reveal additional information to the agents beyond their private information and the information transmitted by the price, because agents receive the optimal bundle only after all choices are made and therefore their behavior is not affected.

The lack of the private information measurability of allocations makes sense in economies in which agents have maxmin preferences. Suppose, for example, that $S = \{a, b, c\}$ and for some $i \in I$, $\mathcal{G}_i^p = \{\{a, b\}, \{c\}\}$ and $x_i : S \to \mathbb{R}_+^\ell$ is an allocation for i. If a is the realized state of nature, agent i receives the informational signal $\{a, b\}$, meaning that she is not able to understand which states between a and b is realized. Since, according to the maxmin expected utility, $i \in I$ considers the worst possible scenario, she expects to receive the bundle $x_i(s)$ such that $u_i(x_i(s)) = \min\{u_i(x_i(a)); u_i(x_i(b))\}$. Therefore, she is indifferent between $x_i(a)$ and $x_i(b)$ because, whatever she will receive *ex post*, she is sure to obtain something ensuring her the lowest possible bound of happiness. Moreover, if we impose allocations to be private information measurable, in the event $\{a, b\}$ agent i is obligated to consider the same bundle in states a and b meaning that $x_i(a) = x_i(b)$. But since she always considers the worst possible scenario, nothing really changes because from the maxmin point of view the private information measurability makes just a meaningless restriction. A similar assumption is made in Correia-da Silva and Hervés-Beloso (2009), who allow agents to choose a plan of lists of bundles and to consume one of the bundles in the list, where agents' lists are merely non empty finite subsets of R_+^ℓ.

The lack of the private information measurability is particularly motivated in economies where all individuals have MEU preferences because de Castro and Yannelis show that every efficient allocation is incentive compatible if and only if all individuals have maxmin preferences (see de Castro and Yannelis 2008). It is known that in ex ante expected utility model (e.g. Radner 1968 and Yannelis 1991), in the one good case the private information measurability of allocations is a necessary and sufficient condition to ensure that trades are incentive compatible (e.g., Krasa and Yannelis 1994; Koutsougeras and Yannelis 1993; Podczeck and Yannelis 2008), and in the multi good case it is a sufficient condition to ensure incentive compatibility. Thus, the private information measurability seems to be a desirable assumption in the ex ante case as it ensures that ex ante private information Pareto optimal allocations are always interim incentive compatible (Hahn and Yannelis 1997).

However, this is not the case with the traditional REE as it is not necessarily incentive compatible, it is not efficient neither implementable (Glycopantis et al. 2005). On the other hand our equilibrium notion universally exists and, under the MEU formulation, it is efficient and incentive compatible.

3.2 The existence theorem

We now show that our notion of REE (universally) exists under mild assumptions. In studies of rational expectations equilibria, it is common to appeal to an artificial family of complete information economies (see e.g., Allen 1981; Einy et al. 2000a, b; De Simone and Tarantino 2010). Given an asymmetric information economy $\mathcal{E}$ described in Sect. 2, since S is finite, there is a finite number of complete information economies $\{\mathcal{E}(s)\}_{s\in S}$. For each fixed s in S, the complete information economy $\mathcal{E}(s)$ is given as follows:

$$\mathcal{E}(s) = \left\{ I, \mathbb{R}^{\ell}_{+}, (u_i(s), e_i(s))_{i\in I} \right\},$$

where $I = \{1, \ldots, n\}$ is still the set of n agents, and for each $i \in I$, $u_i(s) = u_i(s, \cdot) : \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ and $e_i(s) \in \mathbb{R}^{\ell}_{+}$ represent respectively the utility function and the initial endowment of agent i. Let $W(\mathcal{E}(s))$ be the set of *Walrasian equilibrium allocations* of $\mathcal{E}(s)$, and $W(\mathcal{E})$ be the set of *ex post Walrasian equilibrium allocations*, i.e.,

$$W(\mathcal{E}) := \{x : I \times S \to \mathbb{R}^{\ell}_{+} \text{ s.t. } x(s) \in W(\mathcal{E}(s)) \text{ for any } s \in S\}.$$

The idea is to show that the set of V-REE allocations contains all the selections from the Walrasian equilibrium correspondence of the associated family of complete information economies. From the existence of a Walrasian equilibrium in each complete information economy $\mathcal{E}(s)$, we deduce the existence of a V-REE. A related result has been shown by De Simone and Tarantino (2010) and Einy et al. (2000b) but under the additional private information measurability assumption of the utility functions and of the initial endowments. The same assumptions are also used by Correia-da Silva and Hervés-Beloso (2012) to establish the existence of their equilibrium. They assume that preferences are represented by an ex ante expected utility function, where the utility on bundles is continuous, concave, strictly monotone and private information measurable; initial endowments are strictly positive and constant across indistinguishable states; finally any state of nature can be verified by at least one of the agents [see Theorem 1 in Correia-da Silva and Hervés-Beloso (2012)]. Our existence theorem just requires continuous, monotone and quasi-concave state-dependent utility functions and strictly positive endowments. No private information measurability of allocations is imposed on endowments neither on utility function. Moreover, we consider general preferences which may allow for ambiguity and also include as a particular case the Bayesian expected utility. Once, additionally, the agents' initial endowment and utility are private information measurable, our equilibrium coincides with a selection from the Walrasian equilibrium correspondence of the associated family of complete information economies (Proposition 4.8). These private information meascurability requirements are necessary for such equivalence as proved in Example 3.3. A similar result is obtained in CH 2009, in which agents have prudent preferences as they expect to receive the worst of the possibilities in the list, and the equilibrium with prudent expectations coincides with the equilibrium of an associated auxiliary Arrow-Debreu economy. With prudent expectations, agents insure themselves completely against being deceived, and the equilibrium is *ex post* efficient and incentive compatible.

This might be no longer true for subjective expectations equilibria (see CH 2008). A related intuition is behind our efficiency and incentive compatibility results for which we consider the maxmin expected utility of Gilboa and Schmeidler.

Theorem 3.2 *If for any $i \in I$ and $s \in S$ the function $u_i(s, \cdot)$ is quasi-concave and $e_i(s) \gg 0$, then there exists a V-rational expectations equilibrium in $\mathcal{E}$.*

Proof See "Appendix". □

Clearly from Theorem 3.2 we deduce the existence of a maxmin REE once agents' preferences are represented by (6). It is worth noting that according to Definition 3.1 we may also consider the more realistic situation in which different agents have different attitude toward ambiguity. Consequently, Theorem 3.2 guarantees the existence of the equilibrium in economies where some agents' preferences are Bayesian (4) some other are MEU or one of the ambiguous preferences of Sect. 2.2.

We already noticed that the full revealing traditional REE and full revealing V-REE coincide as they are both *ex post* Walrasian equilibrium. Such an equivalence is not true in general since the traditional REE may not exist even in well defined economies (see Example 4.1 below) while the V-REE universally exists. The following example shows that there may exist partial revealing V-REE which is not a Walrasian equilibrium in some complete information economy $\mathcal{E}(s)$.

Example 3.3 Consider an asymmetric information economy $\mathcal{E}$ with three states of nature, $S = \{a, b, c\}$, two goods, $\ell = 2$ (the first good is considered as numeraire) and two agents, $I = \{1, 2\}$ whose characteristics are given as follows:

$$\begin{array}{llll} u_i(a, x, y) = \sqrt{xy} & u_i(b, x, y) = \sqrt{xy} & u_i(c, x, y) = log\left(\frac{1}{4} + xy\right) & \text{for all } i = 1, 2 \\ e_1(a) = e_1(b) = (2, 1) & e_1(c) = (1, 2) & e_2(a) = e_2(c) = (1, 2) & e_2(b) = (2, 1) \\ \mathcal{F}_1 = \{\{a, b\}; \{c\}\} & & \mathcal{F}_2 = \{\{a, c\}; \{b\}\}. & \end{array}$$

Notice that the initial endowment is private information measurable, while the utility functions are not.[10]

The set $W(\mathcal{E})$ of *ex post* Walrasian equilibrium has only one element, i.e.,

$$\begin{array}{lll} (p(a), q(a)) = (1, 1) & (x_1(a), y_1(a)) = \left(\frac{3}{2}, \frac{3}{2}\right) & (x_2(a), y_2(a)) = \left(\frac{3}{2}, \frac{3}{2}\right) \\ (p(b), q(b)) = (1, 2) & (x_1(b), y_1(b)) = (2, 1) & (x_2(b), y_2(b)) = (2, 1) \\ (p(c), q(c)) = \left(1, \frac{1}{2}\right) & (x_1(c), y_1(c)) = (1, 2) & (x_2(c), y_2(c)) = (1, 2)\,. \end{array}$$

Clearly, this equilibrium is also a fully revealing V-REE, since $(p(a), q(a)) \neq (p(b), q(b)) \neq (p(c), q(c))$ and hence $\mathcal{G}_i^p = \sigma(p, q) = \{\{a\}, \{b\}, \{c\}\}$ for any $i = 1, 2$. However if, for any agent i and any state $s \in S$, $V_i(x_i|\mathcal{G}_i^p(s)) = \underline{u}_i^{REE}(s, x_i) = \min_{s' \in \mathcal{G}_i^p(s)} u_i(s', x_i(s'))$, then the following is another V-REE:

$$\begin{array}{lll} (p(a), q(a)) = \left(1, \frac{1}{2}\right) & (x_1(a), y_1(a)) = \left(\frac{5}{4}, \frac{5}{2}\right) & (x_2(a), y_2(a)) = \left(\frac{7}{4}, \frac{1}{2}\right) \\ (p(b), q(b)) = (1, 2) & (x_1(b), y_1(b)) = (2, 1) & (x_2(b), y_2(b)) = (2, 1) \\ (p(c), q(c)) = \left(1, \frac{1}{2}\right) & (x_1(c), y_1(c)) = (1, 2) & (x_2(c), y_2(c)) = (1, 2)\,. \end{array}$$

[10] Hence, this example does not contradict Lemma 8.2 in the "Appendix" and Proposition 4.7.

This is a partially revealing equilibrium, since $(p(a), q(a)) = (p(c), q(c)) \neq (p(b), q(b))$ and hence $\sigma(p, q) = \{\{a, c\}, \{b\}\}$, that is $\mathcal{G}_1^p = \{\{a\}, \{b\}, \{c\}\}$, while $\mathcal{G}_2^p = \mathcal{F}_2$. Notice that the equilibrium allocations are not $\mathcal{G}_i^p$-measurable.

The above example shows that $W(\mathcal{E})$ is in general strictly contained in the set of V-REE allocations. We now observe that in some cases, the set of V-REE coincides with the set of *ex post* Walrasian equilibria. Precisely, consider the next further property which is the strict version of $(A2)$: given an event $F \in \mathcal{F}$ and two functions $f, g : S \to R$,

$(A2^*)$ $f(s') > g(s')$ for some $s' \in F$ implies $V(f|F) > V(h|F)$, where

$$h(s) = \begin{cases} g(s') & \text{if } s = s' \\ f(s) & \text{otherwise.} \end{cases}$$

The following holds.

Proposition 3.4 *Any ex post Walrasian equilibrium is a V-REE. The converse is also true if* $(A2^*)$ *above holds.*

Proof See "Appendix". □

Notice that condition $(A2^*)$ is satisfied by the invariant biseparable preferences and a fortiori by the (general) maxmin function (5) as well as by the Bayesian expected utility, provided that $\mathcal{M}^F$ contains only positive priors (i.e., $\mu(s) > 0$ for any state $s \in F$), otherwise $V(f|F) \geq V(h|F)$ and not strictly preferred (see Sect. 8.5).

Therefore, if we remove the private information measurability requirement from the traditional notion of REE we end up with the notion of *ex post* Walrasian equilibrium. This does not hold for the maxmin REE (see Example 3.3), unless we additionally assume that agents' initial endowment and utility function are private information measurable (see Proposition 4.7).

4 Maxmin rational expectations equilibrium

We now focus on economies in which all market participants have MEU preferences (6) and analyse the notion of maxmin REE comparing it with the traditional (Bayesian) REE. Finally, we establish further properties of the maxmin REE.

4.1 Maxmin REE: comparison with the traditional Bayesian notion

The existence of a maxmin REE follows from Theorem 3.2 as any *ex post* Walrasian equilibrium is a MREE. The converse needs not hold as shown in Example 3.3. The MREE, except at some particular cases (see for example Proposition 4.7), also differs from the traditional REE. Below we consider again the economy described in Kreps's example (Kreps 1977) with two states, two agents and two goods. Endowments are identical and positive. Preferences are state-dependent and such that in state one (two), the agent type one (two) prefers good one relatively more. In an asymmetric information economy in which the preferences of all agents are represented by Bayesian expected utility function [see (4)], since the setup is symmetric, the full information equilibrium price is the same in both states.

Now suppose that agent one can distinguish the states but agent two cannot. There cannot be a fully revealing traditional REE: it would have to coincide with the full information equilibrium, and that equilibrium has a constant price across states, which is not compatible with revelation. Also, there cannot be a non revealing equilibrium. In a non revealing equilibrium with equal prices across states, demand of the uninformed agent would have to be the same across states. But demand of the informed agent would be different across states, and therefore there will not be market clearing. Note that a key reason for the nonexistence of a non revealing equilibrium is that the demand of the uninformed agent is measurable with respect to her private information.

On the other hand, if we impose maxmin evaluation of plans, then we can have a non revealing equilibrium. In such an equilibrium the uniformed agent chooses the worst state out of the two states. Thus, she is indifferent between any two consumption bundles in the better state - her optimal demand is a correspondence. Therefore, we can select an element from the correspondence to clear the market. Note that the allocation is then typically not measurable with respect to the uninformed agent's information and this overcomes the non-existence problem.

Below, we explicitly consider Kreps' example and show that while the traditional REE does not exist, a maxmin rational expectations equilibrium does exist. From this we can conclude that the sets of MREE and REE are different.

Example 4.1 (*Kreps*[11]) There are two agents, two commodities and two equally probable states of nature $S = \{s_1, s_2\}$. The primitives of the economy are:

$$e_1 = \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_1 = \{\{s_1\}, \{s_2\}\};$$
$$e_2 = \left(\left(\frac{3}{2}, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{3}{2}\right)\right) \quad \mathcal{F}_2 = \{\{s_1, s_2\}\}.$$

The utility functions of agents 1 and 2 in states s_1 and s_2 are given as follows

$$u_1(s_1, x_1, y_1) = \log x_1 + y_1 \qquad u_1(s_2, x_1, y_1) = 2\log x_1 + y_1$$
$$u_2(s_1, x_2, y_2) = 2\log x_2 + y_2 \qquad u_2(s_2, x_2, y_2) = \log x_2 + y_2.$$

It is well known that for the above economy, a traditional rational expectations equilibrium does not exist (see Kreps 1977). However we will show below that a maxmin rational expectations equilibrium does exist.

The information generated by the equilibrium price can be either $\{\{s_1\}, \{s_2\}\}$ or $\{\{s_1, s_2\}\}$. In the first case, the MREE coincides with the traditional REE, therefore it

[11] We are grateful to Z. Liu and L. Sun for having checked the computations in Example 4.1.

does not exist. Thus, let us consider the case $\sigma(p) = \{\emptyset, S\}$, i.e., $p^1(s_1) = p^1(s_2) = p$ and $p^2(s_1) = p^2(s_2) = q$.

Since for each s, $\mathcal{G}_1^p(s) = \{s\}$, agent one solves the following constraint maximization problems:

Agent 1 in state s_1:

$$\max_{x_1(s_1), y_1(s_1)} \log x_1(s_1) + y_1(s_1) \quad \text{subject to}$$
$$px_1(s_1) + qy_1(s_1) \leq \frac{3}{2}(p+q).$$

Thus,

$$x_1(s_1) = \frac{q}{p} \quad y_1(s_1) = \frac{3}{2}\frac{p}{q} + \frac{1}{2}.$$

Agent 1 in state s_2:

$$\max_{x_1(s_2), y_1(s_2)} 2\log x_1(s_2) + y_1(s_2) \quad \text{subject to}$$
$$px_1(s_2) + qy_1(s_2) \leq \frac{3}{2}(p+q).$$

Thus,

$$x_1(s_2) = \frac{2q}{p} \quad y_1(s_2) = \frac{3}{2}\frac{p}{q} - \frac{1}{2}.$$

Agent 2 in the event $\{s_1, s_2\}$ maximizes

$$\min\{2logx_2(s_1) + y_2(s_1); logx_2(s_2) + y_2(s_2)\}.$$

Therefore, we can distinguish three cases:

I Case: $2logx_2(s_1) + y_2(s_1) > logx_2(s_2) + y_2(s_2)$. In this case, agent 2 solves the following constraint maximization problem:

max $logx_2(s_2) + y_2(s_2)$ subject to $px_2(s_1) + qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2) + qy_2(s_2) \leq \frac{3}{2}(p+q)$. Thus,

$$x_2(s_2) = \frac{q}{p} \quad y_2(s_2) = \frac{3}{2}\frac{p}{q} + \frac{1}{2}.$$

From feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1, 2) \quad (x_1(s_2), y_1(s_2)) = (2, 1)$$
$$(x_2(s_1), y_2(s_1)) = (2, 1) \quad (x_2(s_2), y_2(s_2)) = (1, 2).$$

Notice that $2logx_2(s_1) + y_2(s_1) = 2log2 + 1 > log1 + 2 = logx_2(s_2) + y_2(s_2)$.

II Case: $2logx_2(s_1) + y_2(s_1) < logx_2(s_2) + y_2(s_2)$. In this case, agent 2 solves the following constraint maximization problem:

max $2logx_2(s_1) + y_2(s_1)$ subject to $px_2(s_1) + qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2) + qy_2(s_2) \leq \frac{3}{2}(p+q)$ Thus,

$$x_2(s_1) = \frac{2q}{p} \quad y_2(s_1) = \frac{3}{2}\frac{p}{q} - \frac{1}{2}.$$

From feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1, 2) \quad (x_1(s_2), y_1(s_2)) = (2, 1)$$
$$(x_2(s_1), y_2(s_1)) = (2, 1) \quad (x_2(s_2), y_2(s_2)) = (1, 2).$$

Clearly, as noticed above, $2log2 + 1 > log1 + 2$. Therefore, in the second case there is no maxmin rational expectations equilibrium.

III Case: $2logx_2(s_1) + y_2(s_1) = logx_2(s_2) + y_2(s_2)$. In this case, agent 2 solves one of the following two constraint maximization problems:

max $logx_2(s_2)+y_2(s_2)$ or max $2logx_2(s_1)+y_2(s_1)$ subject to $px_2(s_1)+qy_2(s_1) \leq \frac{3}{2}(p+q)$ and $px_2(s_2)+qy_2(s_2) \leq \frac{3}{2}(p+q)$. In both cases, from feasibility it follows that $p = q$, and

$$(x_1(s_1), y_1(s_1)) = (1, 2) \quad (x_1(s_2), y_1(s_2)) = (2, 1)$$
$$(x_2(s_1), y_2(s_1)) = (2, 1) \quad (x_2(s_2), y_2(s_2)) = (1, 2).$$

Hence, since $2logx_2(s_1) + y_2(s_1) = 2log2 + 1 > log1 + 2 = logx_2(s_2) + y_2(s_2)$, there is no maxmin rational expectations equilibrium in the third case.

Therefore, we can conclude that the *unique* maxmin REE allocation is given by

$$(x_1(s_1), y_1(s_1)) = (1, 2) \quad (x_1(s_2), y_1(s_2)) = (2, 1)$$
$$(x_2(s_1), y_2(s_1)) = (2, 1) \quad (x_2(s_2), y_2(s_2)) = (1, 2).$$

Observe that the maxmin REE bundles are not $\mathcal{F}_i$-measurable.

Remark 4.2 As we have already observed, the maxmin rational expectations equilibrium allocations may not be $\mathcal{G}_i^p$-measurable. However, if we assume strict quasi-concavity and $\mathcal{F}_i$-measurability of the random utility function of each agent, then the resulting maxmin REE allocations will be $\mathcal{G}_i^p$-measurable, as the following proposition indicates.

Proposition 4.3 *Assume that for all i and for all $s \in S$, $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ and $u_i(s, \cdot)$ is strictly quasi-concave. If (p, x) is a maxmin REE, then $x_i(\cdot)$ is $\mathcal{G}_i^p$-measurable for all $i \in I$, where $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$.*

Proof See "Appendix". □

A similar proposition can be proved for the traditional (Bayesian) rational expectations equilibrium, that is whenever the utility functions are private information measurable and strictly quasi-concave, from the uniqueness of the maximizer, we obtain that the equilibrium allocations must be private information measurable. Moreover, the same holds true with the general MEU formulation (5) provided that for any agent i and state s the set $\mathcal{M}_i^s$ contains only positive priors (i.e., $\mu(s') > 0$ for any $s' \in \Pi_i(s)$ and $\sum_{s' \in \Pi_i(s)} \mu(s') = 1$). See "Counterexamples for a general set of priors" in the "Appendix" for more details.

Remark 4.4 We have already observed that the fully revealing traditional REE and the fully revealing maxmin REE coincide and they are both *ex post* Walrasian equilibria. The converse is not true as shown in the Kreps's example above where an *ex post* Walrasian equilibrium exists and coincides with the unique non-revealing maxmin REE, but the set of fully revealing maxmin REE, as well as of traditional REE, is empty. However, the set of non-revealing MREE is non empty. This is consistent with Lemma 8.1 in the "Appendix". We now present below some hypotheses under which this equivalence still holds.

Proposition 4.5 *Assume that* $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ *for all* $i \in I$. *If* (p, x) *is a traditional REE, then* (p, x) *is a MREE. The converse is also true if* $x_i(\cdot)$ *is* $\mathcal{G}_i^p$*-measurable for all* $i \in I$.

Proof See "Appendix". □

Remark 4.6 The above proposition holds with the general MEU formulation (5) and it remains true if we replace the $\mathcal{G}_i^p$-measurability of the allocations by the strict quasi-concavity of the random utility functions. This follows by combining Propositions 4.3 and 4.5. Note that in Example 4.1, utility functions are not $\mathcal{F}_i$-measurable and therefore Example 4.1 does not fulfill the assumptions of Proposition 4.5.

Proposition 4.7 *Assume that for any* $i \in I$ *and* $s \in S$ *the function* $u_i(s, \cdot)$ *is strictly quasi-concave and* $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$. *Let* x *be a feasible allocation. The following statements are equivalent:*

(1) *x is a maxmin REE*[12]*;*
(2) *x is a traditional REE;*
(3) *x is an ex post Walrasian equilibrium allocation.*

Proof See "Appendix". □

Remark 4.8 If for any $i \in I$ and $s \in S$ $e_i(s) \gg 0$, $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$, and the function $u_i(s, \cdot)$ is strictly quasi-concave, then from Remark 4.6 and Theorem 3.2 it follows that there exists a traditional REE in $\mathcal{E}$. Notice that in Example 4.1, where the traditional REE does not exist, not all the above assumptions are satisfied. In particular, the random utility functions are not private information measurable.

[12] We can consider also the general MEU formulation (5) provided that for all agent i and state s, the set $\mathcal{M}_i^s$ contains only positive priors (see Sect. 8.5).

4.2 Maxmin REE: some properties

In this section we investigate some basic properties of a maxmin rational expectations equilibrium.

The first property of a MREE is about the equilibrium price p.[13] We show that under certain assumptions the equilibrium price is strictly positive in each state of nature, i.e., $p(s) \gg 0$ for all $s \in S$.

Remark 4.9 Recall that in a complete information economy, if the utility function of at least one agent is strictly monotone, the equilibrium price is strictly positive. We prove the same for the MREE prices. Notice that, typically in asymmetric information economies an additional assumption is needed: for each state $s \in S$, there exists an agent $i \in I$ such that $\{s\} \in \mathcal{F}_i$. It implies that $\bigvee_{i \in I} \mathcal{F}_i = \mathcal{F} = 2^S$ which is used in Allen (1981) and Einy et al. (2000b). The converse is not true: in particular in an asymmetric information economy with three states of nature $S = \{a, b, c\}$ and two agents $I = \{1, 2\}$, with $\mathcal{F}_1 = \{\{a, b\}, \{c\}\}$ and $\mathcal{F}_2 = \{\{a, c\}, \{b\}\}$, it is true that $\mathcal{F}_1 \vee \mathcal{F}_2 = \{\{a\}, \{b\}, \{c\}\}$, but $\{a\} \notin \mathcal{F}_i$ for any $i \in \{1, 2\}$. Although this assumption is quite common in the literature (see for example Angeloni and Martins-da Rocha 2009 and Correia-da Silva and Hervés-Beloso 2012), we can prove that MREE prices are strictly positive by dispensing with it.

Proposition 4.10 *Assume that there is at least one agent $i \in I$ such that $u_i(s, \cdot)$ is strictly monotone for any $s \in S$. If (p, x) is a maxmin rational expectations equilibrium, then $p(s) \gg 0$ for any $s \in S$.*

Proof See "Appendix". □

We now show a second property of a MREE: if the utility functions are private information measurable, then for each agent $i \in I$, the maxmin utility at any MREE allocation is constant in each event of the partition $\mathcal{G}_i^p$.

Proposition 4.11 *Assume that $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$. If (p, x) is a maxmin rational expectations equilibrium, then for all i and s, $\underline{u}_i^{REE}(s, x_i) = u_i(s', x_i(s'))$ for all $s' \in \mathcal{G}_i^p(s)$, that is the minimum in the event $\mathcal{G}_i^p(s)$ is obtained in each state s' of the event.*

Proof See "Appendix". □

Notice that if (p, x) is a fully revealing maxmin REE, Proposition 4.11 is trivially satisfied even if the utility functions and the initial endowments are not private information measurable. Moreover Proposition 4.11 holds true even with the general MEU formulation (5) provided that for any agent i and state s, the set $\mathcal{M}_i^s$ contains only positive priors. See "Counterexamples for a general set of priors" in the "Appendix" for more details.

[13] This property stated in Proposition 4.10 holds true even with the general MEU formulation (5).

5 Efficiency of the maxmin REE

We now define the notion of maxmin and *ex post* Pareto optimality and we exhibit conditions which guarantee that any maxmin REE is maxmin efficient and *ex post* Pareto optimal. The results illustrated in this section also holds for the general MEU formulation.[14]

Definition 5.1 A feasible allocation x is said to be *ex post* efficient (or ex post Pareto optimal) if there does not exist an alternative feasible allocation y such that $u_i(s, y_i(s)) \geq u_i(s, x_i(s))$ for all $i \in I$ and for all $s \in S$, with at least a strict inequality.

Definition 5.2 A feasible allocation x is said to be maxmin efficient (or maxmin Pareto optimal) with respect to information structure Π, if there does not exist an alternative feasible allocation y such that $\underline{u}_i^{\Pi_i}(s, y_i) \geq \underline{u}_i^{\Pi_i}(s, x_i)$ for all $i \in I$ and for all $s \in S$, with at least a strict inequality.

Proposition 5.3 *Let Π be an information structure such that for any state s there exists an agent i with $\Pi_i(s) = \{s\}$.*[15] *If for any $i \in I$ and any $s \in S$, $u_i(s, \cdot)$ is strictly monotone or $u_i(s, y) = u_i(s, 0)$ for any $y \in \partial\mathbb{R}^\ell_+$, then any maxmin efficient allocation x with respect to the information structure Π is ex post Pareto optimal. The converse may not be true.*

Proof See "Appendix". □

The assumption that for any state s there exists an agent i such that $\Pi_i(s) = \{s\}$ is fundamental for Proposition 5.3 as shown by Example 8.4 in the "Appendix".

We are now ready to exhibit the conditions under which any MREE is maxmin efficient and also *ex post* Pareto optimal.

Theorem 5.4 *Let (p, x) be a maxmin rational expectations equilibrium. If one of the following conditions holds true:*

1. *$\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for each $i \in I$;*
2. *p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$;*

then x is ex post efficient and maxmin Pareto optimal with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i \in I}$, where $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$ for any $i \in I$.

Moreover, if none of the above conditions is satisfied, a maxmin REE may not be maxmin efficient.

Proof See "Appendix". □

Remark 5.5 The *ex post* Pareto optimality does not follow from Proposition 5.3 because we do not require that the information structure $\mathcal{G}_i^p$ is such that for any state s there exists an agent i with $\mathcal{G}_i^p(s) = \{s\}$, neither that for any $i \in I$ and any $s \in S$, $u_i(s, \cdot)$ is strictly monotone or $u_i(s, y) = u_i(s, 0)$ for any $y \in \partial\mathbb{R}^\ell_+$.

[14] Only to prove the statements of Theorems 5.4 and 5.9 under the first condition (i.e., $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$), any set $\mathcal{M}_i^s$ must contain only positive priors (see Sect. 8.5).

[15] This assumption is quite common in the literature of asymmetric information economies (see for example Angeloni and Martins-da Rocha 2009 and Correia-da Silva and Hervés-Beloso 2012) (see Remark 4.9 in Sect. 4.2).

According to the efficiency concept (Definitions 5.1 and 5.2), an improvement requires a strict utility increase for some pair $(j, \bar{s}) \in I \times S$ and no utility decreases for all $(i, s) \in I \times S$. A weaker notion defined below would require strict utility increases for all agents in all states of nature. Clearly, any maxmin Pareto optimal allocation is weak maxmin efficient. The converse may not be true (see Examples 8.6 and 8.7 and Remark 5.11).

Definition 5.6 A feasible allocation x is said to be weak maxmin efficient (or weak maxmin Pareto optimal) with respect to information structure Π, if there does not exist an alternative feasible allocation y such that $\underline{u}_i^{\Pi_i}(s, y_i) > \underline{u}_i^{\Pi_i}(s, x_i)$ for all $i \in I$ and for all $s \in S$.

Similarly the notion of weak *ex post* efficiency is given as follows.

Definition 5.7 A feasible allocation x is said to be weak *ex post* efficient (or weak *ex post* Pareto optimal) if there does not exist an alternative feasible allocation y such that $u_i(s, y_i(s)) > u_i(s, x_i(s))$ for all $i \in I$ and for all $s \in S$.

Proposition 5.8 *Any weak maxmin efficient allocation x (with respect to any information structure) is weak ex post Pareto optimal. The converse may not be true.*

Proof See "Appendix". □

Notice that, contrary to Proposition 5.3, we need no further assumptions on the information structure neither on agents' utility functions.

We now list the conditions guaranteeing that a maxmin REE is weak maxmin efficient and a fortiori weak *ex post* Pareto optimal (see Proposition 5.8).

Theorem 5.9 *Let (p, x) be a maxmin rational expectations equilibrium. If one of the following conditions holds true:*

(i) *$\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for each $i \in I$;*
(ii) *p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$;*
(iii) *there exists a state of nature $\bar{s} \in S$ such that $\{\bar{s}\} = \mathcal{G}_i^p(\bar{s})$ for all $i \in I$;*
(iv) *the $n - 1$ agents are fully informed.*

then x is weak maxmin Pareto optimal with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i \in I}$, where $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$ for any $i \in I$, and hence weak ex post efficient.

Moreover if none of the above conditions is satisfied, a maxmin REE may not be weak maxmin efficient (and a fortiori it may not be maxmin Pareto optimal).

Proof See "Appendix". □

Remark 5.10 Notice that in the first two cases (i.e., under condition (i) or (ii)), the result easily follows from Theorem 5.4 and from the observation that any allocation maxmin efficient with respect to Π is weak maxmin Pareto optimal with respect to Π. On the other hand, it can be shown that under either condition (iii) or (iv) a maxmin REE allocation is weak maxmin Pareto optimal but it may not be maxmin efficient (see Examples 8.6 and 8.7 in the "Appendix").

Remark 5.11 Notice that in Kreps's example (Example 4.1), one of the two agents is fully informed, hence condition (iv) of Theorem 5.9 is satisfied. This guarantees that the unique maxmin rational expectations equilibrium (MREE) is weak maxmin Pareto optimal and hence weak *ex post* efficient. On the other hand, no condition of Theorem 5.4 is verified and the unique maxmin REE is not maxmin efficient. Indeed consider the following feasible allocation

$$(t_1(s_1), z_1(s_1)) = \left(\frac{5}{4}, 2\right) \quad (t_1(s_2), z_1(s_2)) = (x_1(s_2), y_1(s_2)) = (2, 1)$$
$$(t_2(s_1), z_2(s_1)) = \left(\frac{7}{4}, 1\right) \quad (t_2(s_2), z_2(s_2)) = (x_2(s_2), y_2(s_2)) = (1, 2),$$

and notice that

$$\begin{aligned}
\underline{u}_1^{REE}(s_1, t_1, z_1) &= log\frac{5}{4} + 2 > 2 = \underline{u}_1^{REE}(s_1, x_1, y_1)\\
\underline{u}_1^{REE}(s_2, t_1, z_1) &= 2log2 + 1 = \underline{u}_1^{REE}(s_2, x_1, y_1)\\
\underline{u}_2^{REE}(s_1, t_2, z_2) &= \underline{u}_2^{REE}(s_2, t_2, z_2) = \min\left\{2log\frac{7}{4} + 1; 2\right\} = 2\\
&= \min\{2log2 + 1; 2\} = \underline{u}_2^{REE}(s_2, x_2, y_2) = \underline{u}_2^{REE}(s_1, x_2, y_2).
\end{aligned}$$

Thus, the unique maxmin REE is weak maxmin efficient but not maxmin Pareto optimal with respect to the information structures either $\mathcal{G}^p = (\mathcal{G}_1^p, \mathcal{G}_2^p)$ or $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$, since the equilibrium is non revealing and $\mathcal{G}^p = \mathcal{F}$. On the other hand, the unique non revealing maxmin REE is an *ex post* Walrasian equilibrium and hence it is *ex post* efficient. Indeed assume to the contrary that there exists an alternative feasible allocation (t, z) such that

$$\begin{aligned}
&(i) \quad log(3 - t_2(s_1)) + (3 - z_2(s_1)) \geq 2\\
&(ii) \quad 2log(t_2(s_1)) + z_2(s_1) \geq 2log2 + 1\\
&(iii) \quad 2log(t_1(s_2)) + z_1(s_2) \geq 2log2 + 1\\
&(iv) \quad log(3 - t_1(s_2)) + (3 - z_1(s_2)) \geq 2,
\end{aligned}$$

with at least one strict inequality. If one of (i) and (ii) is strict, then $(3-t_2(s_1))t_2^2(s_1) > 4$ or equivalently that $(t_2(s_1)+1)(t_2(s_1)-2)^2 < 0$ which is a contradiction. Similarly if one of (iii) and (iv) is strict.

Therefore, Kreps's example can also be used to show that a weak maxmin efficient allocation may not be maxmin Pareto optimal and an *ex post* efficient allocation may not be maxmin efficient. Moreover, an *ex post* Walrasian equilibrium allocation, which is always *ex post* efficient, may not be maxmin Pareto optimal.

5.1 Further remarks on the efficiency of maxmin REE

Someone could debate the fact that we have considered the algebra $\mathcal{G}_i^p$ and not $\mathcal{F}_i$. What is the correct definition? It seems to us that it depends on what kind of interpretation or story one has in mind. For example one may say that the notions of efficiency and incentive compatibility are independent of prices and as a consequence agents have to condition their expectations on $\mathcal{F}_i$. This view however can be challenged because at REE each agent in the interim stage behaves like having observed the equilibrium price and conditions herself on the information $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$. Thus, the relevant information for each agent is $\mathcal{G}_i^p$ and not $\mathcal{F}_i$. For this reason we chose to present the definitions of efficiency and incentive compatibility considering the two different private information sets, $\mathcal{F}_i$ and $\mathcal{G}_i^p$.

We now investigate the efficiency of maxmin REE with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$. We do the same for the incentive compatibility (see Sect. 6.2).

Remark 5.12 Clearly, for any non revealing maxmin rational expectations equilibrium the results of Sect. 5 still hold simply because $\mathcal{G}_i^p = \mathcal{F}_i$ for all $i \in I$. In particular notice that the equilibrium in Example 8.5 is non-revealing. On the other hand, any fully revealing maxmin REE is maxmin efficient with respect to $\mathcal{G}_i^p$ and also *ex post* efficient (see Theorem 5.4), but it may not be maxmin efficient with respect to $\mathcal{F}_i$ as the following example shows.

Example 5.13 Consider an asymmetric information economy with two states of nature, $S = \{a, b\}$, two goods, $\ell = 2$ (the first good is considered as numeraire) and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{lll} e_1(a) = (2,1) & e_1(b) = (1,2) & \mathcal{F}_1 = \{\{a\};\{b\}\} \\ e_2(a) = (1,2) & e_2(b) = (1,2) & \mathcal{F}_2 = \{\{a,b\}\} \\ e_3(a) = (2,1) & e_3(b) = (2,1) & \mathcal{F}_3 = \{\{a,b\}\}. \\ u_i(a,x,y) = \sqrt{xy} & u_i(b,x,y) = x^2 y \quad \text{for } i \in \{1,2\} & u_3(\cdot,x,y) = xy. \end{array}$$

Consider the following fully revealing maxmin rational expectations equilibrium

$$(x_1(a), y_1(a)) = \left(\tfrac{13}{8}, \tfrac{13}{10}\right) \; (x_2(a), y_2(a)) = \left(\tfrac{7}{4}, \tfrac{7}{5}\right) \quad (x_3(a), y_3(a)) = \left(\tfrac{13}{8}, \tfrac{13}{10}\right)$$

$$(x_1(b), y_1(b)) = \left(\tfrac{26}{19}, \tfrac{13}{10}\right) \; (x_2(b), y_2(b)) = \left(\tfrac{26}{19}, \tfrac{13}{10}\right) \; (x_3(b), y_3(b)) = \left(\tfrac{24}{19}, \tfrac{12}{5}\right),$$

with $(p(a), q(a)) = \left(1, \frac{5}{4}\right)$ and $(p(b), q(b)) = \left(1, \frac{10}{19}\right)$, which is of course *ex post* efficient since it coincides with an *ex post* Walrasian equilibrium. On the other hand, we now show that it is not maxmin efficient with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$. To this end, consider the following feasible allocation (t, z)

$$(t_1(a), z_1(a)) = \left(\tfrac{13}{8}, \tfrac{13}{10}\right) \; (t_2(a), z_2(a)) = \left(\tfrac{7}{4}, \tfrac{7}{5}\right) \quad (t_3(a), z_3(a)) = \left(\tfrac{13}{8}, \tfrac{13}{10}\right)$$

$$(t_1(b), z_1(b)) = \left(\tfrac{31}{19}, \tfrac{7}{5}\right) \quad (t_2(b), z_2(b)) = \left(\tfrac{25}{19}, \tfrac{6}{5}\right) \; (t_3(b), z_3(b)) = \left(\tfrac{20}{19}, \tfrac{12}{5}\right),$$

and notice that,

$$\underline{u}_1^{\mathcal{F}_1}(a, t_1, z_1) = u_1(a, t_1(a), z_1(a)) = u_1(a, x_1(a), y_1(a)) = \underline{u}_1^{\mathcal{F}_1}(a, x_1, y_1)$$

$$\underline{u}_1^{\mathcal{F}_1}(b, t_1, z_1) = u_1(b, t_1(b), z_1(b)) = \left(\frac{31}{19}\right)^2 \frac{7}{5} > \left(\frac{26}{19}\right)^2 \frac{13}{10}$$

$$= u_1(b, x_1(b), y_1(b)) = \underline{u}_1^{\mathcal{F}_1}(b, x_1, y_1)$$

$$\underline{u}_2^{\mathcal{F}_2}(a, t_2, z_2) = \underline{u}_2^{\mathcal{F}_2}(b, t_2, z_2) = \min\left\{\sqrt{\frac{49}{20}}, \left(\frac{25}{19}\right)^2 \frac{6}{5}\right\} = \sqrt{\frac{49}{20}}$$

$$= \min\left\{\sqrt{\frac{49}{20}}, \left(\frac{26}{19}\right)^2 \frac{13}{10}\right\} = \underline{u}_2^{\mathcal{F}_2}(b, x_2, y_2) = \underline{u}_2^{\mathcal{F}_2}(a, x_2, y_2)$$

$$\underline{u}_3^{\mathcal{F}_3}(a, t_3, z_3) = \underline{u}_3^{\mathcal{F}_3}(b, t_3, z_3) = \min\left\{\frac{169}{80}, \frac{240}{95}\right\} = \frac{169}{80}$$

$$= \min\left\{\frac{169}{80}, \frac{288}{95}\right\} = \underline{u}_3^{\mathcal{F}_3}(b, x_3, y_3) = \underline{u}_3^{\mathcal{F}_3}(a, x_3, y_3).$$

Hence, the equilibrium allocation (x, y) is not maxmin Pareto optimal with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$.

We now list the conditions under which a maxmin REE fully revealing or not is maxmin efficient with respect to the initial private information structure $(\mathcal{F}_i)_{i\in I}$.

Theorem 5.14 *Let (p, x) be a maxmin rational expectations equilibrium. If one of the following conditions holds true:*

(a) *there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{F}_i(\bar{s})$ for all $i \in I$;*
(b) *the $n - 1$ agents are fully informed,*

then x is weak maxmin Pareto optimal with respect to $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$ and with respect to $\mathcal{G}^p = (\mathcal{G}_i^p)_{i\in I}$.

Moreover if none of the above conditions is satisfied, then a maxmin REE may not be weak maxmin efficient with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$ and a fortiori maxmin Pareto optimal.

6 Incentive compatibility of rational expectations equilibrium

We now recall the notion of coalitional incentive compatibility in Krasa and Yannelis (1994).

Definition 6.1 An allocation x is said to be coalitional incentive compatible (CIC) with respect to the information structure $\Pi = (\Pi_i)_{i\in I}$ if the following does not hold: there exist a coalition C and two states a and b such that

$$(i) \quad \Pi_i(a) = \Pi_i(b) \qquad for\ all\ i \notin C,$$
$$(ii) \quad e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^\ell \qquad for\ all\ i \in C,\ and$$
$$(iii) \quad u_i(a, e_i(a) + x_i(b) - e_i(b)) > u_i(a, x_i(a)) \quad for\ all\ i \in C.$$

In order to explain what incentive compatibility means in an asymmetric information economy, let us consider the following two examples.[16]

Example 6.2 Consider an economy with two agents, three equally probable states of nature, denoted by a, b and c, and one good per state denoted by x. The primitives of the economy are given as follows:

$$u_1(\cdot, x_1) = \sqrt{x_1};\ e_1(a,b,c) = (20, 20, 0);\ \mathcal{F}_1 = \{\{a,b\}; \{c\}\}.$$
$$u_2(\cdot, x_2) = \sqrt{x_2};\ e_2(a,b,c) = (20, 0, 20);\ \mathcal{F}_2 = \{\{a,c\}; \{b\}\}.$$

The following risk sharing (Pareto optimal) redistribution of initial endowment $x_1(a,b,c) = (20, 10, 10)$ and $x_2(a,b,c) = (20, 10, 10)$ is not incentive compatible with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$. Indeed, suppose that the realized state of nature is a, agent 1 is in the event $\{a, b\}$ and she reports c, (observe that agent 2 cannot distinguish between a and c). Since agent 2 is not able to identify the lie, she gives to agent 1 ten units. Therefore, the utility of agent 1 when she reports state c is $u_1(a, e_1(a)+x_1(c)-e_1(c)) = u_1(a, 20+10-0) = \sqrt{30}$ which is greater than the realized state of nature a, which is $u_1(a, x_1(a)) = \sqrt{20}$. Hence, the allocation $x_1(a,b,c) = (20, 10, 10)$ and $x_2(a,b,c) = (20, 10, 10)$ is not incentive compatible.

In order to make sure that the equilibrium contracts are stable, we must insist on a coalitional definition of incentive compatibility and not an individual one. As the following example shows, a contract which is individual incentive compatible may not be coalitional incentive compatible and therefore may not be viable.

Example 6.3 Consider an economy with three agents, two goods and three states of nature $S = \{a, b, c\}$. The primitives of the economy are given as follows: for all $i = 1, 2, 3$, $u_i(\cdot, x_i, y_i) = \sqrt{x_i y_i}$ and

$$\mathcal{F}_1 = \{\{a,b,c\}\};\quad e_1(a,b,c) = ((15,0); (15,0); (15,0)).$$
$$\mathcal{F}_2 = \{\{a,b\}, \{c\}\};\quad e_2(a,b,c) = ((0,15); (0,15); (0,15)).$$
$$\mathcal{F}_3 = \{\{a\}, \{b\}, \{c\}\};\ e_3(a,b,c) = ((15,0); (15,0); (15,0)).$$

Consider the following redistribution of the initial endowments:

$$\begin{aligned}\mathbf{x}_1(a,b,c) &= ((8,5), (8,5), (8,13))\\ \mathbf{x}_2(a,b,c) &= ((7,4), (7,4), (12,1))\\ \mathbf{x}_3(a,b,c) &= ((15,6), (15,6), (10,1)).\end{aligned} \tag{12}$$

Notice that the only agent who can misreport either state a or b to agents 1 and 2 is agent 3. Clearly, agent 3 cannot misreport state c since agent 2 would know it. Thus, agent 3 can only lie if either state a or state b occurs. However, agent 3 has no

[16] The reader is also referred to Krasa and Yannelis (1994), Koutsougeras and Yannelis (1993) and Podczeck and Yannelis (2008) for an extensive discussion of the Bayesian incentive compatibility in asymmetric information economies.

incentive to misreport since she gets the same consumption in both states a and b. Hence, the allocation (12) is individual incentive compatible with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3)$, but it is not coalitional incentive compatible with respect to $\mathcal{F}$. Indeed, if c is the realized state of nature, agents 2 and 3 have an incentive to cooperate against agent 1 and report b (notice that agent 1 cannot distinguish between b and c). The coalition $C = \{2, 3\}$ will now be better off, i.e.,

$$\begin{aligned} u_2(c, e_2(c) + \mathbf{x}_2(b) - e_2(b)) &= u_2(c, (0, 15) + (7, 4) - (0, 15)) \\ &= u_2(c, (7, 4)) = \sqrt{28} > \sqrt{12} = u_2(c, \mathbf{x}_2(c)) \\ u_3(c, e_3(c) + \mathbf{x}_3(b) - e_3(b)) &= u_3(c, (15, 0) + (15, 6) - (15, 0)) \\ &= u_3(c, (15, 6)) = \sqrt{90} > \sqrt{10} = u_3(c, \mathbf{x}_3(c)). \end{aligned}$$

In Example 6.2 we have constructed an allocation which is Pareto optimal but it is not individual incentive compatible; while in Example 6.3 we have shown that an allocation, which is individual incentive compatible, need not be coalitional incentive compatible.

In view of Examples 6.2 and 6.3, it is easy to understand the meaning of Definition 6.1. An allocation is coalitional incentive compatible if no coalition of agents C can cheat the complementary coalition (i.e., $I \backslash C$) by misreporting the realized state of nature and make all its members better off. Notice that condition (i) indicates that coalition C can only cheat the agents not in C (i.e., $I \backslash C$) in the states that the agents in $I \backslash C$ cannot distinguish. If $C = \{i\}$ then the above definition reduces to individual incentive compatibility.

6.1 Maxmin incentive compatibility

In this section only,[17] our MEU formulation is a particular form of the original Gilboa-Schmeidler model. Namely, we assume a particular set of probabilities, $\mathcal{C}_i^F$, which comprises all probabilities with support contained in the element F of the partition Π_i, i.e., $F = \Pi_i(s)$. We will prove that the maxmin rational expectations equilibrium is incentive compatible.

Some researchers have expressed the view that this model assumes too much pessimism and that it would be desirable to allow the set $\mathcal{C}_i$ of probabilities to be a strict subset of $\mathcal{C}_i^F$.

There are at least two responses to this criticism. First, we can conceive the partition model as a description of all information that the individual has. If we take this principle seriously, this means that once individual i is informed of its element $\Pi_i(s)$, she knows nothing else. In particular, she has no information about the likelihood or probability of the states inside that partition. If the partition represents her knowledge, she is completely ignorant beyond it, that is, she has no relevant information to rule out any probability in $\mathcal{C}_i^F$. This is related to the literature of complete ignorance that flourished

[17] The existence holds for the general function V, while for the efficiency results we may consider a more general MEU framework by adopting suitable modifications.

in 1950's. For example, Milnor (1954) discusses this hypothesis of complete ignorance in games against nature as follows:

> "Our basic assumption that the player has absolutely no information about Nature may seem too restrictive. However such no-information games may be used as normal form for a wider class of games in which certain types of partial information is allowed. For example if the information consists of bounds for the probabilities of the various states of Nature, then by considering only those mixed strategies for Nature which satisfy these bounds, we construct a new game having no information." (Milnor 1954, p. 49)

Thus, according to this view, we can reduce the partial information that is outside the partition and is represented in some knowledge of the probabilities $\mathcal{C}_i$, in a new model with no information left; this would be the model that we are analyzing.

A second response to this criticism begins by recalling the standard practice in economic theory that an unrealistic assumption is used to capture in a simplistic form a phenomenon that is quite realistic. Even with unrealistic assumptions, economic theory was frequently able to provide good insights about the real world. In our case, the restrictive assumption about the preference is a simplistic way to capture a phenomenon that is universal: indifference among indistinguishable bundles. When people do not have a good reason to prefer an option over other, they are frequently indifferent. The main reason of why our result is true is the indifference between some specific bundles.

Finally, de Castro and Yannelis (2008) showed that every efficient allocation is coalitional incentive compatible if and only if all individuals have maxmin preferences. Thus, the MEU formulation does not reflect pessimistic behavior, but rather incentive compatible behavior. If an agent plays against the nature (e.g., Milnor game), since, nature is not strategic, it makes sense to view the MEU decision making as reflecting pessimistic behavior. However, when you negotiate the terms of a contract under asymmetric information and the other agents have an incentive to misreport the state of nature and benefit, then the MEU provides a mechanism to prevent others from cheating you. This in not pessimism, but incentive compatibility. It is exactly for this reason that the MEU solves the conflict between efficiency and incentive compatibility (see for example de Castro and Yannelis 2008). This conflict seems to be inherent in the Bayesian analysis, where agents must assign probabilities to completely unknown states and those probabilities could be very far from the "true or realized" ones.

In order to prove that the maxmin rational expectations equilibrium is incentive compatible, we need the following definition of maxmin coalitional incentive compatibility, which is an extension of the Krasa and Yannelis (1994) definition to incorporate maxmin preferences (see also de Castro and Yannelis 2008).

Definition 6.4 A feasible allocation x is said to be maxmin coalitional incentive compatible (MCIC) with respect to information structure $\Pi = (\Pi)_{i \in I}$, if the following does not hold: there exist a coalition C and two states a and b such that

$$(i) \quad \Pi_i(a) = \Pi_i(b) \qquad for\ all\ i \notin C,$$
$$(ii) \quad e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^\ell \qquad for\ all\ i \in C,\ and$$

$$(iii) \quad \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) \quad for\ all\ i \in C,$$

where for all $i \in C$,

$$(*) \qquad y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

According to the above definition, an allocation is said to be maxmin coalitional incentive compatible if it is not possible for a coalition to misreport the realized state of nature and have a distinct possibility of making its members better off in terms of maxmin utility.

Remark 6.5 Example 6.2 shows that an efficient allocation may not be incentive compatible in the Krasa-Yannelis sense. We now show that it is not the case in our maxmin utility setting. Precisely, if agents take into account the worst possible state that can occur, then the allocation $x_i(a, b, c) = (20, 10, 10)$ for $i = 1, 2$ in Example 6.2, is maxmin incentive compatible. Indeed, if a is the realized state of nature, agent 1 does not have an incentive to report state c and benefit, because when she misreports she gets:

$$\begin{aligned} \underline{u}_1(a, y_1) &= \min\{u_1(a, e_1(a) + x_1(c) - e_1(c)); u_1(b, x_1(b))\} \\ &= \min\{\sqrt{30}, \sqrt{10}\} = \sqrt{10}. \end{aligned}$$

When agent 1 does not misreport, she gets:

$$\underline{u}_1(a, x_1) = \min\{u_1(a, x_1(a)); u_1(b, x_1(b))\} = \min\{\sqrt{20}, \sqrt{10}\} = \sqrt{10}.$$

Consequently, agent 1 does not gain by misreporting. Similarly, one can easily check that agent 2, when a is the realized state of nature, does not have an incentive to report state b and benefit.

Proposition 6.6 *If x is CIC with respect to the information structure $\Pi = (\Pi_i)_{i \in I}$, then it is also maxmin CIC with respect to Π. The converse may not be true.*

Proof See "Appendix". □

Theorem 6.7 *Any maxmin rational expectations equilibrium (p, x) is maxmin coalitional incentive compatible with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i \in I}$, where $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$ for any $i \in I$.*

Proof See "Appendix". □

6.2 Further remarks on the incentive compatibility of maxmin REE

In this section we consider the incentive compatibility with respect to the initial private information $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$ since the same considerations made in Sect. 5.1 apply. In what follows, by the term "(private) incentive compatible", we mean incentive compatibility with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$.

Remark 6.8 Clearly, any non revealing maxmin rational expectations equilibrium is (private) maxmin CIC, simply because $\mathcal{G}_i^p = \mathcal{F}_i$ for all $i \in I$, and hence the result follows from Theorem 6.7. Example 6.10 below shows that a fully revealing maxmin REE may not be (private) maxmin CIC. This suggests that a weaker notion of maxmin CIC is needed.

Definition 6.9 A feasible allocation x is said to be weak maxmin coalitional incentive compatible (weak MCIC) with respect to information structure Π, if the following does not hold: there exist a coalition C and two states a and b such that

$$\begin{aligned}
&(I) \quad \Pi_i(a) = \Pi_i(b) \qquad for\ all\ i \notin C,\\
&(II) \quad u_i(a, x_i(a)) = u_i(a, x_i(b)) \quad for\ all\ i \notin C,\\
&(III) \quad e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^{\ell} \qquad for\ all\ i \in C,\ and\\
&(IV) \quad \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) \quad for\ all\ i \in C,
\end{aligned}$$

where for all $i \in C$,

$$(*) \qquad y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

Condition (II) of Definition 6.9 does not necessarily mean that $x_i(\cdot)$ is Π_i-measurable for all $i \notin C$, neither that $x_i(a) = x_i(b)$. It just guarantees, together with (I), that individuals not in coalition C are not able to detect a misreport by coalition C.

Clearly, any maxmin CIC allocation is also weak maxmin CIC whatever is the information structure Π, but the converse may not be true as shown by the following example.

Example 6.10 We consider the Example 3.1 in Glycopantis et al. (2005) that we recall below.[18] There are two agents $I = \{1, 2\}$, two commodities and three states of nature $S = \{a, b, c\}$. The primitives of the economy are given as follows

$$\begin{array}{llll}
e_1(a) = e_1(b) = (7, 1) & e_1(c) = (4, 1) & \mathcal{F}_1 = \{\{a, b\}, \{c\}\} & u_1(\cdot, x_1, y_1) = \sqrt{x_1 y_1}\\
e_2(b) = e_2(c) = (1, 7) & e_2(a) = (1, 10) & \mathcal{F}_2 = \{\{a\}, \{b, c\}\} & u_2(\cdot, x_2, y_2) = \sqrt{x_2 y_2}.
\end{array}$$

In this economy the unique traditional REE is the following:

$$\begin{array}{lll}
(p_1(a), p_2(a)) = \left(1, \frac{8}{11}\right) & (x_1(a), y_1(a)) = \left(\frac{85}{22}, \frac{85}{16}\right) & (x_2(a), y_2(a)) = \left(\frac{91}{22}, \frac{91}{16}\right)\\
(p_1(b), p_2(b)) = (1, 1) & (x_1(b), y_1(b)) = (4, 4) & (x_2(b), y_2(b)) = (4, 4)\\
(p_1(c), p_2(c)) = \left(1, \frac{5}{8}\right) & (x_1(c), y_1(c)) = \left(\frac{37}{16}, \frac{37}{10}\right) & (x_2(c), y_2(c)) = \left(\frac{43}{16}, \frac{43}{10}\right).
\end{array}$$

Notice that (p, x) is a fully revealing traditional REE and hence it is also a maxmin REE. Moreover, x is weak (private) maxmin CIC, but it is not (private) maxmin CIC.

[18] We thank Liu Zhiwei for having suggested this example to us.

Indeed, take $C = \{2\}$ and the two states a and b, and observe that

$$\begin{aligned}
&\mathcal{F}_1(a) = \mathcal{F}_1(b) \\
&(e_2^1(a) + x_2(b) - e_2^1(b), e_2^2(a) + y_2(b) - e_2^2(b)) \\
&\quad = (1 + 4 - 1, 10 + 4 - 7) = (4, 7) \gg 0 \\
&u_2(a, e_2^1(a) + x_2(b) - e_2^1(b), e_2^2(a) + y_2(b) - e_2^2(b)) \\
&\quad = \sqrt{28} > \sqrt{\frac{91^2}{352}} = u_2(a, x_2(a), y_2(a)).
\end{aligned}$$

Hence, x is not (private) maxmin CIC, but there do not exist two states s_1 and s_2 and an agent i, such that

$$\begin{aligned}
\mathcal{F}_i(s_1) &= \mathcal{F}_i(s_2) \\
\sqrt{x_i(s_1)y_i(s_1)} &= \sqrt{x_i(s_2)y_i(s_2)}.
\end{aligned}$$

Therefore, x is weak (private) maxmin coalitional incentive compatible.

Proposition 6.11 *Assume that $\sigma(e_i) \subseteq \mathcal{F}_i$ for all $i \in I$ and let (p, x) be a maxmin rational expectations equilibrium. If one of the following conditions holds true:*

1. *$\sigma(u_i) \subseteq \mathcal{F}_i$ for any $i \in I$*[19]*;*
2. *p is fully revealing, i.e., $\sigma(p) = \mathcal{F}$;*

then x is weak (private) maxmin coalitional incentive compatible.

Proof See "Appendix". □

Remark 6.12 As a corollary of Theorem 6.7 we deduce that any maxmin rational expectations equilibrium is maxmin individual incentive compatible. Although in Kreps's example, the utility functions are not private information measurable, the unique maxmin rational expectations equilibrium is (private) maxmin coalitional incentive compatible, since the equilibrium price p is non revealing (see Remarks 6.8). Indeed if state s_1 occurs and agent 1 announces s_2, then

$$\begin{aligned}
&u_1(s_1, e_1^1(s_1) + x_1(s_2) - e_1^1(s_2), e_1^2(s_1) + y_1(s_2) - e_1^2(s_2)) \\
&\quad = log2 + 1 < 2 \\
&\quad = u_1(s_1, x_1(s_1), y_1(s_1)).
\end{aligned}$$

If state s_2 occurs and agent 1 announces s_1, then

$$\begin{aligned}
&u_1(s_2, e_1^1(s_2) + x_1(s_1) - e_1^1(s_1), e_1^2(s_2) + y_1(s_1) - e_1^2(s_1)) \\
&\quad = 2 < 2log2 + 1 = u_1(s_2, x_1(s_2), y_1(s_2)).
\end{aligned}$$

[19] Notice that the private information measurability assumption of utility functions is not too strong when we deal with coalitional incentive compatibility notions (see for example Koutsougeras and Yannelis 1993; Krasa and Yannelis 1994; Angeloni and Martins-da Rocha 2009 where the utility functions are assumed to be state-independent, and therefore $\mathcal{F}_i$-measurable).

On the other hand, in Example 6.10 both hypotheses of Proposition 6.11 are satisfied and the maxmin REE is weak (private) maxmin CIC. However, as it has been already observed, it is not (private) maxmin CIC.

7 Open questions

We conclude this paper with some open questions.

Throughout we have used the assumption that there is a finite number of states. We conjecture that the main existence theorem can be extended to infinitely many states of nature and even to an infinite dimensional commodity space. Some preliminary work in this direction can be found in Bhowmik et al. (2014) and Bhowmik and Cao (2016).

It is also of interest to know if the results of this paper could be extended to a continuum of agents, or to a more general setup such as mixed markets.

Based on the Bayesian expected utility formulation, Sun et al. (2012) show that with a continuum of agents, whose private signals are independent conditioned on the macro states of nature, a REE universally exists, it is incentive compatible and efficient. These results have been obtained by means of the law of large numbers. It is of interest to know if the theorems of this paper can be extended in such a framework which makes the law of large numbers applicable.

In view of the recent work Cea-Echenique et al. (2017), it will be of interest to see if one can modify the notion of REE and obtain the results presented in this paper by allowing individuals' preferences to depend endogenously on the information transmitted by prices (price-dependent utility function).

Furthermore, it is of interest to know under what conditions the core-value-REE equivalence theorems hold for the maxmin expected utility framework. For the ex ante case some existence and equivalence results are obtained in He and Yannelis (2015) and Angelopoulos and Koutsougeras (2015).

8 Appendix

8.1 Proofs of Section 3

In order to prove the existence theorem, we show below that the set of V-REE allocations and a fortiori of maxmin REE allocations contains all the selections from the Walrasian equilibrium correspondence of the associated family of complete information economies.

Lemma 8.1 *If (p, x) is an ex post Walrasian equilibrium, then (p, x) is a V-REE, and in particular it is a maxmin REE.*

Proof Let (p, x) an *ex post* Walrasian equilibrium, we want to show that (p, x) is a V-REE. First, notice that x is feasible in the economy $\mathcal{E}$ because so is $x(s)$ in the economy $\mathcal{E}(s)$ for each s, and p is a price function because for any $s \in S$, $p(s) > 0$. Consider the algebra generated by p denoted by $\sigma(p)$, and for each agent

i let $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$. We show that (p, x) is a V-REE for $\mathcal{E}$. Clearly, for all $i \in I$ and $s \in S$, $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$, hence $x_i \in B_i(s, p)$. It remains to prove that x_i maximizes $V_i(\cdot|\mathcal{G}_i^p(s))$ on $B_i(s, p)$. Assume, on the contrary, that there exists an alternative allocation y such that for some agent i and some state s,

$$\begin{aligned} V_i(y_i|\mathcal{G}_i^p(s)) > V_i(x_i|\mathcal{G}_i^p(s)) \text{ and } y_i \in B_i(s, p) \text{ that is} \\ p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s') \quad \text{for all } s' \in \mathcal{G}_i^p(s). \end{aligned} \tag{13}$$

From $(A2)$ it follows that there exists a state $\bar{s} \in \mathcal{G}_i^p(s)$ such that

$$u_i(\bar{s}, y_i(\bar{s})) > u_i(\bar{s}, x_i(\bar{s})).$$

Since $(p(\bar{s}), x(\bar{s}))$ is a Walrasian equilibrium for $\mathcal{E}(\bar{s})$, it follows that, $p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s})$, which clearly contradicts (13). □

Proof of Theorem 3.2 Since S is finite, there is a finite number of complete information economies $\mathcal{E}(s) = \{I, \mathbb{R}_+^\ell, (u_i(s), e_i(s))_{i \in I}\}$, where for any $i \in I$ and any $s \in S$, $u_i(s) := u_i(s, \cdot) : \mathbb{R}_+^\ell \to \mathbb{R}$ is continuous, monotone, quasi-concave; and $e_i(s) \gg 0$. For any $s \in S$, let $W(\mathcal{E}(s))$ the set of Walrasian equilibrium allocations for the economy $\mathcal{E}(s)$. The above assumptions ensure that for any $s \in S$, $W(\mathcal{E}(s)) \neq \emptyset$ and hence the set $W = \left\{x : I \times S \to \mathbb{R}_+^\ell \mid x(s) \in W(\mathcal{E}(s)) \text{ for all } s \in S\right\}$ is non empty. An element of W is an *ex post* Walrasian equilibrium allocation and from Lemma 8.1 it is a V-REE.□

Lemma 8.1 states that any *ex post* Walrasian equilibrium is a V-REE. The converse is not true (see Example 3.3) unless the strict version of $(A2)$ holds, as stated in Proposition 3.4.

Proof of Proposition 3.4 One inclusion is shown in Lemma 8.1. In order to prove the converse, let (p, x) a V-REE and consider for any agent $i \in I$ the algebra $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$. Condition $(A2^*)$ implies that for any $s \in S$ the equilibrium price $p(s)$ is positive in $\mathcal{E}(s)$, i.e., $p(s) > 0$ for any $s \in S$. Clearly, feasibility and budget constrains hold. Assume to the contrary that for some state s the pair $(x(s), p(s))$ is not a Walrasian equilibrium for the complete information economy $\mathcal{E}(s)$. This means that there exists an alternative bundle $y \in \mathbb{R}_+^\ell$ such that for some agent j

$$\begin{aligned} &(i) \quad u_j(s, y) > u_j(s, x_j(s)) \\ &(ii) \quad p(s) \cdot y \leq p(s) \cdot e_i(s). \end{aligned}$$

If $\mathcal{G}_j^p(s) = \{s\}$, from $(A1)$ we get the contradiction. If $\mathcal{G}_j^p(s) \backslash \{s\} \neq \emptyset$, let $z_j(s) = y$ and $z_j(s') = x_j(s')$ for any $s' \in \mathcal{G}_j^p(s) \backslash \{s\}$. Condition $(A2^*)$ implies that $V_j(z_j|\mathcal{G}_j^p(s)) > V_j(x_j|\mathcal{G}_j^p(s))$, and hence there must exist $\bar{s} \in \mathcal{G}_j^p(s)$ such that

$$p(\bar{s}) \cdot z_j(\bar{s}) > p(\bar{s}) \cdot e_j(\bar{s}).$$

This is impossible by the definition of z_j. □

8.2 Proofs of Section 4

Proof of Proposition 4.3 Assume to the contrary that there exist an agent $i \in I$ and two states $a, b \in S$ such that $a \in \mathcal{G}_i^p(b)$ and $x_i(a) \neq x_i(b)$. Consider $z_i(s) = \alpha x_i(a) + (1-\alpha)x_i(b)$ for all $s \in \mathcal{G}_i^p(b)$, where $\alpha \in (0,1)$, and notice that z_i is constant in the event $\mathcal{G}_i^p(b)$. Moreover,

$$\underline{u}_i^{REE}(b, z_i) = \min_{s \in \mathcal{G}_i^p(b)} u_i(s, z_i(s)) = \min_{s \in \mathcal{G}_i^p(b)} u_i(s, \alpha x_i(a) + (1-\alpha)x_i(b)).$$

Since $u_i(\cdot, y)$ is $\mathcal{G}_i^p$-measurable for all $y \in \mathbb{R}_+^\ell$, from strict quasi-concavity of u_i it follows that

$$\begin{aligned}\underline{u}_i^{REE}(b, z_i) &= u_i(b, \alpha x_i(a) + (1-\alpha)x_i(b)) > \min\{u_i(b, x_i(a)), u_i(b, x_i(b))\}\\ &= \min\{u_i(a, x_i(a)), u_i(b, x_i(b))\} \geq \min_{s \in \mathcal{G}_i^p(b)} u_i(s, x_i(s))\\ &= \underline{u}_i^{REE}(b, x_i).\end{aligned}$$

Since (p, x) is a maxmin rational expectations equilibrium it follows that $z_i \notin B_i(b, p)$, that is, there exists a state $s_i \in \mathcal{G}_i^p(b)$ such that

$$\begin{aligned}p(s_i) \cdot z_i(s_i) > p(s_i) \cdot e_i(s_i) \;\Rightarrow\; \alpha p(s_i) \cdot x_i(a)\\ +(1-\alpha)p(s_i) \cdot x_i(b) > p(s_i) \cdot e_i(s_i).\end{aligned}$$

Moreover, since $p(\cdot)$ and $e_i(\cdot)$ are $\mathcal{G}_i^p$-measurable and $p(s) \cdot x_i(s) \leq p(s) \cdot e_i(s)$ for all $s \in S$, it follows that $p(s_i) \cdot e_i(s_i) > p(s_i) \cdot e_i(s_i)$, which is a contradiction. □

Proof of Proposition 4.5 Clearly if $\sigma(e_i) \subseteq \mathcal{G}_i^p$ and $y_i(\cdot)$ is $\mathcal{G}_i^p$-measurable, then $p(s) \cdot y_i(s) \leq p(s) \cdot e_i(s)$ is equivalent to $p(s') \cdot y_i(s') \leq p(s') \cdot e_i(s')$ for all $s' \in \mathcal{G}_i^p(s)$. Thus, all we need to show is that the maxmin utility and the (Bayesian) interim expected utility coincide. Since for all $i \in I$, $\sigma(u_i) \subseteq \mathcal{F}_i$ and $\mathcal{F}_i \subseteq \mathcal{G}_i^p$, then $\sigma(u_i) \subseteq \mathcal{G}_i^p$.

Moreover, since for each $i \in I$, $x_i(\cdot)$ is $\mathcal{G}_i^p$-measurable it follows that for all $i \in I$ and $s \in S$, both maxmin and interim utility function are equal to the *ex post* utility function. That is,

$$\underline{u}_i^{REE}(s, x_i) = \min_{s' \in \mathcal{G}_i^p(s)} u_i(s', x_i(s')) = u_i(s, x_i(s)) \tag{14}$$

and

$$E_\pi(u_i(\cdot, x(\cdot))|\mathcal{G}_i^p(s)) = \sum_{s' \in \mathcal{G}_i^p(s)} u_i(s', x_i(s')) \frac{\pi_i(s')}{\pi_i\left(\mathcal{G}_i^p(s)\right)} = u_i(s, x_i(s)). \tag{15}$$

From (14) and (15) it follows that for all i and s, $\underline{u}_i^{REE}(s, x_i) = E_\pi(u_i(\cdot, x(\cdot))|\mathcal{G}_i^p(s))$. Therefore, we can conclude that if (p, x) is a traditional REE, then (p, x) is a MREE; the converse is also true if $x_i(\cdot)$ is $\mathcal{G}_i^p$-measurable for all $i \in I$. □

From Lemma 8.1 it follows that any ex post Walrasian equilibrium is a maxmin REE. The converse is not true (see Example 3.3) unless agents' utility functions and initial endowments are private information measurable. The next lemma, which is useful for the proof of Proposition 4.7, holds true for the general MEU formulation (5) provided that for any agent i and state s, the set $\mathcal{M}_i^s$ contains only positive priors (see Sect. 8.5).

Lemma 8.2 *If $(u_i, e_i) \subseteq \mathcal{F}_i$ for all $i \in I$, then any ex post Walrasian equilibrium is a maxmin REE and vice versa.*

Proof One inclusion is shown in Lemma 8.1 and no private information measurability assumption is needed. In order to prove the converse, let (p, x) a maxmin REE and consider for any agent $i \in I$ the algebra $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$. The monotonicity assumption on agents' utility function ensures that for any $s \in S$ the equilibrium price $p(s)$ is positive in $\mathcal{E}(s)$, i.e., $p(s) > 0$ for any $s \in S$. Clearly, feasibility and the budget constraint hold. Assume to the contrary that for some state s the pair $(x(s), p(s))$ is not a Walrasian equilibrium for the complete information economy $\mathcal{E}(s)$. This means that there exist an agent j and an alternative bundle $y \in \mathbb{R}_+^\ell$ such that

$$\begin{aligned} &(i) \quad u_j(s, y) > u_j(s, x_j(s)), \\ &(ii) \quad p(s) \cdot y \leq p(s) \cdot e_i(s). \end{aligned}$$

Let $z_j(s') = y$ for any $s' \in \mathcal{G}_j^p(s)$, and notice that since $u_j(\cdot, z)$ is $\mathcal{F}_j$-measurable and a fortiori $\mathcal{G}_j^p$-measurable, from (i) it follows that $\underline{u}_j^{REE}(s, z_j) = u_j(s, y) > u_j(s, x_j(s)) \geq \underline{u}_i^{REE}(s, x_i)$. Recall that (p, x) is a maxmin rational expectations equilibrium, thus there exists $\bar{s} \in \mathcal{G}_j^p(s)$ such that $p(\bar{s}) \cdot z_j(\bar{s}) > p(\bar{s}) \cdot e_j(\bar{s})$. Since $e_j(\cdot)$ and $p(\cdot)$ are $\mathcal{G}_j^p$-measurable, it follows that $p(s) \cdot y > p(s) \cdot e_i(s)$, which contradicts (ii) above. □

Remark 8.3 Proposition 4.7 states that if in addition $u_i(s, \cdot)$ is strict quasi-concave for all $i \in I$ and $s \in S$, the *ex post* Walrasian equilibria coincide also with the (traditional) rational expectations equilibrium (see also Einy et al. 2000b and De Simone and Tarantino 2010).

Proof of Proposition 4.7 The equivalence between (1) and (2) is obtained by combining Propositions 4.3 and 4.5 (see Remark 4.6). The equivalence between (1) and (3) is instead stated in Lemma 8.2. □

Proof of Proposition 4.10 For each $s \in S$, let

$$H(s) = \{h \in \{1, \ldots, \ell\} \ : \ p^h(s) = 0\},$$

and let

$$\bar{S} = \{s \in S \ : \ H(s) \neq \emptyset\}.$$

Since (p, x) is a maxmin REE, we consider the information generated by the equilibrium price, that is the algebra $\sigma(p)$. Clearly, $H(\cdot)$ is $\sigma(p)$-measurable,[20] because $p(s_1) = p(s_2)$ whenever $\sigma(p)(s_1) = \sigma(p)(s_2)$. Moreover, since for any $i \in I, \sigma(p) \subseteq \mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$, it follows that for all $i \in I$

$$H(\cdot) \text{ is } \mathcal{G}_i^p\text{-measurable.} \tag{16}$$

Now, assume to the contrary that $\bar{S}$ is non empty and let $\bar{s} \in \bar{S}$. Hence, $H(\bar{s}) \neq \emptyset$, i.e., there exists at least a "free" good h such that $p^h(\bar{s}) = 0$. Let $i \in I$ be the agent such that $u_i(s, \cdot)$ is strictly monotone for any $s \in S$; and define the following allocation:

$$z_i^h(s) = \begin{cases} x_i^h(s) + K & \text{if } s \in \mathcal{G}_i^p(\bar{s}) \text{ and } h \in H(s) \\ x_i^h(s) & \text{otherwise,} \end{cases}$$

where $K > 0$.

Notice that for any $s \in \mathcal{G}_i^p(\bar{s})$, since $H(s) = H(\bar{s}) \neq \emptyset$ [see (16)], from the strict monotonicity it follows that $u_i(s, z_i(s)) > u_i(s, x_i(s))$ for all $s \in \mathcal{G}_i^p(\bar{s})$, and hence

$$\underline{u}_i^{REE}(\bar{s}, z_i) > \underline{u}_i^{REE}(\bar{s}, x_i).$$

Since (p, x) is a maxmin REE, $z_i \notin B_i(\bar{s}, p)$, that is there exists a state $s_i \in \mathcal{G}_i^p(\bar{s})$ such that

$$p(s_i) \cdot [z_i(s_i) - e_i(s_i)] > 0.$$

From (16), it follows that $H(s_i) = H(\bar{s}) \neq \emptyset$, and therefore

$$\begin{aligned} 0 &< p(s_i) \cdot [z_i(s_i) - e_i(s_i)] \\ &= \sum_{h \in H(s_i)} p^h(s_i)[x_i^h(s_i) + K - e_i^h(s_i)] + \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] \\ &= 0 + \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] \\ &= \sum_{h \in H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] + \sum_{h \notin H(s_i)} p^h(s_i)[x_i^h(s_i) - e_i^h(s_i)] \\ &= p(s_i) \cdot [x_i(s_i) - e_i(s_i)] \leq 0. \end{aligned}$$

This is a contradiction, hence $p(s) \gg 0$ for each $s \in S$. □

Proof of Proposition 4.11 Let (p, x) be a maxmin rational expectations equilibrium and define for each agent $i \in I$ and state $s \in S$ the following set:

$$M_i(s) = \left\{ s' \in \mathcal{G}_i^p(s) : \ \underline{u}_i^{REE}(s, x_i) = u_i(s', x_i(s')) \right\}.$$

[20] We mean that $H(s_1) = H(s_2)$ if $\sigma(p)(s_1) = \sigma(p)(s_2)$.

Clearly, since S is finite, for all $i \in I$ and $s \in S$, the set $M_i(s)$ is nonempty, i.e., $M_i(s) \neq \emptyset$. Moreover, if $s' \in \mathcal{G}_i^p(s) \backslash M_i(s)$ it means that $\underline{u}_i^{REE}(s, x_i) < u_i(s', x_i(s'))$. Thus, we want to show that for all $i \in I$ and $s \in S$, $M_i(s) = \mathcal{G}_i^p(s)$.

Assume to the contrary that there exist an agent $j \in I$ and a state $\bar{s} \in S$ such that $\mathcal{G}_j^p(\bar{s}) \backslash M_j(\bar{s}) \neq \emptyset$. Notice that

$$\underline{u}_j^{REE}(\bar{s}, x_j) < u_j(s, x_j(s)) \quad \text{for any } s \in \mathcal{G}_j^p(\bar{s}) \backslash M_j(\bar{s}).$$

Fix $s' \in \mathcal{G}_j^p(\bar{s}) \backslash M_j(\bar{s})$ and define the following allocation

$$y_j(s) = \begin{cases} x_j(s) & \text{if } s \in \mathcal{G}_j^p(\bar{s}) \backslash M_j(\bar{s}) \\ x_j(s') & \text{if } s \in M_j(\bar{s}). \end{cases}$$

Since the utility functions are assumed to be private information measurable, it follows that $u_j(s, y_j(s)) > \underline{u}_j^{REE}(\bar{s}, x_j)$ for any $s \in \mathcal{G}_j(\bar{s})$, and hence $\underline{u}_j^{REE}(\bar{s}, y_j) > \underline{u}_j^{REE}(\bar{s}, x_j)$. Recall that (p, x) is a maxmin REE, therefore there exists $s \in \mathcal{G}_j^p(\bar{s})$ such that $p(s) \cdot y_j(s) > p(s) \cdot e_j(s)$. If $s \in M_j(\bar{s})$, then $p(s) \cdot x_j(s') > p(s) \cdot e_j(s)$. Since $p(\cdot)$ and $e_j(\cdot)$ are both $\mathcal{G}_j^p$-measurable, it follows that $p(s') = p(s)$ and $e_j(s') = e_j(s)$. This implies that $p(s') \cdot x_j(s') > p(s') \cdot e_j(s')$, which is clearly a contradiction. On the other hand, if $s \in G_j(\bar{s}) \backslash M_j(\bar{s})$, we have that $p(s) \cdot x_j(s) > p(s) \cdot e_j(s)$ which is a contradiction as well. Therefore, for each $i \in I$ and $s \in S$, $M_i(s) = \mathcal{G}_i^p(s)$. □

8.3 Proofs of Section 5

Proof of Proposition 5.3 Let x be a maxmin Pareto optimal allocation with respect to the information structure Π and assume to the contrary that there exists a feasible allocation y such that $u_i(s, y_i(s)) \geq u_i(s, x_i(s))$ for all $i \in I$ and all $s \in S$ with at least one strict inequality.

Let $j \in I$ and $\bar{s} \in S$ such that $u_j(\bar{s}, y_j(\bar{s})) > u_j(\bar{s}, x_j(\bar{s}))$. Hence, $y_j(\bar{s}) > 0$ and if $u_j(\bar{s}, t) = u_j(\bar{s}, 0)$ for any $t \in \partial\mathbb{R}_+^\ell$, then $y_j(\bar{s}) \gg 0$. Thanks to continuity of $u_j(\bar{s}, \cdot)$ there exists $\epsilon \in (0, 1)$ for which $u_j(\bar{s}, \epsilon y_j(\bar{s})) > u_j(\bar{s}, x_j(\bar{s}))$. Consider the feasible allocation z given by $z_i(s) = y_i(s)$ for any $i \in I$ and $s \in S \backslash \{\bar{s}\}$; while in $\bar{s}$

$$z_i(\bar{s}) = \begin{cases} \epsilon y_j(\bar{s}) & \text{if } i = j \\ y_i(\bar{s}) + \frac{1-\epsilon}{n-1} y_j(\bar{s}) & \text{otherwise.} \end{cases}$$

From the strict monotonicity it follows that the feasible allocation z is such that

$$u_i(s, z_i(s)) \geq u_i(s, x_i(s)) \quad \text{for any } i \in I \text{ and } s \in S,$$
$$u_i(\bar{s}, z_i(\bar{s})) > u_i(\bar{s}, x_i(\bar{s})) \quad \text{for any } i \in I.$$

The same happens if $u_j(\bar{s}, t) = u_j(\bar{s}, 0)$ for any $t \in \partial\mathbb{R}_+^\ell$ because $y_j(\bar{s}) \gg 0$ and $u_i(\bar{s}, \cdot)$ is monotone.

Let $k \in I$ be such that $\Pi_k(\bar{s}) = \{\bar{s}\}$, then

$$\begin{aligned} &\underline{u}_i^{\Pi_i}(s, z_i) \geq \underline{u}_i^{\Pi_i}(s, x_i) \quad \text{for any } i \in I \text{ and } s \in S, \\ &\underline{u}_k^{\Pi_k}(\bar{s}, z_k) = u_k(\bar{s}, z_k(\bar{s})) > u_k(\bar{s}, x_k(\bar{s})) = \underline{u}_k^{\Pi_k}(\bar{s}, x_k). \end{aligned}$$

Therefore, x is not maxmin efficient with respect to the information structure Π, which is a contradiction. We now show that the converse may not be true.[21] To this end consider an asymmetric information economy with two agents $I = \{1, 2\}$, two goods and two states $S = \{a, b\}$. The primitives are as follows:

$$\begin{array}{ll} \Pi_1 = \{\{a\}, \{b\}\} & \Pi_2 = \{\{a, b\}\} \\ e_1(a) = (1, 2) & e_2(a) = (1, 1) \\ e_1(b) = (2, 1) & e_2(b) = (1, 1) \\ u_i(a, x, y) = \sqrt{xy} & u_i(b, x, y) = xy \end{array}$$

Notice that since the first agent is fully informed, the information structure Π satisfies the assumption that for any state s there exists an agent i such that $\Pi_i(s) = \{s\}$. The following feasible allocation

$$(x_i(a), y_i(a)) = \left(1, \frac{3}{2}\right) \quad (x_i(b), y_i(b)) = \left(\frac{3}{2}, 1\right) \quad \text{for any } i \in I$$

is *ex post* efficient. Indeed assume to the contrary the existence of an alternative feasible allocation (t, z) such that $t_i(s)z_i(s) \geq \frac{3}{2}$ for all $i \in I$ and $s \in S$, with at least one strict inequality.

Without loss of generality let $t_1(a)z_1(a) > \frac{3}{2}$, which means that[22] $z_1(a) > \frac{3}{2t_1(a)}$. This together with feasibility imply that

$$(2 - t_1(a))\left(3 - \frac{3}{2t_1(a)}\right) > (2 - t_1(a))\,(3 - z_1(a)) = t_2(a)z_2(a) \geq \frac{3}{2},$$

which causes the contradiction $(t_1(a) - 1)^2 < 0$. Hence, (x, y) is *ex post* Pareto optimal. We now show that it is not maxmin efficient with respect to the information structure Π. To this end consider the following feasible allocation

$$\begin{aligned} &(t_i(a), z_i(a)) = (x_i(a), y_i(a)) \quad \text{for any } i \in I, \\ &(t_1(b), z_1(b)) = \left(\frac{7}{4}, 1\right) \\ &(t_2(b), z_2(b)) = \left(\frac{5}{4}, 1\right), \end{aligned}$$

[21] Kreps's example can also be used to show that an *ex post* efficient allocation may not be maxmin Pareto optimal (see Remark 5.11).

[22] Notice that $(t_1(a), z_1(a)) \gg 0$ because $t_1(a)z_1(a) > \frac{3}{2} > 0$.

and notice that

$$\underline{u}_1^{\Pi_1}(a, t_1, z_1) = \underline{u}_1^{\Pi_1}(a, x_1, y_1)$$
$$\underline{u}_1^{\Pi_1}(b, t_1, z_1) = \frac{7}{4} > \frac{3}{2} = \underline{u}_1^{\Pi_1}(b, x_1, y_1)$$
$$\underline{u}_2^{\Pi_2}(a, t_2, z_2) = \underline{u}_2^{\Pi_2}(b, t_2, z_2) = \min\left\{\sqrt{\frac{3}{2}}, \frac{5}{4}\right\} = \sqrt{\frac{3}{2}}$$
$$= \min\left\{\sqrt{\frac{3}{2}}, \frac{3}{2}\right\} = \underline{u}_2^{\Pi_2}(b, x_2, y_2) = \underline{u}_2^{\Pi_2}(a, x_2, y_2).$$

Thus, the allocation (x, y) is *ex post* efficient but not maxmin Pareto optimal with respect to the information structure Π. □

The next example shows that the assumption that for any state s there exists an agent $i \in I$ such that $\Pi_i(s) = \{s\}$ is crucial in the proof of Proposition 5.3.

Example 8.4 Consider an asymmetric information economy with two agents $I = \{1, 2\}$, two goods and three states $S = \{a, b, c\}$, whose primitives are given as follows:

$$\begin{array}{ll} \Pi_1 = \{\{a\}, \{b, c\}\} & \Pi_2 = \{\{a, b\}, \{c\}\} \\ e_1(a) = (4, 4) & e_2(a) = (0, 0) \\ e_1(b) = (2, 2) & e_2(b) = (2, 2) \\ e_1(c) = (0, 0) & e_2(c) = (4, 4), \end{array}$$

$u_i(\cdot, x, y) = xy$ for any $i \in I$. Notice that $\{b\} \neq \Pi_i(b)$ for any $i \in I$. The following feasible allocation $(x_i(s), y_i(s)) = e_i(s)$ for any i and any $s \neq b$; $(x_1(b), y_1(b)) = (1, 3)$ and $(x_2(b), y_2(b)) = (3, 1)$ is not *ex post* efficient since it is blocked by the initial endowment, but it is maxmin Pareto optimal with respect to the information structure (Π_1, Π_2). Indeed, assume by the way of contradiction the existence of an alternative feasible allocation (t, z) such that

$$\begin{array}{ll} (i) & t_1(a)z_1(a)) \geq 16 \\ (ii) & \min\{t_1(b)z_1(b), t_1(c)z_1(c)\} \geq \min\{3, 0\} = 0 \\ (iii) & \min\{t_2(a)z_2(a), t_2(b)z_2(b)\} \geq \min\{0, 3\} = 0 \\ (iv) & t_2(c)z_2(c) \geq 16, \end{array}$$

with at least one strict inequality. If one of (i) and (iii) is strict, then $(4 - t_1(a))\left(4 - \frac{16}{t_1(a)}\right) > 0$ or equivalently that $(t_1(a) - 4)^2 < 0$ which is a contradiction. Similarly if one of (ii) and (iv) is strict.

Proof of Theorem 5.4 Let (p, x) be a maxmin rational expectations equilibrium.

I CASE: If $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ for each $i \in I$, Lemma 8.2 ensures that x is an *ex post* Walrasian equilibrium allocation and therefore it is *ex post* efficient. We now show that it is also maxmin Pareto optimal. To this end, assume to the contrary that there exists an alternative feasible allocation y such that $\underline{u}_i^{REE}(s, y_i) \geq \underline{u}_i^{REE}(s, x_i)$ for all $i \in I$ and all $s \in S$, with at least one strict inequality. Proposition 4.11 implies that for any agent $i \in I$ and any state $s \in S$

$$u_i(s, y_i(s)) \geq \underline{u}_i^{REE}(s, y_i) \geq \underline{u}_i^{REE}(s, x_i) = u_i(s, x_i(s)),$$

with at least one strict inequality. This means that x is not ex post efficient which is a contradiction.

II CASE: Assume that p is fully revealing. Clearly since $\mathcal{G}_i^p(s) = \{s\}$ for all i and s, maxmin Pareto optimality with respect to the information structure $\mathcal{G}^p$ coincides with the *ex post* efficiency. We have already observed that in this case a maxmin REE is an *ex post* Walrasian equilibrium and hence it is both *ex post* and maxmin efficient.

Example 8.5 and Remark 5.11 show that if none of the above conditions is satisfied, a maxmin REE may not be maxmin efficient. □

Proof of Proposition 5.8 Let x be a weak maxmin efficient allocation and assume, on the contrary, that there exists an alternative feasible allocation y such that $u_i(s, y_i(s)) > u_i(s, x_i(s))$ for all $i \in I$ and all $s \in S$. Thus, for each agent $i \in I$ whatever her information partition is Π_i, it follows that $\underline{u}_i^{\Pi_i}(s, y_i) > \underline{u}_i^{\Pi_i}(s, x_i)$ for each state s. Hence, we get a contradiction since x is weak maxmin Pareto optimal. In order to show that the converse may not be true, consider an economy with two agents, three states of nature, $S = \{a, b, c\}$, and two goods, such that

$$u_i(a, x_i, y_i) = \sqrt{x_i y_i} \quad u_i(b, x_i, y_i) = x_i y_i \quad u_i(c, x_i, y_i) = x_i^2 y_i \quad \text{for all } i = 1, 2.$$
$$e_1(a) = (2, 1) \quad e_2(a) = e_1(b) = e_2(b) = e_1(c) = e_2(c) = (1, 2)$$
$$\Pi_1 = \{\{a, c\}, \{b\}\} \quad \Pi_2 = \{\{a\}, \{b, c\}\}.$$

Consider the following feasible allocation:

$$(x_1(a), y_1(a)) = \left(3, \frac{1}{3}\right) \quad (x_2(a), y_2(a)) = \left(0, \frac{8}{3}\right)$$
$$(x_1(b), y_1(b)) = (1, 2) \quad (x_2(b), y_2(b)) = (1, 2)\,,$$
$$(x_1(c), y_1(c)) = (2, 1) \quad (x_2(c), y_2(c)) = (0, 3)\,.$$

Notice that it is weak *ex post* efficient. Indeed, if on the contrary there exists (t, z) such that

$$u_i(s, t_i(s), z_i(s)) > u_i(s, x_i(s), y_i(s)) \quad \text{for all } i \in I \text{ and all } s \in S,$$

in particular,

$$\begin{cases} t_1(b)z_1(b) > 2 \\ t_2(b)z_2(b) > 2 \\ t_1(b) + t_2(b) = 2 \\ z_1(b) + z_2(b) = 4, \end{cases}$$

then[23]

$$\begin{cases} z_1(b) > \frac{2}{t_1(b)} \\ (2-t_1(b))(2t_1(b)-1) > t_1(b). \end{cases}$$

This implies that $(t_1(b)-1)^2 < 0$, which is impossible. Thus, the above allocation is weak *ex post* Pareto optimal, but it is not weak maxmin efficient with respect to the information structure Π, since it is (maxmin) blocked by the following feasible allocation:

$$\begin{aligned} (t_1(a), z_1(a)) &= \left(\frac{5}{4}, \frac{5}{2}\right) \quad (t_2(a), z_2(a)) = \left(\frac{7}{4}, \frac{1}{2}\right) \\ (t_1(b), z_1(b)) &= \left(1, \frac{8}{3}\right) \quad (t_2(b), z_2(b)) = \left(1, \frac{4}{3}\right) \\ (t_1(c), z_1(c)) &= \left(\frac{3}{4}, 2\right) \quad (t_2(c), z_2(c)) = \left(\frac{5}{4}, 2\right). \end{aligned}$$

Indeed,

$$\begin{aligned} \underline{u}_1^{\Pi_1}(a, t_1, z_1) &= \underline{u}_1^{\Pi_1}(c, t_1, z_1) = \min\left\{\sqrt{\frac{25}{8}}, \frac{9}{8}\right\} = \frac{9}{8} \\ &> 1 = \min\{1, 4\} = \underline{u}_1^{\Pi_1}(c, x_1, y_1) = \underline{u}_1^{\Pi_1}(a, x_1, y_1) \\ \underline{u}_1^{\Pi_1}(b, t_1, z_1) &= u_1(b, t_1(b), z_1(b)) = \frac{8}{3} \\ &> 2 = u_1(b, x_1(b), y_1(b)) = \underline{u}_1^{\Pi_1}(b, x_1, y_1) \\ \underline{u}_2^{\Pi_2}(a, t_2, z_2) &= u_2(a, t_2(a), z_2(a)) = \sqrt{\frac{7}{8}} \\ &> 0 = u_2(a, x_2(a), y_2(a)) = \underline{u}_2^{\Pi_2}(a, x_2, y_2) \\ \underline{u}_2^{\Pi_2}(b, t_2, z_2) &= \underline{u}_2^{\Pi_2}(c, t_2, z_2) = \min\left\{\frac{4}{3}, \frac{25}{8}\right\} = \frac{4}{3} \\ &> 0 = \min\{2, 0\} = \underline{u}_2^{\Pi_2}(c, x_2, y_2) = \underline{u}_2^{\Pi_2}(b, x_2, y_2). \end{aligned}$$

□

Proof of Theorem 5.9 Clearly in the first two cases the result easily follows from Theorem 5.4 and from the observation that any allocation maxmin efficient with respect to Π is weak maxmin Pareto optimal with respect to Π.

Let (p, x) be a maxmin rational expectations equilibrium, and assume to the contrary that there exists an alternative feasible allocation y such that $\underline{u}_i^{REE}(s, y_i) > \underline{u}_i^{REE}(s, x_i)$ for all $i \in I$ and all $s \in S$.

III CASE: there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{G}_i^p(\bar{s})$ for all $i \in I$.

[23] Clearly, $(t_i(b), z_i(b)) \gg (0, 0)$ for each $i = 1, 2$.

Since for each $i \in I$, $\{\bar{s}\} = \mathcal{G}_i^p(\bar{s})$; it follows that $\underline{u}_i^{REE}(\bar{s}, y_i) = u_i(\bar{s}, y_i(\bar{s})) > u_i(\bar{s}, x_i(\bar{s})) = \underline{u}_i^{REE}(\bar{s}, x_i)$ for all $i \in I$. Hence, since (p, x) is a MREE, for each agent i there exists at least one state $s_i \in \mathcal{G}_i^p(\bar{s}) = \{\bar{s}\}$ (that is $s_i = \bar{s}$ for all $i \in I$) such that $p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s})$. Therefore,

$$\sum_{i \in I} p(\bar{s})[y_i(\bar{s}) - e_i(\bar{s})] > 0,$$

which contradicts the feasibility of y.

IV CASE: $n - 1$ agents are fully informed.

Since (p, x) is a MREE and y is preferred by anyone to x, it follows that for any state $s \in S$ and any agent $i \in I$ there exists at least one state $s_i \in \mathcal{G}_i^p(s)$ such that $p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i)$. Let j be the unique not fully informed agent, and consider the state s_j for which $p(s_j) \cdot y_j(s_j) > p(s_j) \cdot e_j(s_j)$. Since each agent $i \neq j$ is fully informed, it follows that $\mathcal{G}_i^p(s_j) = \{s_j\}$ for all $i \neq j$. Thus,

$$p(s_j) \cdot y_i(s_j) > p(s_j) \cdot e_i(s_j) \quad \text{for all } i \in I.$$

Hence,

$$\sum_{i \in I} p(s_j) \cdot y_i(s_j) > \sum_{i \in I} p(s_j) \cdot e_i(s_j),$$

which is a contradiction.

Example 8.5 below shows that if no condition of Theorem 5.9 is satisfied, then a maxmin REE may not be weak maxmin efficient (and a fortiori it may not be maxmin Pareto optimal). □

Example 8.5 Consider an asymmetric information economy with three states of nature, $S = \{a, b, c\}$, two goods, $\ell = 2$ (the first good is considered as numeraire) and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{lll}
e_1(a) = e_1(b) = (2, 1) & e_1(c) = (3, 1) & \mathcal{F}_1 = \{\{a, b\}; \{c\}\} \\
e_2(a) = e_2(c) = (1, 2) & e_2(b) = (2, 2) & \mathcal{F}_2 = \{\{a, c\}; \{b\}\} \\
e_3(b) = e_3(c) = (2, 1) & e_3(a) = (3, 1) & \mathcal{F}_3 = \{\{a\}; \{b, c\}\}. \\
u_1(a, x, y) = \sqrt{xy} & u_1(b, x, y) = log(xy) & u_1(c, x, y) = \sqrt{xy}, \\
u_2(a, x, y) = log(xy) & u_2(b, x, y) = \sqrt{xy} & u_2(c, x, y) = \sqrt{xy}, \\
u_3(a, x, y) = \sqrt{xy} & u_3(b, x, y) = \sqrt{xy} & u_3(c, x, y) = log(xy).
\end{array}$$

Consider the following maxmin rational expectations equilibrium

$$\begin{array}{llll}
(p(a), q(a)) = \left(1, \frac{3}{2}\right) & (x_1(a), y_1(a)) = \left(\frac{7}{4}, \frac{7}{6}\right) & (x_2(a), y_2(a)) = \left(2, \frac{4}{3}\right) & (x_3(a), y_3(a)) = \left(\frac{9}{4}, \frac{3}{2}\right) \\
(p(b), q(b)) = \left(1, \frac{3}{2}\right) & (x_1(b), y_1(b)) = \left(\frac{7}{4}, \frac{7}{6}\right) & (x_2(b), y_2(b)) = \left(\frac{5}{2}, \frac{5}{3}\right) & (x_3(b), y_3(b)) = \left(\frac{7}{4}, \frac{7}{6}\right) \\
(p(c), q(c)) = \left(1, \frac{3}{2}\right) & (x_1(c), y_1(c)) = \left(\frac{9}{4}, \frac{3}{2}\right) & (x_2(c), y_2(c)) = \left(2, \frac{4}{3}\right) & (x_3(c), y_3(c)) = \left(\frac{7}{4}, \frac{7}{6}\right),
\end{array}$$

and notice that it is a non revealing equilibrium, since $(p(a), q(a)) = (p(b), q(b)) = (p(c), q(c))$ and hence $\sigma(p, q) = \{\{a, b, c\}\}$, that is $\mathcal{G}_i^p = \mathcal{F}_i$ for any $i \in I$. Moreover, notice that no condition of Theorems 5.4 and 5.9 is satisfied. We now show that the equilibrium allocation is not weak maxmin Pareto optimal with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i \in I}$ and a fortiori it is neither maxmin efficient. Indeed, consider the following feasible allocation

$$(t_1(a), z_1(a)) = \left(\frac{20}{12}, \frac{13}{12}\right) \quad (t_2(a), z_2(a)) = \left(\frac{25}{12}, \frac{16}{12}\right) \quad (t_3(a), z_3(a)) = \left(\frac{27}{12}, \frac{19}{12}\right)$$
$$(t_1(b), z_1(b)) = \left(\frac{22}{12}, \frac{14}{12}\right) \quad (t_2(b), z_2(b)) = \left(\frac{30}{12}, \frac{21}{12}\right) \quad (t_3(b), z_3(b)) = \left(\frac{20}{12}, \frac{13}{12}\right)$$
$$(t_1(c), z_1(c)) = \left(\frac{28}{12}, \frac{18}{12}\right) \quad (t_2(c), z_2(c)) = \left(\frac{23}{12}, \frac{15}{12}\right) \quad (t_3(c), z_3(c)) = \left(\frac{21}{12}, \frac{15}{12}\right),$$

and notice that,

$$\begin{aligned}
\underline{u}_1^{REE}(a, t_1, z_1) &= \underline{u}_1^{REE}(b, t_1, z_1) = \min\left\{\sqrt{\frac{260}{144}}, log\frac{308}{144}\right\} = log\frac{308}{144} > log\frac{49}{24} \\
&= \min\left\{\sqrt{\frac{49}{24}}, log\frac{49}{24}\right\} = \underline{u}_1^{REE}(a, x_1, y_1) = \underline{u}_1^{REE}(b, x_1, y_1), \\
\underline{u}_1^{REE}(c, t_1, z_1) &= u_1(c, t_1(c), z_1(c)) = \sqrt{\frac{504}{144}} > \sqrt{\frac{27}{8}} \\
&= u_1(c, x_1(c), y_1(c)) = \underline{u}_1^{REE}(c, x_1, y_1), \\
\underline{u}_2^{REE}(a, t_2, z_2) &= \underline{u}_2^{REE}(c, t_2, z_2) = \min\left\{log\frac{400}{144}, \sqrt{\frac{345}{144}}\right\} = log\frac{400}{144} > log\frac{8}{3} \\
&= \min\left\{log\frac{8}{3}, \sqrt{\frac{8}{3}}\right\} = \underline{u}_2^{REE}(a, x_2, y_2) = \underline{u}_2^{REE}(c, x_2, y_2), \\
\underline{u}_2^{REE}(b, t_2, z_2) &= u_2(b, t_2(b), z_2(b)) = \sqrt{\frac{630}{144}} > \sqrt{\frac{25}{6}} = u_2(b, x_2(b), y_2(b)) \\
&= \underline{u}_2^{REE}(b, x_2, y_2), \\
\underline{u}_3^{REE}(a, t_3, z_3) &= u_3(a, t_3(a), z_3(a)) = \sqrt{\frac{513}{144}} > \sqrt{\frac{27}{8}} = u_3(a, x_3(a), y_3(a)) \\
&= \underline{u}_3^{REE}(a, x_3, y_3), \\
\underline{u}_3^{REE}(b, t_3, z_3) &= \underline{u}_3^{REE}(c, t_3, z_3) = \min\left\{\sqrt{\frac{260}{144}}, log\frac{315}{144}\right\} = log\frac{315}{144} > log\frac{49}{24} \\
&= \min\left\{\sqrt{\frac{49}{24}}, log\frac{49}{24}\right\} = \underline{u}_3^{REE}(b, x_3, y_3) = \underline{u}_3^{REE}(c, x_3, y_3).
\end{aligned}$$

Hence, the equilibrium allocation (x, y) is not weak maxmin Pareto optimal with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i \in I}$.

The following example shows that if there exists a state that everybody may distinguish (see condition (iii) of Theorem 5.9) then according to Theorem 5.9, a maxmin REE allocation is weak maxmin efficient with respect to the information structure $(\mathcal{G}_i^p)_{i\in I}$, but it is not maxmin Pareto optimal.

Example 8.6 Consider an asymmetric information economy with five states of nature, $S = \{a, b, c, d, f\}$, two goods and two agents, $I = \{1, 2\}$ whose characteristics are given as follows:

$$\begin{array}{lll} e_1(a)=e_1(b)=(1,2) & e_1(c)=e_1(d)=e_1(f)=(2,1) & \mathcal{F}_1=\{\{a,b\};\{c,d\};\{f\}\} \\ e_2(a)=e_2(c)=e_2(d)=e_2(f)=(2,1) & e_2(b)=(1,2) & \mathcal{F}_2=\{\{a,c\};\{b\};\{d,f\}\}. \\ u_i(a,x,y)=u_i(c,x,y)=\sqrt{xy} & u_i(b,x,y)=u_i(d,x,y)=log(xy) & u_i(f,x,y)=xy. \end{array}$$

Consider the following maxmin rational expectations equilibrium

$$\begin{array}{lll} (p(a),q(a))=(1,1) & (x_1(a),y_1(a))=\left(\frac{3}{2},\frac{3}{2}\right) & (x_2(a),y_2(a))=\left(\frac{3}{2},\frac{3}{2}\right) \\ (p(b),q(b))=\left(1,\frac{1}{2}\right) & (x_1(b),y_1(b))=(1,2) & (x_2(b),y_2(b))=(1,2) \\ (p(c),q(c))=(1,2) & (x_1(c),y_1(c))=(2,1) & (x_2(c),y_2(c))=(2,1) \\ (p(d),q(d))=(1,2) & (x_1(d),y_1(d))=(2,1) & (x_2(d),y_2(d))=(2,1) \\ (p(f),q(f))=(1,2) & (x_1(f),y_1(f))=(2,1) & (x_2(f),y_2(f))=(2,1)\,, \end{array}$$

and notice that $\sigma(p,q) = \{\{a\},\{b\},\{c,d,f\}\}$ and hence, $\mathcal{G}_1^p = \{\{a\},\{b\},\{c,d\},\{f\}\}$ and $\mathcal{G}_2^p = \{\{a\},\{b\},\{c\},\{d,f\}\}$.

For any $i \in I$ the equilibrium allocation (x_i, y_i) is $\mathcal{G}_i^p$-measurable but not $\mathcal{F}_i$-measurable. Moreover notice that the utility functions are not $\mathcal{F}_i$-measurable neither $\mathcal{G}_i^p$-measurable, the equilibrium price is not fully revealing, and no agent is fully informed. On the other hand, there exists a state s such that $\mathcal{G}_i^p(s) = \{s\}$ for any agent i, for example states a and b, but such a condition does not hold for the initial information structure $(\mathcal{F}_i)_{i\in I}$. Thus, only condition (iii) of Theorem 5.9 is satisfied. From this it follows that the equilibrium allocation (x, y) is weak efficient with respect to the information structure $(\mathcal{G}_i^p)_{i\in I}$. We now show that x is not maxmin efficient with respect to the information structure $(\mathcal{G}_i^p)_{i\in I}$. To this end, consider the following feasible allocation

$$\begin{array}{l} (t_i(s),z_i(s)) = (x_i(s),y_i(s)) \quad \text{for any } i=\{1,2\} \text{ and any } s\in\{a,b,d\} \\ (t_1(c),z_1(c)) = \left(\dfrac{3}{2},1\right) \quad (t_2(c),z_2(c)) = \left(\dfrac{5}{2},1\right) \\ (t_1(f),z_1(f)) = \left(\dfrac{5}{2},1\right) \quad (t_2(f),z_2(f)) = \left(\dfrac{3}{2},1\right), \end{array}$$

and notice that,

$$\begin{aligned} \underline{u}_i^{REE}(s,t_i,z_i) &= \underline{u}_i^{REE}(s,x_i,y_i) \quad \text{for any } i\in\{1,2\} \text{ and any } s\in\{a,b\} \\ \underline{u}_1^{REE}(c,t_1,z_1) &= \underline{u}_1^{REE}(d,t_1,z_1) = \min\left\{\sqrt{\frac{3}{2}}, log2\right\} = log2 \\ &= \min\{\sqrt{2}, log2\} = \underline{u}_1^{REE}(d,x_1,y_1) = \underline{u}_1^{REE}(c,x_1,y_1) \end{aligned}$$

$$\underline{u}_2^{REE}(c, t_2, z_2) = u_2(c, t_2(c), z_2(c)) = \sqrt{\frac{5}{2}}$$
$$> \sqrt{2} = u_2(c, x_2(c), y_2(c)) = \underline{u}_2^{REE}(c, x_2, y_2)$$
$$\underline{u}_1^{REE}(f, t_1, z_1) = u_1(f, t_1(f), z_1(f)) = \frac{5}{2}$$
$$> 2 = u_1(f, x_1(f), y_1(f)) = \underline{u}_1^{REE}(f, x_1, y_1)$$
$$\underline{u}_2^{REE}(d, t_2, z_2) = \underline{u}_2^{REE}(f, t_2, z_2) = \min\left\{log2, \frac{3}{2}\right\} = log2$$
$$= \min\{log2, 2\} = \underline{u}_2^{REE}(f, x_2, y_2) = \underline{u}_2^{REE}(d, x_2, y_2).$$

Hence, the equilibrium allocation is not maxmin Pareto optimal with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i\in I}$.

The next example shows that if all agents except one are fully informed (i.e., condition (iv) of Theorem 5.9 holds), then a maxmin REE allocation is weak maxmin efficient with respect to the information structure $(\mathcal{G}_i^p)_{i\in I}$ but it may not be maxmin Pareto optimal.

Example 8.7 Consider an asymmetric information economy with two states of nature, $S = \{a, b\}$, two goods and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{ll} e_1(a) = e_1(b) = \left(\frac{1}{3}, \frac{1}{3}\right) & \mathcal{F}_1 = \{\{a\}; \{b\}\} \\ e_2(a) = e_2(b) = \left(\frac{1}{3}, \frac{1}{3}\right) & \mathcal{F}_2 = \{\{a\}; \{b\}\}. \\ e_3(a) = e_3(b) = \left(\frac{1}{3}, \frac{1}{3}\right) & \mathcal{F}_3 = \{\{a, b\}\}. \\ u_i(a, x, y) = \sqrt{xy} & u_i(b, x, y) = xy \text{ for all } i \in I. \end{array}$$

Notice that for any $i \in I$ $e_i(\cdot)$ is $\mathcal{F}_i$-measurable, while u_i is not. Two agents are fully informed. The initial endowment is a non-revealing maxmin rational expectations equilibrium and there does not exist a state s such that $\mathcal{G}_i^p(s) = \{s\}$ for any i, neither $\mathcal{F}_i(s) = \{s\}$ for any i. Thus, only condition (iv) of Theorem 5.9 is satisfied. From this it follows that the equilibrium allocation e is weak maximin efficient with respect to the information structure $(\mathcal{G}_i^p)_{i\in I}$, and since it is a non-revealing maxmin REE it is also weak maximin efficient with respect to the information structure $(\mathcal{F}_i)_{i\in I}$ (because $\mathcal{G}_i^p = \mathcal{F}_i$ for any $i \in I$). We now show that e is not maxmin efficient with respect to the information structure $(\mathcal{G}_i^p)_{i\in I}$ and hence neither with respect to $(\mathcal{F}_i)_{i\in I}$. To this end, consider the following feasible allocation

$$(t_i(a), z_i(a)) = \left(\frac{5}{12}, \frac{5}{12}\right) \quad \text{for any } i \in \{1, 2\},$$
$$(t_3(a), z_3(a)) = \left(\frac{1}{6}, \frac{1}{6}\right),$$
$$(t_i(b), z_i(b)) = \left(\frac{1}{3}, \frac{1}{3}\right) \quad \text{for any } i \in \{1, 2, 3\}.$$

Notice that,

$$\begin{aligned}
\underline{u}_i^{REE}(a, t_i, z_i) &= \frac{5}{12} > \frac{1}{3} = \underline{u}_i^{REE}(a, x_i, y_i) \quad \text{for any } i \in \{1, 2\} \\
\underline{u}_i^{REE}(b, t_i, z_i) &= \underline{u}_i^{REE}(b, x_i, y_i) \quad \text{for any } i \in \{1, 2\} \\
\underline{u}_3^{REE}(a, t_3, z_3) &= \underline{u}_3^{REE}(b, t_3, z_3) = \min\left\{\frac{1}{6}, \frac{1}{9}\right\} = \frac{1}{9} \\
&= \min\left\{\frac{1}{3}, \frac{1}{9}\right\} = \underline{u}_3^{REE}(a, x_3, y_3) = \underline{u}_3^{REE}(b, x_3, y_3).
\end{aligned}$$

Hence, the equilibrium allocation e is not maxmin Pareto optimal with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i \in I}$ neither with respect to $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$.

Proof of Theorem 5.14 Let (p, x) be a maxmin rational expectations equilibrium and assume to the contrary that there exists an alternative feasible allocation y such that

$$\underline{u}_i(s, y_i) > \underline{u}_i(s, x_i) \quad \text{for all } i \in I \text{ and } s \in S. \tag{17}$$

(a) CASE: If there exists a state of nature $\bar{s} \in S$, such that $\{\bar{s}\} = \mathcal{F}_i(\bar{s})$ for all $i \in I$, then in particular from (17) it follows that for all $i \in I$

$$\underline{u}_i^{REE}(\bar{s}, y_i) = u_i(\bar{s}, y_i(\bar{s})) = \underline{u}_i(\bar{s}, y_i) > \underline{u}_i(\bar{s}, x_i) = u_i(\bar{s}, x_i(\bar{s})) = \underline{u}_i^{REE}(\bar{s}, x_i).$$

Thus, since (p, x) is a maxmin rational expectations equilibrium for all $i \in I$ there exists a state $s_i \in \mathcal{G}_i^p(\bar{s}) = \{\bar{s}\}$ (i.e., $s_i = \bar{s}$ for all $i \in I$) such that $p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i)$, that is

$$p(\bar{s}) \cdot y_i(\bar{s}) > p(\bar{s}) \cdot e_i(\bar{s}) \quad \text{for all } i \in I.$$

Hence,

$$p(\bar{s}) \cdot \sum_{i \in I} [y_i(\bar{s}) - e_i(\bar{s})] > 0,$$

which contradicts the feasibility of the allocation y. Thus, x is weak maxmin efficient with respect to the information structure $\mathcal{F}$. Moreover, notice that if there is a state of nature $\bar{s}$ such that $\mathcal{F}_i(\bar{s}) = \{\bar{s}\}$ for all $i \in I$, then a fortiori $\mathcal{G}_i^p(\bar{s}) = \{\bar{s}\}$ for all $i \in I$. This means that condition (iii) of Theorem 5.9 is satisfied and hence x is maxmin Pareto optimal also with respect to the information structure $\mathcal{G}^p$.

(b) CASE: If the $n-1$ agents are fully informed, condition (iv) of Theorem 5.9 holds and hence x is weak maxmin efficient with respect to the information structure $\mathcal{G}^p$. We want to show that x is maxmin Pareto optimal also with respect to the information structure $\mathcal{F}$. To this end, assume without loss of generality that 1 is the unique non fully informed agent and let s be a state of nature. From (17) it follows in particular that there exists $\bar{s} \in \mathcal{F}_1(s)$ such that

$$\underline{u}_1^{REE}(\bar{s}, y_1) \geq \underline{u}_1(\bar{s}, y_1) > \underline{u}_1(\bar{s}, x_1) = \underline{u}_1^{REE}(\bar{s}, x_1).$$

Since (p, x) is a maxmin REE, there exists a state $s' \in \mathcal{G}_1^p(\bar{s})$ such that

$$p(s') \cdot y_1(s') > p(s') \cdot e_1(s'). \tag{18}$$

Any agent $i \neq 1$ is fully informed, then (17) implies that

$$\underline{u}_i^{REE}(s', y_i) = u_i(s', y_i(s')) = \underline{u}_i(s', y_i) > \underline{u}_i(s', x_i) = u_i(s', x_i(s')) = \underline{u}_i^{REE}(s', x_i).$$

Thus, for all $i \neq 1$ there exists a state $s_i \in \mathcal{G}_i^p(s') = \{s'\}$ (i.e., $s_i = s'$ for all $i \neq 1$, because they are all fully informed) such that $p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i)$, that is

$$p(s') \cdot y_i(s') > p(s') \cdot e_i(s') \quad \text{for all } i \neq 1. \tag{19}$$

Hence, from (18) and (19) it follows that

$$p(s') \cdot \sum_{i \in I} [y_i(s') - e_i(s')] > 0,$$

which contradicts the feasibility of the allocation y.

We now show that if none of the above conditions is satisfied, then a maxmin REE may not be weak maxmin efficient with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i \in I}$ and a fortiori may not be maxmin Pareto optimal. To this end, consider an asymmetric information economy with two states of nature, $S = \{a, b\}$, two goods, $\ell = 2$ (the first good is considered as numeraire) and three agents, $I = \{1, 2, 3\}$ whose characteristics are given as follows:

$$\begin{array}{lll} e_1(a) = (2, 1) & e_1(b) = (1, 2) & \mathcal{F}_1 = \{\{a\}; \{b\}\} \\ e_2(a) = (1, 2) & e_2(b) = (1, 2) & \mathcal{F}_2 = \{\{a, b\}\} \\ e_3(a) = (2, 1) & e_3(b) = (2, 1) & \mathcal{F}_3 = \{\{a, b\}\}. \end{array}$$
$$u_1(s, x, y) = x^2 y, \; u_2(s, x, y) = \sqrt{xy}, \; u_3(s, x, y) = xy, \quad \text{for any } s \in S.$$

Notice that agents' initial endowments and utility functions are private information measurable. Consider the following fully revealing maxmin rational expectations equilibrium

$$(p(a), q(a)) = (1, 1) \quad (p(b), q(b)) = \left(1, \frac{11}{17}\right)$$
$$(x_1(a), y_1(a)) = (2, 1) \quad (x_2(a), y_2(a)) = \left(\tfrac{3}{2}, \tfrac{3}{2}\right) \quad (x_3(a), y_3(a)) = \left(\tfrac{3}{2}, \tfrac{3}{2}\right)$$
$$(x_1(b), y_1(b)) = \left(\tfrac{26}{17}, \tfrac{13}{11}\right) \; (x_2(b), y_2(b)) = \left(\tfrac{39}{34}, \tfrac{39}{22}\right) \; (x_3(b), y_3(b)) = \left(\tfrac{45}{34}, \tfrac{45}{22}\right).$$

The above fully revealing maxmin REE is maxmin efficient (and a fortiori weak maxmin Pareto optimal) with respect to the information structure $\mathcal{G}^p = (\mathcal{G}_i^p)_{i \in I}$ (see Theorem 5.4). Of course it is also *ex post* efficient since it coincides with an *ex post* Walrasian equilibrium. On the other hand, we now show that it is not weak maxmin efficient (and a fortiori it is not maxmin Pareto optimal) with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$. To this end, consider the following feasible allocation (t, z)

$$(t_1(a), z_1(a)) = \left(\tfrac{33}{16}, 1\right) \quad (t_1(b), z_1(b)) = \left(\tfrac{105}{68}, \tfrac{13}{11}\right)$$

$$(t_2(a), z_2(a)) = \left(\tfrac{22}{16}, \tfrac{3}{2}\right) \quad (t_2(b), z_2(b)) = \left(\tfrac{79}{68}, \tfrac{39}{22}\right)$$

$$(t_3(a), z_3(a)) = \left(\tfrac{25}{16}, \tfrac{3}{2}\right) \quad (t_3(b), z_3(b)) = \left(\tfrac{88}{68}, \tfrac{45}{22}\right),$$

and notice that,

$$\begin{aligned}
\underline{u}_1(a, t_1, z_1) &= u_1(a, t_1(a), z_1(a)) = \left(\frac{33}{16}\right)^2 > 4 \\
&= u_1(a, x_1(a), y_1(a)) = \underline{u}_1(a, x_1, y_1) \\
\underline{u}_1(b, t_1, z_1) &= u_1(b, t_1(b), z_1(b)) = \left(\frac{105}{68}\right)^2 \frac{13}{11} > \left(\frac{26}{17}\right)^2 \frac{13}{11} \\
&= u_1(b, x_1(b), y_1(b)) = \underline{u}_1(b, x_1, y_1) \\
\underline{u}_2(a, t_2, z_2) &= \underline{u}_2(b, t_2, z_2) = \min\left\{\sqrt{\frac{22}{16}\frac{3}{2}}, \sqrt{\frac{79}{68}\frac{39}{22}}\right\} = \sqrt{\frac{79}{68}\frac{39}{22}} > \sqrt{\frac{39}{34}\frac{39}{22}} \\
&= \min\left\{\frac{3}{2}, \sqrt{\frac{39}{34}\frac{39}{22}}\right\} = \underline{u}_2(b, x_2, y_2) = \underline{u}_2(a, x_2, y_2) \\
\underline{u}_3(a, t_3, z_3) &= \underline{u}_3(b, t_3, z_3) = \min\left\{\frac{25}{16}\frac{3}{2}, \frac{88}{68}\frac{45}{22}\right\} = \frac{25}{16}\frac{3}{2} > \frac{9}{4} \\
&= \min\left\{\frac{9}{4}, \frac{45}{34}\frac{45}{22}\right\} = \underline{u}_3(b, x_3, y_3) = \underline{u}_3(a, x_3, y_3).
\end{aligned}$$

Hence, the equilibrium allocation (x, y) is not weak maxmin Pareto optimal with respect to the information structure $\mathcal{F} = (\mathcal{F}_i)_{i\in I}$. □

8.4 Proofs of Section 6

Before proving Proposition 6.6 the following lemma is needed.

Lemma 8.8 *Condition (iii) and $(*)$ in the Definition 6.4, imply that for all $i \in C$,*

$$u_i(a, x_i(a)) = \min_{s\in\Pi_i(a)} u_i(s, x_i(s)) = \underline{u}_i^{\Pi_i}(a, x_i),$$

and

$$u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad for \ all \ s \in \Pi_i(a)\backslash\{a\}.$$

Proof Assume, on the contrary, there exist an agent $i \in C$ and a state $s_1 \in \Pi_i(a)\setminus\{a\}$ such that $\underline{u}_i^{\Pi_i}(a, x_i) = \min_{s\in\Pi_i(a)} u_i(s, x_i(s)) = u_i(s_1, x_i(s_1))$.

Notice that

$$\underline{u}_i^{\Pi_i}(a, y_i) = \min\{u_i(a, e_i(a) + x_i(b) - e_i(b)); \min_{s\in\Pi_i(a)\setminus\{a\}} u_i(s, x_i(s))\}.$$

If, $u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $u_i(a, y_i(a)) \le u_i(s_1, x_i(s_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This contradicts (iii) in Definition 6.4. On the other hand, if there exists $s_2 \in \Pi_i(a)\setminus\{a\}$ such that $u_i(s_2, x_i(s_2)) = \underline{u}_i^{\Pi_i}(a, y_i)$, then in particular $\underline{u}_i^{\Pi_i}(a, y_i) = u_i(s_2, x_i(s_2)) \le u_i(s_1, x_i(s_1)) = \underline{u}_i^{\Pi_i}(a, x_i)$. This again contradicts (iii) in Definition 6.4. Thus, for each member i of C, there does not exist a state $s \in \Pi_i(a)\setminus\{a\}$ such that $\underline{u}_i^{\Pi_i}(a, x_i) = u_i(s, x_i(s))$. This means that

$$u_i(a, x_i(a)) = \min_{s\in\Pi_i(a)} u_i(s, x_i(s)) = \underline{u}_i^{\Pi_i}(a, x_i),$$

and

$$u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad \text{for all } s \in \Pi_i(a)\setminus\{a\}.$$

□

Proof of Proposition 6.6 Let x be a CIC with respect to the information structure Π and assume to the contrary that there exist a coalition C and two states a and b such that

$$\begin{aligned} &(i) \quad \Pi_i(a) = \Pi_i(b) \qquad \text{for all } i \notin C,\\ &(ii) \quad e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^{\ell} \qquad \text{for all } i \in C, \text{ and}\\ &(iii) \quad \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) \quad \text{for all } i \in C, \end{aligned}$$

where for all $i \in C$,

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a\\ x_i(s) & \text{otherwise.}\end{cases}$$

Notice that from (iii) and Lemma 8.8 it follows that for all $i \in C$,

$$u_i(a, e_i(a) + x_i(b) - e_i(b)) = u_i(a, y_i(a)) \ge \underline{u}_i^{\Pi_i}(a, y_i) > \underline{u}_i^{\Pi_i}(a, x_i) = u_i(a, x_i(a)).$$

Hence, x is not CIC with respect to the information structure Π, which is a contradiction. For the converse, we construct the following counterexample. Consider the economy, described in Example 6.2, with two agents, three states of nature, denoted by a, b and c, and one good per state denoted by x. Assume that

$$u_1(\cdot, x_1) = \sqrt{x_1};\; e_1(a, b, c) = (20, 20, 0);\; \mathcal{F}_1 = \{\{a, b\}; \{c\}\}.$$
$$u_2(\cdot, x_2) = \sqrt{x_2};\; e_2(a, b, c) = (20, 0, 20);\; \mathcal{F}_2 = \{\{a, c\}; \{b\}\}.$$

Consider the allocation

$$x_1(a,b,c) = (20, 10, 10)$$
$$x_2(a,b,c) = (20, 10, 10).$$

We have already noticed that such an allocation is not Krasa-Yannelis incentive compatible with respect to the initial private information structure $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2)$ (see Example 6.2), but it is maxmin CIC with respect to $\mathcal{F}$ (see Remark 6.5). □

Proof of Theorem 6.7 Let (p, x) be a maxmin rational expectations equilibrium. Since agents take into account the information generated by the equilibrium price p, the private information of each individual i is given by $\mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$. Thus, for each agent $i \in I$, $\Pi_i = \mathcal{G}_i$ and $\underline{u}_i^{\Pi_i} = \underline{u}_i^{REE}$. Assume to the contrary that (p, x) is not maxmin CIC. This means that there exist a coalition C and two states $a, b \in S$ such that

$$\begin{aligned} &(i) \quad \mathcal{G}_i^p(a) = \mathcal{G}_i^p(b) \qquad \text{for all } i \notin C, \\ &(ii) \quad e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}_+^\ell \qquad \text{for all } i \in C, \text{ and} \\ &(iii) \quad \underline{u}_i^{REE}(a, y_i) > \underline{u}_i^{REE}(a, x_i) \quad \text{for all } i \in C, \end{aligned}$$

where for all $i \in C$,

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

Notice that condition (i) implies that $p(a) = p(b)$, meaning that the equilibrium price is partially revealing.[24] Clearly, if p is fully revealing, since for any $i \in I$, $\mathcal{G}_i^p = \mathcal{F}$, then there does not exist a coalition C and two states a and b such that $\mathcal{G}_i^p(a) = \mathcal{G}_i^p(b)$ for all $i \notin C$. Therefore, any fully revealing MREE is maxmin coalitional incentive compatible. On the other hand, since (p, x) is a maxmin rational expectations equilibrium, it follows from (iii) that for all $i \in C$ there exists a state $s_i \in \mathcal{G}_i^p(a)$ such that

$$p(s_i) \cdot y_i(s_i) > p(s_i) \cdot e_i(s_i) \geq p(s_i) \cdot x_i(s_i).$$

By the definition of y_i, it follows that for all $i \in C$, $s_i = a$, that is $p(a) \cdot y_i(a) > p(a) \cdot e_i(a)$, and hence $p(a) \cdot [x_i(b) - e_i(b)] > 0$. Furthermore, since $p(a) = p(b)$ it follows that $p(b) \cdot x_i(b) > p(b) \cdot e_i(b)$. This contradicts the fact that (p, x) is a maxmin rational expectations equilibrium. □

[24] Notice that for all i, $\sigma(p) \subseteq \mathcal{G}_i^p = \mathcal{F}_i \vee \sigma(p)$. Thus, for all i, $p(\cdot)$ is $\mathcal{G}_i^p$-measurable. Therefore, condition (i) implies that $p(a) = p(b)$.

Proof of Proposition 6.11 Let (p, x) be a maxmin REE and assume to the contrary that there exist a coalition C and two states $a, b \in S$ such that

$$\begin{aligned}&(I)\quad \mathcal{F}_i(a) = \mathcal{F}_i(b) \qquad \text{for all } i \notin C,\\&(II)\quad u_i(a, x_i(a)) = u_i(a, x_i(b)) \quad \text{for all } i \notin C,\\&(III)\quad e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} \qquad \text{for all } i \in C, \text{ and}\\&(IV)\quad \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) \quad \text{for all } i \in C,\end{aligned}$$

where for all $i \in C$,

$$y_i(s) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } s = a \\ x_i(s) & \text{otherwise.} \end{cases}$$

If (p, x) is a non revealing MREE, then the proposition holds true with no additional assumptions on utility functions (see Remark 6.8).

I CASE: Assume that $\sigma(u_i) \subseteq \mathcal{F}_i$ for any $i \in I$. Observe that if p is partially revealing and $\mathcal{G}_i^p(a)\setminus\{a\} \neq \emptyset$ for some agent i in C, then the allocation x is (private) maxmin coalitional incentive compatible and hence weak (private) maxmin CIC. Indeed, from Lemma 8.8 and condition (IV), it follows that

$$\underline{u}_i^{REE}(a, x_i) = \underline{u}_i(a, x_i) = u_i(a, x_i(a)) < u_i(s, x_i(s)) \quad \text{for all } s \in \mathcal{F}_i(a)\setminus\{a\}.$$

In particular the above inequality holds for all $s \in \mathcal{G}_i(a)\setminus\{a\}$, and this contradicts Proposition 4.11. Moreover, if for some agent $i \notin C$, $\mathcal{G}_i^p(a) = \mathcal{G}_i^p(b)$, then it follows that $p(a) = p(b)$, and hence p is partially revealing. However, even if utility functions are not private information measurable, we can conclude that x is (private) maxmin coalitional incentive compatible and hence weak (private) maxmin CIC. In fact, from (IV) and Lemma 8.8, it follows that for all $i \in C$,

$$\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i).$$

Therefore, since (p, x) is a maxmin REE, from the definition of the allocation y, it follows that for each $i \in C$, $p(a) \cdot y_i(a) > p(a) \cdot e_i(a)$, and hence $p(a) \cdot x_i(b) > p(a) \cdot e_i(b)$, which is a contradiction because $p(a) = p(b)$.

Thus, let us assume that $\mathcal{G}_i^p(a) = \{a\}$ for all $i \in C$ and $\mathcal{G}_i^p(a) \neq \mathcal{G}_i^p(b)$ for any $i \notin C$. Again from (IV) and Lemma 8.8, it follows that for all $i \in C$,

$$\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i),$$

while from (II) it follows that for all $i \notin C$,

$$\begin{aligned}\underline{u}_i^{REE}(a, y_i) &= \min\left\{\min_{s\in\mathcal{G}_i^p(a)\setminus\{a\}} u_i(s, x_i(s)), u_i(a, y_i(a))\right\}\\ &= \min\left\{\min_{s\in\mathcal{G}_i^p(a)\setminus\{a\}} u_i(s, x_i(s)), u_i(a, x_i(b))\right\}\\ &= \min\left\{\min_{s\in\mathcal{G}_i^p(a)\setminus\{a\}} u_i(s, x_i(s)), u_i(a, x_i(a))\right\}\\ &= \underline{u}_i^{REE}(a, x_i).\end{aligned}$$

Moreover, y is feasible. Indeed, for each state $s \neq a$, y is feasible because so is x. On the other hand, if $s = a$, then

$$\sum_{i\in I} y_i(a) = \sum_{i\in I} e_i(a) + \sum_{i\in I} x_i(b) - \sum_{i\in I} e_i(b) = \sum_{i\in I} e_i(a).$$

Hence, there exists a feasible allocation y such that

$$\underline{u}_i^{REE}(s, y_i) \geq \underline{u}_i^{REE}(s, x_i) \quad \text{for all } i \in I \text{ and all } s \in S,$$

with a strict inequality for each $i \in C$ in state a. Since x is a maxmin REE and $\mathcal{G}_i^p(a) = \{a\}$ for all $i \in C$, it follows that

$$p(a) \cdot y_i(a) > p(a) \cdot e_i(a) \quad \text{for any } i \in C.$$

Moreover, since y is feasible, there exists at least one agent $j \notin C$ such that

$$p(a) \cdot y_j(a) < p(a) \cdot e_j(a).$$

Notice that

$$p(s) \cdot y_j(a) < p(s) \cdot e_j(s) \qquad \text{for all } s \in \mathcal{G}_j^p(a), \tag{20}$$

because $p(\cdot)$ and $e_j(\cdot)$ are $\mathcal{G}_j$-measurable. Define the allocation[25] z_j as follows:

$$z_j(s) = y_j(a) + \frac{\mathbf{1}p(s) \cdot [e_j(s) - y_j(a)]}{\sum_{h=1}^{\ell} p^h(s)} \qquad \text{for any } s \in \mathcal{G}_j^p(a),$$

where $\mathbf{1}$ is the vector with ℓ components each of them equal to one, i.e., $\mathbf{1} = (1, \ldots, 1)$. Notice that $z_j(\cdot)$ is constant in the event $\mathcal{G}_j^p(a)$; for any $s \in \mathcal{G}_j^p(a)$, $z_j(s) \gg y_j(a)$ and $p(s) \cdot z_j(s) = p(s) \cdot e_j(s)$. Therefore, since (p, x) is a maxmin REE and $u_j(\cdot, x)$ is $\mathcal{F}_j$-measurable, from the monotonicity of $u_j(a, \cdot)$, it follows that

[25] Notice that for any $s \in \mathcal{G}_j(a)$, $\sum_{h=1}^{\ell} p^h(s) > 0$, because $p(s) \in \mathbb{R}_+^{\ell}\setminus\{0\}$ for any $s \in S$.

$$\underline{u}_j^{REE}(a, x_j) \geq \underline{u}_j^{REE}(a, z_j) = u_j(a, z_j(a)) > u_j(a, y_j(a)) \geq \underline{u}_j^{REE}(a, y_j) \\ = \underline{u}_j^{REE}(a, x_j),$$

a contradiction.

II CASE: Assume now that the equilibrium price p is fully revealing; hence $\mathcal{G}_i^p(a) = \{a\}$ for any $i \in I$. From (IV) and Lemma 8.8 it follows that for all $i \in C$,

$$\underline{u}_i^{REE}(a, y_i) \geq \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i),$$

and hence

$$p(a) \cdot y_i(a) > p(a) \cdot e_i(a) \quad \text{for any } i \in C.$$

while from (II) it follows that for all $i \notin C$,

$$\underline{u}_i^{REE}(a, y_i) = u_i(a, x_i(b)) = u_i(a, x_i(a)) = \underline{u}_i^{REE}(a, x_i).$$

Since, we have already observed that y is feasible, we conclude that for some agent $j \notin C$,

$$p(a) \cdot y_j(a) < p(a) \cdot e_j(a).$$

Define the following bundle[26]

$$z_j(a) = y_j(a) + \frac{\mathbf{1} p(a) \cdot [e_j(a) - y_j(a)]}{\sum_{h=1}^{\ell} p^h(a)} \gg y_j(a),$$

where $\mathbf{1}$ is the vector with ℓ components each of them equal to one, i.e., $\mathbf{1} = (1, \ldots, 1)$. Notice that $p(a) \cdot z_j(a) = p(a) \cdot e_j(a)$ and

$$\underline{u}_j^{REE}(a, z_j) = u_j(a, z_j(a)) > u_j(a, y_j(a)) = \underline{u}_j^{REE}(a, y_j) = \underline{u}_j^{REE}(a, x_j),$$

contradicts the fact that x is a maxmin REE allocation. □

8.5 Counterexamples for a general set of priors

As we commented above, Propositions 4.3, 4.7, 4.11, Theorems 5.4, 5.9, and Lemma 8.2 are valid for the general MEU models, provided that all priors are strictly positive. In this section, we give counterexamples to these results if some priors are not strictly positive.

[26] Notice that $\sum_{h=1}^{\ell} p^h(a) > 0$, because agents' utility functions are monotone and consequently $p(s) \in \mathbb{R}_+^{\ell} \setminus \{0\}$ for any $s \in S$.

Consider the following asymmetric information economy:

$I = \{1, 2, 3\}$	$S = \{a, b, c, d\}$	$\ell = 2$	
$\mathcal{F}_1 = \{\{a, b, c\}, \{d\}\}$	$\mathcal{F}_2 = \{\{a, b, c, d\}\}$	$\mathcal{F}_3 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$	
$e_1(s) = (1, 3)$ for all $s \in \{a, b, c\}$	$e_1(d) = (2, 2)$	$e_2(s) = (2, 1)$ for all $s \in S$	
$e_3(a) = (1, 4)$	$e_3(b) = (2, 6)$	$e_3(c) = (0, 2)$	
$e_3(d) = (1, 7)$	$u_i(s, x, y) = \sqrt{xy}$ $\forall i$ and $\forall s \in S$.		

Notice that $\sigma(u_i, e_i) \subseteq \mathcal{F}_i$ and $u_i(s, \cdot)$ is concave. For any $i \in I$ and any $F \in \mathcal{F}_i$, let $\mathcal{C}_i^F$ be the set of all priors with support contained in F. Clearly if $F = \{s\}$ then $\mathcal{C}_i^{\{s\}}$ consists of only one measure assigning one to $\{s\}$. Let $\mathcal{M}_1^F = \{\alpha : S \to [0, 1] : \alpha(a)+\alpha(b) = 1\}$ if $F = \{a, b, c\}$ and $\mathcal{M}_2^F = \{\alpha : S \to [0, 1] : \alpha(a) + \alpha(b) + \alpha(d) = 1\}$ for $F = S$. $\mathcal{M}_1^F$ and $\mathcal{M}_2^F$ are proper subsets of $\mathcal{C}_1^F$ and $\mathcal{C}_2^F$ and they do not contain only positive priors.

Consider the following allocation $\{(x_i^*(s), y_i^*(s))\}_{i \in I, s \in S}$

$$(x_1^*(a), y_1^*(a)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_2^*(a), y_2^*(a)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_3^*(a), y_3^*(a)) = \left(\tfrac{3}{2}, 3\right)$$

$$(x_1^*(b), y_1^*(b)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_2^*(b), y_2^*(b)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_3^*(b), y_3^*(b)) = \left(\tfrac{5}{2}, 5\right)$$

$$(x_1^*(c), y_1^*(c)) = \left(\tfrac{9}{4}, \tfrac{1}{2}\right) \quad (x_2^*(c), y_2^*(c)) = \left(\tfrac{1}{4}, \tfrac{9}{2}\right) \quad (x_3^*(c), y_3^*(c)) = \left(\tfrac{1}{2}, 1\right)$$

$$(x_1^*(d), y_1^*(d)) = \left(\tfrac{3}{2}, 3\right) \quad (x_2^*(d), y_2^*(d)) = \left(\tfrac{5}{4}, \tfrac{5}{2}\right) \quad (x_3^*(d), y_3^*(d)) = \left(\tfrac{9}{4}, \tfrac{9}{2}\right)$$

and the following price $(p(s), q(s)) = \left(1, \frac{1}{2}\right)$ for all $s \in S$. Thus, (p, q) is non revealing and hence $\mathcal{G}_i^p = \mathcal{F}_i$ for all i.

We now show that the allocation above is a MREE where agents' preferences are represented by (the general) maxmin expected utility (5). Indeed, $\{(x_i^*(s), y_i^*(s))\}_{i \in I, s \in S}$ is feasible and it satisfies the budget constraints. Moreover it maximizes the MEU subject to the budget constraint. Indeed, assume to the contrary that

I case $(i = 1$ and $s \in \{a, b, c\})$
there exists a random bundle $(x_1(s), y_1(s))$ such that

$$\inf_{\alpha \in \mathcal{M}_1^F} \sum_{s' \in \{a,b,c\}} \sqrt{x_1(s') y_1(s')} \alpha(s') > \inf_{\alpha \in \mathcal{M}_1^F} \sum_{s' \in \{a,b,c\}} \sqrt{x_1^*(s') y_1^*(s')} \alpha(s')$$

and $x_1(s) + \frac{1}{2} y_1(s) \leq 1 + \frac{3}{2}$ for any $s \in \{a, b, c\}$. Since for all $\alpha \in \mathcal{M}_1^F$, $\alpha(c) = 0$ and there exists $\beta \in \mathcal{M}_1^F$ such that $\beta(a) = 1$ and $\beta(b) = \beta(c) = 0$, it follows in particular that $\sqrt{x_1(a) y_1(a)} > \sqrt{\frac{25}{8}}$ and $x_1(a) + \frac{1}{2} y_1(a) \leq \frac{5}{2}$. Thus, $\frac{1}{2}(5 - y_1(a)) y_1(a) > \frac{25}{8}$, i.e., $\left(y_1(a) - \frac{5}{2}\right)^2 < 0$, a contradiction.

II case $(i = 1 \text{ and } s = d)$
there exists a random bundle $(x_1(d), y_1(d))$ such that $\sqrt{x_1(d)y_1(d)} > \sqrt{\frac{9}{2}}$ and $x_1(d) + \frac{1}{2}y_1(d) \leq 3$. This implies that $(3 - \frac{1}{2}y_1(d))y_1(d) > \frac{9}{2}$, i.e., $(y_1(a) - 3)^2 < 0$, a contradiction.

III case $(i = 2 \text{ and } s \in S)$
there exists a random bundle $(x_2(s), y_2(s))$ such that $\inf_{\alpha \in \mathcal{M}_2^F} \sum_{s' \in S} \sqrt{x_2(s')y_2(s')}\alpha(s') > \inf_{\alpha \in \mathcal{M}_2^F} \sum_{s' \in S} \sqrt{x_2^*(s')y_2^*(s')}\alpha(s')$, and $x_2(s) + \frac{1}{2}y_2(s) \leq 2 + \frac{1}{2}$ for all $s \in S$. Since for all $\alpha \in \mathcal{M}_2^F$, $\alpha(c) = 0$ and there exists $\beta \in \mathcal{M}_2^F$ such that $\beta(a) = 1$ and $\beta(b) = \beta(c) = \beta(d) = 0$, it follows in particular that $\sqrt{x_2(a)y_2(a)} > \sqrt{\frac{25}{8}}$ and $x_2(a) + \frac{1}{2}y_2(a) \leq \frac{5}{2}$. As in the first case, this implies a contradiction.

IV case $(i = 3 \text{ and } s = a)$
there exists a random bundle $(x_3(a), y_3(a))$ such that $\sqrt{x_3(a)y_3(a)} > \sqrt{\frac{9}{2}}$ and $x_3(a) + \frac{1}{2}y_3(a) \leq 3$. As in the second case, this implies a contradiction.

V case $(i = 3 \text{ and } s = b)$
there exists a random bundle $(x_3(b), y_3(b))$ such that $\sqrt{x_3(b)y_3(b)} > \sqrt{\frac{25}{2}}$ and $x_3(b) + \frac{1}{2}y_3(b) \leq 5$. This implies that $(5 - \frac{1}{2}y_3(b))y_3(b) > \frac{25}{2}$, i.e., $(y_3(b) - 5)^2 < 0$, a contradiction.

VI case $(i = 3 \text{ and } s = c)$
there exists a random bundle $(x_3(c), y_3(c))$ such that $\sqrt{x_3(c)y_3(c)} > \sqrt{\frac{1}{2}}$ and $x_3(c) + \frac{1}{2}y_3(c) \leq 1$. This implies that $\left(1 - \frac{1}{2}y_3(c)\right) y_3(c) > \frac{1}{2}$, i.e., $(y_3(c) - 1)^2 < 0$, a contradiction.

VII case $(i = 3 \text{ and } s = d)$
there exists a random bundle $(x_3(d), y_3(d))$ such that $\sqrt{x_3(d)y_3(d)} > \sqrt{\frac{81}{8}}$ and $x_3(d) + \frac{1}{2}y_3(d) \leq \frac{9}{2}$. This implies that $\left(\frac{9}{2} - \frac{1}{2}y_3(d)\right) y_3(d) > \frac{81}{8}$, i.e., $\left(y_3(d) - \frac{9}{2}\right)^2 < 0$, a contradiction.

Notice that

- the allocation $(x_i^*(\cdot), y_i^*(\cdot))$ is not $\mathcal{G}_i^p$-measurable. Thus, this is a counterexample to Proposition 4.3 for the general MEU case if the set of priors contains priors that are not strictly positive.[27]
- agents' utilities are not constant in the event $\mathcal{G}_i^p(s)$. Thus, this is a counterexample to Proposition 4.11 for the general MEU case if the set of priors contains priors that are not strictly positive.

[27] Actually Proposition 4.3 requires strict quasi-concavity, while u_i is concave and satisfies a weaker condition according to which the inequality $u(\alpha x + (1-\alpha)y) > \min\{u(x), u(y)\}$ holds when $u(x) \neq u(y)$.

- the allocation $(x_i^*(\cdot), y_i^*(\cdot))$ is not *ex post* efficient, since it is blocked by

$$\begin{aligned}
(t_i(s), z_i(s)) &= (x_i^*(s), y_i^*(s)) \ \forall i \in I \text{ if } s \neq c, \text{ and} \\
(t_i(c), z_i(c)) &= \left(\frac{5}{4}, \frac{5}{2}\right) \ \forall i \in \{1, 2\} \\
(t_3(c), z_3(c)) &= (x_3^*(c), y_3^*(c)) = \left(\frac{1}{2}, 1\right).
\end{aligned}$$

  Indeed (t, z) is feasible, and $u_i(s, t, z) = u_i(s, x^*, y^*)$ for all $i \in I$ if $s \neq c$, and

$$\begin{aligned}
u_1(t_1(c), z_1(c)) &= \sqrt{\frac{25}{8}} > \sqrt{\frac{9}{8}} = u_1(x_1^*(c), y_1^*(c)) \\
u_2(t_2(c), z_2(c)) &= \sqrt{\frac{25}{8}} > \sqrt{\frac{9}{8}} = u_2(x_2^*(c), y_2^*(c)) \\
u_3(t_3(c), z_3(c)) &= u_3(x_3^*(c), y_3^*(c))
\end{aligned}$$

  Thus, this is a counterexample to Theorem 5.4 for the general MEU case if the set of priors contains priors that are not strictly positive.
- the allocation $(x_i^*(\cdot), y_i^*(\cdot)$ is not maxmin efficient, since it is blocked by

$$\begin{aligned}
(t_i(s), z_i(s)) &= (x_i^*(s), y_i^*(s)) \ \forall i \in I \text{ if } s \neq c, \text{ and} \\
(t_i(c), z_i(c)) &= (0, 0) \ \forall i \in \{1, 2\} \\
(t_3(c), z_3(c)) &= (3, 6).
\end{aligned}$$

  Indeed (t, z) is feasible, and $u_i(s, t, z) = u_i(s, x^*, y^*)$ for all $i \in I$ if $s \neq c$, and

$$\inf_{\alpha \in \mathcal{M}_1^c} \sum_{s' \in \{a,b,c\}} \sqrt{t_1(s')z_1(s')}\alpha(s') = \sqrt{\frac{25}{8}} = \inf_{\alpha \in \mathcal{M}_1^c} \sum_{s' \in \{a,b,c\}} \sqrt{x_1^*(s')y_1^*(s')}\alpha(s')$$

$$\inf_{\alpha \in \mathcal{M}_2^c} \sum_{s' \in S} \sqrt{t_2(s')z_2(s')}\alpha(s') = \sqrt{\frac{25}{8}} = \inf_{\alpha \in \mathcal{M}_2^c} \sum_{s' \in S} \sqrt{x_2^*(s')y_2^*(s')}\alpha(s')$$

$$u_3(t_3(c), z_3(c)) = \sqrt{18} > \sqrt{\frac{1}{2}} = u_3(x_3^*(c), y_3^*(c))$$

  Thus, this is a counterexample to Theorem 5.9 for the general MEU case if the set of priors contains measures that are not strictly positive.
- the allocation $(x_i^*(\cdot), y_i^*(\cdot))$ is not an ex post Walrasian equilibrium allocation. Indeed consider for example agent 2 in state c and the bundle $\left(\frac{5}{4}, \frac{5}{2}\right)$ which is such that

$$\sqrt{\frac{25}{8}} > \sqrt{\frac{9}{8}} \quad \text{and} \quad \frac{5}{4} + \frac{5}{4} = 2 + \frac{1}{2}.$$

Thus, this is a counterexample to Proposition 4.7 and Lemma 8.2 for the general MEU case if the set of priors contains priors that are not strictly positive.

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