Econ Theory (2011) 48:519–548
DOI 10.1007/s00199-011-0637-3

SYMPOSIUM

Core and equilibria under ambiguity

Luciano I. de Castro · Marialaura Pesce · Nicholas C. Yannelis

Received: 28 April 2011 / Accepted: 16 May 2011 / Published online: 29 July 2011

Abstract This paper introduces new core and Walrasian equilibrium notions for an asymmetric information economy with non-expected utility preferences. We prove existence and incentive compatibility results for the notions we introduce. We also discuss a framework for ex ante, interim and ex post preferences.

Keywords Maximin core · Maximin Walrasian equilibrium · Incentive compatibility · Ambiguity, ex ante, interim and ex post preferences

JEL Classification D51 · D6 · D8

We are grateful to Monique Florenzano, Maria Gabriella Graziano and Fabio Maccheroni for very helpful comments.

L. I. de Castro (✉)
Department of Managerial Economics and Decision Sciences,
Northwestern University, Evanston, IL 60208, USA
e-mail: decastro.luciano@gmail.com

M. Pesce
Dipartimento di Matematica e Statistica and CSEF,
Universitá di Napoli Federico II, 81026 Napoli, Italy
e-mail: marialaura.pesce@unina.it

N. C. Yannelis
Economics—School of Social Sciences, The University of Manchester,
Oxford Road, Manchester M13 9PL, UK
e-mail: nyanneli@illinois.edu; nicholasyannelis@manchester.ac.uk

1 Introduction

Ellsberg (1961)'s seminal paper generated a huge literature considering non-expected utility preferences, beginning with Gilboa and Schmeidler (1989) and Schmeidler (1989). In an early realization of the importance of these developments, Machina (1989, p. 1623) observed that "non-expected utility models of individual decision making can be used *to conduct analyses of standard economic decisions under uncertainty*, such as insurance, gambling, investment or search." However, he foresaw that "unless and until economists are able to use these new models as engines of inquiry into basic economic questions, they—and the laboratory evidence that has inspired them—will remain on a shelf." Unfortunately, for a long time Machina's research program seemed to have been largely ignored, at least in the field of general equilibrium with asymmetric information.[1] The main objective of this paper is to advance Machina's program in this field. We consider an asymmetric information economy with non-expected utility preferences and introduce new core and Walrasian equilibrium notions which include as a special case the ones of Radner (1968) and Yannelis (1991).

To understand why these definitions are not trivial variations of the Arrow-Debreu concepts, it may be instructive to recall the "state contingent model". This model is an enhancement of the deterministic model of Arrow-Debreu-MacKenzie which allows for the initial endowments and utility functions to depend on an exogenously given state space. In this case, agents make contracts before the state of nature is realized, and ex post, i.e., once the state of nature is realized, agents fulfill their contracts and consumption takes place. Of course one must assume that there is an exogenous enforcer—a government or a court—which makes sure that the agreements made ex ante are fulfilled ex post; otherwise, agents may renege on their ex ante contracts. The existence and optimality results continue to hold for the state contingent model.

Radner (1968) introduced private information into the Arrow-Debreu's state contingent model. In particular, each agent is now allowed to have her own private information which was modeled as a partition of the exogenously given state space and assumed that the allocation of each agent is measurable with respect to her private information, i.e., allocations are private information measurable. Although Radner continued to give the state contingent interpretation of Arrow-Debreu, clearly such a story is not appealing now because if the government or court will enforce the contacts ex post, why should agents write measurable contacts? After all measurability reduces efficiency. By now it is known that the private measurability assumption guarantees that the contacts are incentive compatible and thus enforceable (see for example Koutsougeras and Yannelis 1993; Krasa and Yannelis 1994; Angeloni and Martins-da Rocha 2009, among others for a discussion of this issue). Thus, if one assumes that agents are subjective utility maximizers and allocations are private information measurable, then the resulting Walrasian equilibrium notion of Radner (1968) and the private core notion of Yannelis (1991) result in

[1] There are, of course, a few (but recent) notable exceptions, beginning with Correia-da Silva and Hervés-Beloso (2009) and followed by Condie and Ganguli (2009, 2010) and de Castro and Yannelis (2008, 2010).

outcomes which are incentive compatible and private information measurable efficient (in other words restricted efficient). Of course, we know that it is not possible to write contacts using the standard expected utility which are first best efficient and incentive compatible simultaneously.

It should be noted that the fundamental problem in mechanism design and equilibrium under asymmetric information is the conflict between efficiency and incentive compatibility. The recent work by de Castro and Yannelis (2008, 2010) has discussed this problem in settings not restricted to the expected utility framework. Once we consider a special form of the maximin expected utility of Gilboa and Schmeidler (1989), the conflict between efficiency and incentive compatibility ceases to hold—see details in the study by de Castro and Yannelis (2010).

In this paper, we consider an asymmetric information economy where the agents have general non-expected preferences and introduce new core and Walrasian equilibrium notions. We recapture the state contingent model of Arrow-Debreu but in terms of a much more general class of preferences. One of the advantages of our new modeling is that whenever we specialize the non-expected utility to the maximin expected utility, we will guarantee that any maximin efficient allocation is incentive compatible. Hence, any maximin core and maximin Walrasian equilibrium which is maximin efficient is also incentive compatible.

According to the maximin core, agents maximize interim expected utility taking into account what is the worse possible state to occur. The latter works like a prevention mechanism for any coalition of agents, not to be cheated by any other coalition. Although agents in a coalition have their own private information, they do not need to share it. Specifically, each member of the coalition calculates her expected utility based on her own private information. In that sense, this notion resembles the private core of Yannelis (1991), but there are two main differences: first, allocations need not be measurable with respect to the private information of each individual and second, the expected utility functional form is now different as we are using the maximin expected utility and not the subjective expected utility (SEU). A formal comparison of the two concepts is given in Sect. 3, which indicates that although those concepts are quite different, once we impose private information measurability on allocations and utility functions, both notions coincide.

It should be noted that the private core results in allocations that are incentive compatible. However, the private information measurability of allocations restricts the efficiency of the private core and although we have a solution of the consistency of efficiency and incentive compatibility, this solution amounts to "second best" efficiency. In other words, the private core does provide a solution to the inconsistency between efficiency and incentive compatibility, but there is a welfare loss associated with this solution. To the contrary, our approach provides a framework to analyze equilibrium notions which are first best efficient and also incentive compatible.

Koutsougeras and Yannelis (1993) and Krasa and Yannelis (1994) suggest that for efficient contacts to be viable, they must be *coalitional* incentive compatible and not just *individual* incentive compatible. Of course, coalitional incentive compatible allocations are a fortiori individual incentive compatible. Thus, we will work with a notion of coalitional incentive compatibility which is an extension of the one of

Krasa and Yannelis (1994), de Castro and Yannelis (2008, 2010). We show that the maximin core notion introduced in this paper is maximin coalitional incentive compatible.

Our paper also introduces new Walrasian equilibrium notions that are based on the maximin expected utility formulation and proves their existence and efficiency. It should be noted that Correia-da Silva and Hervés-Beloso (2009) were the first to study MEU into the Walrasian model; however, their notion is different than ours.

Another contribution of this paper is the discussion of ex ante, interim and ex post preferences. We offer simple axioms that allow to establish useful relationship between them.

The paper proceeds as follows. Section 2 describes the model and establishes some basic results about the considered preferences. Section 3 defines and compares the private core and the maximin core. We introduce and discuss our notions of equilibrium in Sect. 4. Our analysis is particularized to the maximin preferences in Sect. 5, where we establish incentive compatibility and existence of equilibrium. Section 6 is a brief conclusion.

2 Differential information economy and preferences

This section describes our model, beginning in Sect. 2.1, that lays down basic notation. Sect. 2.2 describes the class of preferences that each individual will be assumed to have, but without referring to any specific individual. Then, in Sect. 2.3, we describe the economy.

2.1 Notation

In what follows, Ω is the finite set of states of nature and $\mathcal{F} = 2^{\Omega}$ is the algebra of all the events. Let Π be a partition of Ω, which generates the algebra $\mathcal{G} \subseteq \mathcal{F}$, that is, Π (and hence $\mathcal{G}$) is coarser than $\mathcal{F}$. Let $\Pi(\omega)$ denote the element of Π that contains $\omega \in \Omega$.[2]

The set of consumption bundles for all individuals is a convex set $X \subseteq \mathbb{R}^{\ell}_{+}$, for most of our results X can be assumed to be equal to $\mathbb{R}^{\ell}_{+}$, i.e., $X = \mathbb{R}^{\ell}_{+}$. Let L denote the set of functions $f : \Omega \to X$. Since Ω is finite, L is a subset of a finite dimensional Euclidean space. Therefore, there is no ambiguity about its topology. For each $E \subset \Omega$, let L_E be the set of functions $f : E \to X$. Therefore, we can identify L with $\times_{E\in\Pi} L_E$, that is, for each $f \in L$, there is one (and only one) profile $(f_E)_{E\in\Pi} \in \times_{E\in\Pi} L_E$ such that $f(\omega) = f_E(\omega)$ if $\omega \in E$. We will use this notation repeatedly, that is, given any function $f \in L$, we will denote by $f_E \in L_E$ the restriction of $f : \Omega \to X$ to $E \subset \Omega$. Also, given $f, g \in L$ and $E \subset \Omega$, we will write (f_E, g_{E^c}) for the function that is valued $f(\omega)$ if $\omega \in E$ and $g(\omega)$ otherwise. Given $x \in X$ and $E \subset \Omega$, we will also denote by x the function $f : E \to X$ defined by $f(\omega) = x$ for every $\omega \in E$. This standard abuse of notation will not cause confusion.

[2] Although this will not be essential for the discussion in this subsection, we clarify that later the partition Π will be substituted by the private information partition of each agent.

Given a collection $\tilde{\Pi}$ of elements of the partition Π, let $\tilde{\Pi}^c$ denote $\Pi \setminus \tilde{\Pi}$ and $L^{\tilde{\Pi}}$ denote the set of profiles $(f_E)_{E\in\tilde{\Pi}} \in \times_{E\in\tilde{\Pi}} L_E$. Therefore, we may write $L = L^{\Pi} = \times_{E\in\Pi} L_E$.

2.2 Preferences

We will consider three kinds of preferences: ex ante, interim and ex post. The ex ante preference is a binary relation $\succcurlyeq$ on L. The interim preferences form a profile $(\succcurlyeq_E)_{E\in\Pi}$, such that $\succcurlyeq_E$ is a binary relation on L_E, for each $E \in \Pi$. Correspondingly, the ex post preferences form a profile $(\succcurlyeq_\omega)_{\omega\in\Omega}$, where each $\succcurlyeq_\omega$ is a binary relation on $L_{\{\omega\}} = X$.

The objective of this subsection is to define properties and study the relationship between these preferences in such a way that can serve as a foundation for a satisfactory theory of asymmetric information with special preferences (and not only expected utility). Although the facts collected in this section are based on known results, we are not aware of papers explicitly discussing general ex ante, interim and ex post preferences and their relation as we do here.[3]

It is clear from this discussion that an ex ante, interim or ex post preference can be abstractly denoted by $\succcurlyeq_E$ where $E \subset \Omega$.[4] Therefore, we can make the following assumptions:

Axiom 1 (*Weak Order*) $\succcurlyeq_E$ is non-trivial, complete and transitive.

Axiom 2 (*Continuity*) The sets $\{g \in L_E : g \succcurlyeq_E f\}$ and $\{g \in L_E : f \succcurlyeq_E g\}$ are closed for any $f \in L_E$.

Let us begin by observing a trivial consequence of these axioms.

Proposition 2.1 *Assume that the ex ante, interim and ex post preferences satisfy Axioms 1 and 2. Then, there exist continuous functions $U : L \to \mathbb{R}$; $u : \Pi \times L \to \mathbb{R}$ and $\tilde{u} : \Omega \times X \to \mathbb{R}$ such that for all $f, g \in L$, $E \in \Pi$ and $\omega \in \Omega$:*

$$f \succcurlyeq g \iff U(f) \geqslant U(g); \quad (1)$$

$$f_E \succcurlyeq_E g_E \iff u(E, f) \geqslant u(E, g); \quad (2)$$

$$f(\omega) \succcurlyeq_\omega g(\omega) \iff \tilde{u}(\omega, f(\omega)) \geqslant \tilde{u}(\omega, g(\omega)). \quad (3)$$

Moreover, these functions are unique up to monotonic increasing transformations.[5]

---

[3] Luce and Krantz (1971) discusses a conditional expected utility which were followed by Drèze and Rustichini (1999) and others, but their motivation is quite different from ours. There is also a relevant discussion in the study by Drèze and Rustichini (2004) and useful results can be found in the study by Koopmans (1960). More recently, Hanany and Klibanoff (2009) discuss update of preferences, which can be seen as related to our ex ante, interim and ex post preferences. However, their focus is on dynamic consistency.

[4] More clearly, the notation has the following meaning: if $E = \Omega$, $\succcurlyeq_E$ is an ex ante preference (and we write $\succcurlyeq$ instead of $\succcurlyeq_\Omega$); if $E \in \Pi$, $\succcurlyeq_E$ is an interim preference and if $E = \{\omega\}$ for some $\omega \in \Omega$, $\succcurlyeq_E$ is an ex post preference. The axioms are supposed to hold for the three cases, for the respectively relevant sets.

[5] The interim preferences could be more properly represented by a profile of functions $u_E : L_E \to \mathbb{R}$, that is, instead of (2), we could write $f_E \succcurlyeq_E g_E \iff u_E(f_E) \geqslant u_E(g_E)$. Depending on the context,

Proof It is an immediate consequence of the classical Debreu's result (Debreu 1954, Theorem II) applied separately to each of these preferences. □

The above result is useful to setting the notation that we are going to use in the rest of the paper, but of course, the existence of continuous functions representing the ex ante, interim and ex post preference is just the initial step toward our objective. What interests us the most is the consistency requirement between these preferences.[6]

Axiom 3 (*Ex ante/Interim Consistency*) For any $E \in \Pi$ and $f, g, h \in L$,

$$f_E \succcurlyeq_E g_E \implies (f_E, h_{E^c}) \succcurlyeq (g_E, h_{E^c}).$$

Axiom 4 (*Interim/Ex post Consistency*) For any $E \in \Pi$, $\omega \in E$ and $f, g, h \in L$,

$$f(\omega) \succcurlyeq_\omega g(\omega) \implies (f(\omega), h_{E\setminus\{\omega\}}) \succcurlyeq_E (g(\omega), h_{E\setminus\{\omega\}}).$$

We have the following:

Proposition 2.2 *Assume that the preferences satisfy Axioms 1 and 2 and let U, u and $\tilde{u}$ be the functions given by Proposition 2.1.*

1. *If Axiom 3 holds, then there exists a continuous and monotonic function $A : \mathbb{R}^{|\Pi|} \to \mathbb{R}$ such that $U(f) = A(u(E, f)_{E\in\Pi})$.*
2. *If Axiom 4 holds, then there exists a continuous and monotonic function $I : \mathbb{R}^{|E|} \to \mathbb{R}$ such that $u(E, f) = I(\tilde{u}(\omega, f(\omega))_{\omega\in E})$.*

Proof We prove only the first statement; the proof of the second is analogous. Fix $h \in L$. Using the notation discussed in footnote 5, (2) means that for any $f, g \in L$, $f_E \succcurlyeq_E g_E \iff u_E(f_E) \geqslant u_E(g_E)$. Therefore, by Axiom 3 and (1),

$$u_E(f_E) \geqslant u_E(g_E) \implies (f_E, h_{E^c}) \succcurlyeq (g_E, h_{E^c}) \iff U(f_E, h_{E^c}) \\ \geqslant U(g_E, h_{E^c}). \tag{4}$$

In particular, $u_E(f_E) = u_E(g_E) \implies U(f_E, h_{E^c}) = U(g_E, h_{E^c})$. Since h is arbitrary, this allows us to write: $U(f) = A^1(u_E(f_E), f_{E^c})$. Because of (4), this function A^1 is monotonic increasing in its first entry. Using $A^1(u_E(f_E), f_{E^c})$ in (4) for $E' \neq E$, $E' \in \Pi$, we obtain $A^2(u_E(f_E), u_{E'}(f_{E'}), f_{(E\cup E')^c})$, monotonic increasing in the first two entries. Repeating this argument for each $E \in \Pi$, we obtain $U(f) = A\,(u_E(f_E)_{E\in\Pi})$, as we wanted. □

Footnote 5 continued
one or other form is more convenient. Observe also that although the second entry of u is on L, the only important part for defining $u(E, f)$ is f_E, that is, if $f, g \in L$ are such that $f(\omega) = g(\omega)$ for all $\omega \in E$ then $u(E, f) = u(E, g)$.

[6] We were not able to find any suitable statement of these axioms in our framework. The closest that we were able to find was that of Koopmans (1960).

We can call the functions A and I given by the above proposition the ex ante and interim aggregators. For most purposes, the above properties and characterizations are enough. However, for some applications, it will be useful to obtain a more precise characterization of the ex ante aggregator. For this, we need some new definition.

Fix $\tilde{\Pi} \subset \Pi$ and $h = (h_E)_{E \in \tilde{\Pi}} \in L^{\tilde{\Pi}}$. Let the **preference given** h, denoted $\succcurlyeq_h$, be the binary order induced on $L^{\tilde{\Pi}^c}$, that is, for any profiles $f, g \in L^{\tilde{\Pi}^c}$:

$$f \succcurlyeq_h g \iff (f, h) \succcurlyeq (g, h).$$

Consider the following axiom.

Axiom 5 (*Independence*) Given a collection $\tilde{\Pi}$ of elements of the partition Π, the preference given h does not depend on $h \in L^{\tilde{\Pi}}$.

Proposition 2.3 *Assume that the preferences satisfy Axioms 1–5 and assume that* Π *has at least three elements. Then, there exist continuous functions* $U : L \to \mathbb{R}$ *and* $u : \Pi \times L \to \mathbb{R}$ *such that* $U(f) = \sum_{E \in \Pi} u(E, f)$ *represents* $\succcurlyeq$, *that is,*

$$f \succcurlyeq g \iff \sum_{E \in \Pi} u(E, f) \geqslant \sum_{E \in \Pi} u(E, g). \tag{5}$$

Proof By Axiom 1, $\succcurlyeq_E$ is not trivial for each $E \in \Pi$. Thus, we have all the assumptions of (Debreu, 1960, Theorem 3), which implies the conclusion. $\square$

In the above theorem, we can relax the assumption that Π has three elements. This is important in some examples. For doing this, it is enough to require that the preferences satisfy the hexagon condition given by Karni and Safra (1998). The reader can consult that paper for more details. Another relevant comment is that some specific formulations of state-dependent utility (not restricted to the separability condition presented in Proposition 2.3) can be found in Cerreia et al. (2011).

Below, where Π will represent the information partition of the decision maker, we will refer to the function $\tilde{u} : \Omega \times \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ as the **ex post utility function** and to $u : \Pi \times L \to \mathbb{R}$ as the **interim utility function**. Although the first argument of u is a set, we will sometimes abuse notation and write $u : \Omega \times L \to \mathbb{R}$, with the proviso that u is Π-measurable, that is, $u(\omega, \cdot) = u(\omega', \cdot)$ whenever $\Pi(\omega) = \Pi(\omega')$.

Notice that the state-dependent utility is consistent with any kind of priors. That is, if π is a probability measure on Ω, such that $\pi(\{E\}) > 0$ for every $E \in \Pi$, then we can define $u'(\omega, f) = \frac{u(\omega, f)}{\pi(E)}$. In this case, we can write (5) as

$$f \succcurlyeq g \iff \sum_{E \in \Pi} u'(E, f)\pi(E) \geqslant \sum_{E \in \Pi} u'(E, g)\pi(E). \tag{6}$$

In what follows, we will denote by $\mathbb{P}$ the system of ex ante, interim and ex post preferences. In Sect. 2.4 below, we exemplify some relevant systems of ex ante and interim preferences.

2.3 Differential information economy

For all $i \in I$, we define the following:

- $\mathcal{F}_i$ is a partition[7] of $(\Omega, \mathcal{F})$ denoting the **private information** of agent i, that is, if $\omega \in \Omega$ is the state of nature that is going to be realized, agent i observes $\mathcal{F}_i(\omega)$ the element of $\mathcal{F}_i$ which contains ω.
- $L_i \subset L$ is the set of agent i's **private measurable consumption allocations**:

$$L_i = \{x_i \in L : \ x_i(\cdot) \text{ is } \mathcal{F}_i\textit{-measurable}\}.$$

- $\mathbb{P}_i$ is the **system of ex ante, interim and ex post preferences** of agent i and satisfying Axioms 1, 2, 3 and 4 of Sect. 2.2.[8]
- $e_i : \ \Omega \to X$ is agent i's **random initial endowment** of physical resources.

We assume that $e_i \in L_i$.

A differential information exchange economy $\mathcal{E}$ is a set

$$\mathcal{E} = \{(\Omega, \mathcal{F}); \ X; \ (\mathcal{F}_i, \mathbb{P}_i, e_i) : i \in I = \{1, \ldots, n\}\}.$$

As usual, we can interpret the above economy as a three time period model (ex ante or $t = 0$, interim or $t = 1$ and ex post or $t = 2$). At the ex ante stage, it is common knowledge only the above description of the economy. At the interim stage, $t = 1$, agent i only knows that the realized state belongs to the event $\mathcal{F}_i(\omega^*)$, where ω^* is the true state at $t = 2$. We will consider two main situations of trade: either ex ante or interim. In the ex ante case, agent i chooses bundles in L according to the preference $\succcurlyeq_i$ and write contracts for delivery of those bundles. Similarly, in the interim case, agent i chooses bundles in L according to the preference $\succcurlyeq_i^{\mathcal{F}_i(\omega)}$ when the state is ω. At the ex post stage ($t = 2$), agents execute the contracts and consumption takes place.

A function $x : \Omega \to X^n$ written as $x = (x_1, \ldots, x_n)$ is said to be a **random consumption vector or allocation**. Let $\bar{L} = \times_{i \in I} L_i$. An allocation $x \in L^n$ is said to be **feasible** if

$$\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega) \quad \text{for all } \omega \in \Omega.$$

2.4 Examples of preferences

Before we conclude this section, it seems useful to specify important examples of the preferences discussed above.

[7] By an abuse of notation we will still denote by $\mathcal{F}_i$ the algebra that the partition $\mathcal{F}_i$ generates.

[8] Occasionally, we will assume also additive separation and use the representation (6). It will be clear from the context what representation we are using.

2.4.1 Expected utility (EU)

We define now the (Bayesian or subjective expected utility) ex ante and interim expected utility. For each i, let $(\Omega, \mathcal{F}, \pi_i)$ be a probability space and Π_i be any partition of Ω. For each agent i and for any allocation $x_i : \Omega \to X$, agent i's **ex ante expected utility** function is given by

$$V_i(x_i) = \sum_{\omega \in \Omega} \tilde{u}_i(\omega, x_i(\omega))\pi_i(\omega).$$

For any allocation $x_i : \Omega \to X$, agent i's **interim expected utility** function with respect to Π_i at x_i in state ω is given by

$$v_i(x_i|\Pi_i)(\omega) = \sum_{\omega' \in \Omega} \tilde{u}_i(\omega', x_i(\omega'))\pi_i(\omega'|\omega),$$

where

$$\pi_i(\omega'|\omega) = \begin{cases} 0 & \text{for } \omega' \notin \Pi_i(\omega) \\ \frac{\pi_i(\omega')}{\pi_i(\Pi_i(\omega))} & \text{for } \omega' \in \Pi_i(\omega). \end{cases}$$

We can also express the interim expected utility using conditional probability as

$$v_i(x_i|\Pi_i)(\omega) = \sum_{\omega' \in \Pi_i(\omega)} \tilde{u}_i(\omega', x_i(\omega')) \frac{\pi_i(\omega')}{\pi_i\left(\Pi_i(\omega)\right)}.$$

2.4.2 Maximin preferences

The maximin interim utility of each agent i with respect to Π_i of Ω at an allocation $x_i : \Omega \to X$ in state ω is given by

$$u_i(\Pi_i(\omega), x_i) = \underline{u}_i(\omega, x_i) \equiv \min_{\omega' \in \Pi_i(\omega)} \tilde{u}_i(\omega', x_i(\omega')).$$

We will abuse notation by writing both $\underline{u}_i^{\Pi_i}(\omega, x_i)$ and $\underline{u}_i^{\Pi_i}(E, x_i)$, but no confusion should arise. The maximin ex ante utility is just an expectation of this value, that is,

$$U_i(x_i) \equiv \sum_{E \in \Pi_i} \underline{u}_i(E, x_i)\pi_i(E).$$

3 General core versus private core

Below we recall the notion of private core (see Yannelis 1991).

Definition 3.1 A feasible allocation x is said to be an **interim private core allocation** for the economy $\mathcal{E}$ if for all $i \in I$, $x_i(\cdot)$ is $\mathcal{F}_i$-measurable, and there do not exist a coalition S and an allocation y such that

(i) $y_i(\cdot)$ is $\mathcal{F}_i$-measurable for all $i \in S$
(ii) $v_i(y_i|\mathcal{F}_i)(\omega) > v_i(x_i|\mathcal{F}_i)(\omega)$ for all $i \in S$ and for all $\omega \in \Omega$
(iii) $\sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega)$ for all $\omega \in \Omega$.

Definition 3.2 Moreover, if we replace condition (ii) in Definition 3.1 with

$$V_i(y_i) > V_i(x_i) \quad \text{for all } i \in S,$$

the feasible allocation x is said to be an **ex ante private core allocation** for the economy $\mathcal{E}$.

Another notion of interim core present in the literature has been introduced by Hahn and Yannelis (1997) which we recall below.

Definition 3.3 A feasible allocation x is said to be a **weak interim private core allocation** for the economy $\mathcal{E}$ if for all $i \in I$, $x_i(\cdot)$ is $\mathcal{F}_i$-measurable and there do not exist a coalition S, a state $\bar{\omega}$ and an allocation y such that

(i) $y_i(\cdot)$ is $\mathcal{F}_i$-measurable for all $i \in S$
(ii) $v_i(y_i|\mathcal{F}_i)(\bar{\omega}) > v_i(x_i|\mathcal{F}_i)(\bar{\omega})$ for all $i \in S$ and
(iii) $\sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega)$ for all ω.

Clearly, any weak interim private core allocation belongs to the interim private core. It is still an open question if a weak interim private core allocation exists. On the other hand, it is known that the ex ante as well as the interim private core is non-empty under standard assumptions (see Angeloni and Martins-da Rocha 2009; Yannelis 1991). It is easy to check that any ex ante private core allocation cannot be privately blocked in the interim stage, and the converse may not hold, as the next proposition states.

Proposition 3.4 *Any ex ante private core allocation belongs to the interim private core. The converse may not hold. Moreover, the weak interim private core may not be included into the ex ante private core.*

Proof See Appendix. □

The private information measurability assumption of allocations is an exogenous theoretical requirement that may be difficult to justify in real economies, and furthermore, it reduces efficiency (see de Castro and Yannelis 2010). However, it does guarantee incentive compatibility (see Koutsougeras and Yannelis 1993). By specializing our preferences to the maximin one, we will show in Sect. 5.2 that even if allocations are not private information measurable, any maximin efficient allocation is incentive compatible. We now define the notion of core with general preferences and without private information measurability hypothesis on allocations.

Definition 3.5 A feasible allocation x is said to be an **interim core allocation** for the economy $\mathcal{E}$ if there do not exist a coalition S and an allocation y such that

$$\begin{aligned} &(i) \quad u_i(\omega, y_i) > u_i(\omega, x_i) \quad \text{for all } i \in S \text{ and } \omega \in \Omega, \\ &(ii) \quad \sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega) \quad \text{for all } \omega \in \Omega. \end{aligned}$$

Notice that the above notion is related to the one given by Yannelis (1991), since members of the coalition S prefer the allocation y in each state ω.

Definition 3.6 If we replace condition (i) in Definition 3.5 with

$$U_i(y_i) > U_i(x_i) \qquad \text{for all } i \in S,$$

the feasible allocation x is said to be an **ex ante core allocation** for the economy $\mathcal{E}$.

The same relationship of Proposition 3.4 holds true between interim and ex ante core allocations, i.e., the ex ante core is included in the interim core.

Proposition 3.7 *Any ex ante core allocation is in the interim core.*

Proof See Appendix. □

We have already remarked that any private (ex ante as well as interim) core allocation exists under standard assumptions. We are now ready to show that the same existence results hold for general preferences. Precisely, by using Scarf's Theorem (see Scarf 1967), we will prove that an ex ante core allocation exists. Clearly, the non-emptiness of the ex ante core implies the existence of an interim core allocation.

Theorem 3.8 *Assume that for all $i \in I$, $U_i(\cdot)$ is continuous and concave and that X is compact.*[9] *Then, the ex ante core is non-empty.*

Proof See Appendix. □

Corollary 3.9 *Assume that for all $i \in I$, $U_i(\cdot)$ is continuous and concave and that X is compact. Then, the interim core is non-empty.*

Proof This directly follows from Theorem 3.8 and Proposition 3.7. □

We now compare the notions of private and general core in the interim and ex ante stage. We will show that the private core may not be a subset of the general core.

Proposition 3.10 *Assume that for all $i \in I$ and $t \in \mathbb{R}^{\ell}_{+}$, $\tilde{u}_i(\cdot, t)$ is $\mathcal{F}_i$-measurable. Then, any ex ante core allocation x, where each $x_i(\cdot)$ is $\mathcal{F}_i$-measurable, belongs to the ex ante private core. The converse may not be true.*

Proof See Appendix. □

Proposition 3.11 *Assume that for all $i \in I$ and $t \in \mathbb{R}^{\ell}_{+}$, $\tilde{u}_i(\cdot, t)$ is $\mathcal{F}_i$-measurable. Then, any interim core allocation x, where each $x_i(\cdot)$ is $\mathcal{F}_i$-measurable, belongs to the interim private core. The converse may not be true.*

Proof See Appendix. □

[9] Notice that for all $i \in I$ and $\omega \in \Omega$, X is also non-empty, since it contains at least i's initial endowment. Moreover, one may take X to be the order interval, i.e., $X = [0, \max_{\omega \in \Omega} \sum_{i \in I} e_i(\omega)]$, which is clearly non-empty, convex and compact. Alternatively, one can use standard truncation arguments to relax the compactness assumption. See Florenzano (1989) and Allouch and Florenzano (2004) among others. Also, Theorem 3.8 holds in infinite dimensional commodity spaces, but such generalizations go beyond the purposes of our paper.

3.1 The particular case of maximin preferences

We now introduce a notion of interim core related to the one given in Definition 3.3 by using the maximin formulation. Despite the fact that the weak interim private core (see Definition 3.3) may be empty, whenever we allow agents in the same definition to have MEU preferences, the corresponding maximin core of Definition 3.3 is non-empty.

Definition 3.12 A feasible allocation x is said to be a *maximin core allocation* for the economy $\mathcal{E}$, if there do not exist a coalition S, a state $\bar{\omega}$ and an allocation y such that

$$(i)\quad \underline{u}_i(\bar{\omega}, y_i) > \tilde{u}_i(\bar{\omega}, x_i(\bar{\omega})) \geq \underline{u}_i(\bar{\omega}, x_i) \quad \text{for all } i \in S,$$
$$(ii)\quad \sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega) \quad \text{for all } \omega \in \Omega.$$

If in the above definition, the coalition S is replaced by the grand coalition I, the allocation x is said to be **maximin efficient**. It is obvious that the maximin core is included into the set of maximin Pareto optimal allocations. Moreover, one might give also a stronger notion of maximin core by requiring that the blocking allocation is preferred by each member of coalition S in each state of nature, i.e., replace (i) by

$$(i')\quad \underline{u}_i(\omega, y_i) > \tilde{u}_i(\omega, x_i(\omega)) \geq \underline{u}_i(\omega, x_i) \quad \text{for all } i \in S \text{ and for all } \omega \in \Omega.$$

Obviously such a core contains properly the maximin core as the following example illustrates.

Example 3.13 Consider a differential information economy with three equiprobable state of nature, i.e., $\Omega = \{a, b, c\}$ with $\pi_i(\omega) = \frac{1}{3}$ for each i and ω. There are two agents asymmetrically informed and only one good. Moreover, the primitives of the economy are given as follows:

$$e_1 = (5, 5, 0) \quad \mathcal{F}_1 = \{\{a, b\}; \{c\}\} \quad u_1(\cdot, x_1) = \sqrt{x_1}$$
$$e_2 = (5, 0, 5) \quad \mathcal{F}_2 = \{\{a, c\}; \{b\}\} \quad u_2(\cdot, x_2) = \sqrt{x_2}.$$

One can easily prove that the allocation $x_1 = (5, 4, 1)$ and $x_2 = (5, 1, 4)$ cannot be maximin blocked in each state of nature. However, it is not a maximin core allocation, since it is blocked by agent 1, i.e., $S = \{1\}$, in state b via the initial endowment, since

$$\underline{u}_1(b, e_1) = \min\{\sqrt{5}, \sqrt{5}\} = \sqrt{5} > 2 = \tilde{u}_1(b, x_1(b)) = \min\{\sqrt{5}, \sqrt{4}\} = \underline{u}_1(b, x_1).$$

Clearly, x is also blocked by agent 2 i.e., $S = \{2\}$, in state c still via the initial endowment.

Remark 3.14 An alternative notion of maximin core could be the one according to which there does not exist a coalition S and an allocation y such that

$$(i)\quad \underline{u}_i(\omega, y_i) > \underline{u}_i(\omega, x_i) \quad \text{for all } i \in S \quad \text{and} \quad \omega \in \Omega,$$
$$(ii)\quad \sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega) \quad \text{for all } \omega \in \Omega.$$

One can easily prove that this core is non empty and moreover, if there exists a state of nature ω^* that everybody can distinguish, i.e., $\{\omega^*\} = \mathcal{F}_i(\omega^*)$ for all $i \in I$, it contains the set of maximin interim Walrasian equilibrium allocations (see Definition 5.1).

4 General Walrasian equilibrium versus Walrasian expectations equilibrium

We define a **price vector** as a non-zero function $p : \Omega \to \mathbb{R}^{\ell}_{+}$, such that $p(\cdot)$ is $\mathcal{F}$-measurable.

We define below the notion of a Walrasian expectations equilibrium in the sense of Radner (1968).

Definition 4.1 A pair (p, x), where p is a price vector and x is a feasible allocation, is said to be an **ex ante Walrasian expectations equilibrium (WEE)** if for each i, $x_i(\cdot)$ is $\mathcal{F}_i$-measurable and maximizes

$$V_i(x_i) = \sum_{\omega \in \Omega} \tilde{u}_i(\omega, x_i(\omega))\pi_i(\omega)$$

subject to the ex ante budget set, i.e.,

$$B_i(p) = \left\{ x_i \in L_i \; : \; \sum_{\omega \in \Omega} p(\omega) \cdot x_i(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot e_i(\omega) \right\}.$$

It is known that a WEE belongs to the ex ante private core; therefore, it is second best efficient and also under standard assumptions it exists (see Angeloni and Martins-da Rocha 2009). We now define the related notion of an ex ante Walrasian equilibrium (WE).

Definition 4.2 A pair (p, x) is said to be an **ex ante Walrasian equilibrium (WE)** if p is a price vector and x is a feasible allocation, such that for each i, x_i maximizes the ex ante expected utility $U_i(x_i)$, subject to the ex ante budget set $B_i(p)$.

The above ex ante WE notion is first best efficient (but may not be incentive compatible) and one can prove adopting standard arguments that it exists. We will prove that for the interim case, whenever we specialize the utility into the maximin formulation, then a maximin interim Walrasian equilibrium exists.

We now define the related Walrasian equilibrium concept in asymmetric information economies with the standard Bayesian subjective expected utility functions.

Definition 4.3 An allocation x is said to be an **interim Walrasian expectations equilibrium allocation (IWEE)** if there exists a price vector p such that

(i) $x_i(\cdot)$ is $\mathcal{F}_i$−measurable for all $i \in I$,

(ii) for all i and ω, $x_i(\omega)$ maximizes

$$v_i(x_i|\mathcal{F}_i)(\omega) = \sum_{\omega' \in \mathcal{F}_i(\omega)} \tilde{u}_i(\omega', x_i(\omega')) \frac{\pi_i(\omega')}{\pi_i\,(\mathcal{F}_i(\omega))}$$

subject to the interim budget set, i.e.,

$$\sum_{\omega' \in \mathcal{F}_i(\omega)} p(\omega') \cdot x_i(\omega') \frac{\pi_i(\omega')}{\pi_i\,(\mathcal{F}_i(\omega))} \leq \sum_{\omega' \in \mathcal{F}_i(\omega)} p(\omega') \cdot e_i(\omega') \frac{\pi_i(\omega')}{\pi_i\,(\mathcal{F}_i(\omega))}$$

(iii) $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$ for all $\omega \in \Omega$.

The above concept is different from the rational expectations equilibrium, since agents do not take into account the information generated by prices. An interim Walrasian expectations equilibrium seems to be very similar to the notion of Bayesian Walrasian equilibrium introduced by Balder and Yannelis (2009), but condition (iii) is replaced by

$$p(\omega) \cdot \sum_{i \in I} [x_i(\omega) - e_i(\omega)] = \max_{1 \leq h \leq \ell} \sum_{i \in I} [x_i^h(\omega) - e_i^h(\omega)]_h \quad \text{for all } \omega \in \Omega.$$

It is proved in the study by Balder and Yannelis (2009) that the set of interim Walrasian expectations equilibria may be empty; while a Bayesian Walrasian equilibrium always exists under standard assumptions.

The problem of the existence of an interim Walrasian expectations equilibrium is deeply linked to the private information measurability requirement of allocations. We now introduce the notion of an interim Walrasian equilibrium with general preferences and we will show that by using the maximin formulation, such an equilibrium exists.

Definition 4.4 A feasible allocation x is said to be an **interim Walrasian equilibrium allocation (IWE)** if there exists a price vector p such that for all $i \in I$ and $\omega \in \Omega$, x_i maximizes the interim utility function $u_i(\omega, \cdot)$ subject to the interim budget set

$$B_i(\omega, p) = \{y_i \in L : \quad p(\omega') \cdot y_i(\omega') \leq p(\omega') \cdot e_i(\omega') \text{ for all } \omega' \in \mathcal{F}_i(\omega)\}.$$

5 Maximin preferences: existence and incentive compatibility results

In this section, we particularize our notions to the setting of the maximin preferences studied by de Castro and Yannelis (2008), which is a particular case of Gilboa and Schmeidler (1989)'s MEU preferences (see the definition of maximin preferences on Sect. 2.4.2). In this section, we show the existence of IWE for maximin preferences. We first particularize the equilibrium notion to this case.

Definition 5.1 A feasible allocation x is said to be a **maximin interim Walrasian equilibrium allocation (MIWE)** if there exists a price vector p such that for all $i \in I$ and $\omega \in \Omega$,

$$\underline{u}_i(\omega, x_i) = \max_{y_i \in B_i(\omega, p)} \underline{u}_i(\omega, y_i), \quad \text{where}$$
$$B_i(\omega, p) = \{y_i \in L : \quad p(\omega') \cdot y_i(\omega') \leq p(\omega') \cdot e_i(\omega') \text{ for all } \omega' \in \mathcal{F}_i(\omega)\}.$$

Before we prove the existence of a maximin interim Walrasian equilibrium (MIWE), we wish to compare it with the interim Walrasian expectations equilibrium allocation (IWEE).

Proposition 5.2 *An interim Walrasian expectations equilibrium (IWEE) and a maximin interim Walrasian equilibrium (MIWE) may not be comparable.*

Proof See Appendix. □

In order to prove that the set of MIWE is non-empty, the following proposition plays a crucial role.

Proposition 5.3 *If (p, x) is an ex post Walrasian equilibrium, then (p, x) is a maximin interim Walrasian equilibrium.*

Proof See Appendix. □

It is well known that an ex post Walrasian equilibrium exists.

We are now ready to prove that despite the fact that the set of IWEE may be empty, a maximin IWE always exists, as the following theorem states.

Theorem 5.4 *Assume that for each agent i and each state ω, $e_i(\omega) \gg 0$ and $u_i(\omega, \cdot)$ is continuous, concave and monotone. Then, a maximin interim Walrasian equilibrium exists.*

Proof It directly follows from the existence of an ex post Walrasian equilibrium and of Proposition 5.3. □

The next proposition shows that any ex post Walrasian equilibrium allocation belongs to the maximin core.

Proposition 5.5 *Any ex post Walrasian equilibrium allocation belongs to the maximin core.*

Proof See Appendix. □

The above proposition simply implies the non-emptiness of the maximin core, as the following corollary states, and that any ex post Walrasian equilibrium allocation is maximin efficient.

Corollary 5.6 *Assume that for each agent i and each state ω, $e_i(\omega) \gg 0$ and $u_i(\omega, \cdot)$ is continuous, concave and monotone. Then, the maximin core is non-empty.*

Proof This directly follows from the existence of an ex post Walrasian equilibrium and of Proposition 5.5. □

The non-emptiness of the maximin core may also be proved by showing that it contains the set of maximin REE, which is non-empty, as it has been shown in the study by de Castro et al. (2010). We recall below the notion of maximin REE.

Definition 5.7 Let p be a price vector and $\sigma(p)$ the information generated by the price p, i.e., the finest algebra such that $p(\cdot)$ is measurable. For each agent $i \in I$, consider the algebra $\mathcal{G}_i = \mathcal{F}_i \vee \sigma(p)$. A price vector p and a feasible allocation x are said to be a **maximin rational expectations equilibrium** (MREE) for the economy $\mathcal{E}$ if:

(i) for all $i \in I$ and for all $\omega \in \Omega$ the allocation $x_i \in B_i^{\text{REE}}(\omega, p)$, where

$$B_i^{\text{REE}}(\omega, p) = \left\{ y_i \in L \ : \ p(\omega') \cdot y_i(\omega') \leq p(\omega') \cdot e_i(\omega') \text{ for all } \omega' \in \mathcal{G}_i(\omega) \right\};$$

(ii) for all $i \in I$ and for all $\omega \in \Omega$, $\underline{u}_i^{\text{REE}}(\omega, x_i) = \max_{y_i \in B_i^{\text{REE}}(\omega, p)} \underline{u}_i^{\text{REE}}(\omega, y_i)$, where

$$\underline{u}_i^{\text{REE}}(\omega, y_i) = \min_{\omega' \in \mathcal{G}_i(\omega)} \tilde{u}_i(\omega', y_i(\omega')).$$

Conditions (i) and (ii) indicate that each individual maximizes her maximin expected utility conditioned on her private information and the information the equilibrium prices have generated, subject to the budget constraint.

It is easy to show that any ex post Walrasian equilibrium is a maximin REE, and therefore the non-emptiness of the set of ex post Walrasian equilibria implies the existence of a maximin REE. Contrary to the Bayesian REE, which may not exist, may not be efficient or incentive compatible (see Glycopantis and Yannelis 2005, p. 31 and also Example 9.1.1, p.43), in the study by de Castro et al. (2010), it is shown that any maximin REE exists, it is maximin Pareto optimal and also incentive compatible. We prove below that any maximin REE belongs to the maximin core.

Theorem 5.8 *If for any $i \in I$, and $t \in R_+^\ell$, $\tilde{u}_i(\cdot, t)$ is $\mathcal{F}_i$-measurable,*[10] *then any maximin REE allocation belongs to the maximin core.*

Proof See Appendix. □

From the above proposition, it follows that any maximin REE is maximin Pareto optimal. It is an open question if a maximin IWE is maximin efficient. We now introduce a different notion of maximin equilibrium which is maximin Pareto optimal.

Definition 5.9 A feasible allocation x is said to be a **maximin Walrasian equilibrium allocation (MWE)** if there exists a price vector p such that for all $i \in I$ and $\omega \in \Omega$,

$$\underline{u}_i(\omega, x_i) = \max_{y_i \in B_i^*(\omega, p)} \underline{u}_i(\omega, y_i), \quad \text{where,}$$
$$B_i^*(\omega, p) = \{ y_i \in L : \quad p(\omega) \cdot y_i(\omega) \leq p(\omega) \cdot e_i(\omega) \}.$$

[10] Notice that the private information measurability assumption of the utility does not imply that the maximin utility coincides with the ex post one, since the allocation may not be measurable.

Clearly, any maximin Walrasian equilibrium is a maximin IWE, as the following proposition indicates.

Proposition 5.10 *Any maximin Walrasian equilibrium is a maximin IWE.*

Proof See Appendix. □

We show that any MWE is maximin efficient. We first define the notion of maximin Pareto optimality.[11]

Definition 5.11 A feasible allocation x is said to be **maximin efficient (or maximin Pareto optimal)** if there do not exist a state $\bar{\omega}$ and an allocation $y \in L$ such that

$$\begin{aligned} &\text{(i)} \quad \underline{u}_i(\bar{\omega}, y_i) > \tilde{u}_i(\bar{\omega}, x_i(\bar{\omega})) \geq \underline{u}_i(\bar{\omega}, x_i) \quad \text{for all } i \in I \text{ and} \\ &\text{(ii)} \quad \sum_{i \in I} y_i(\omega) = \sum_{i \in I} e_i(\omega) \quad \text{for all } \omega \in \Omega. \end{aligned}$$

The following proposition guarantees that any MWE is maximin efficient.

Proposition 5.12 *Any maximin Walrasian equilibrium allocation is maximin efficient.*

Proof Appendix. □

5.1 Incentive compatibility notions

We now recall the notion of coalitional incentive compatibility of Krasa and Yannelis (1994).

Definition 5.13 An allocation x is said to be **coalitional incentive compatible (CIC)** if the following does not hold: there exist a coalition S and two states a and b such that

$$\begin{aligned} &\text{(i)} \quad \mathcal{F}_i(a) = \mathcal{F}_i(b) \quad \text{for all } i \notin S, \\ &\text{(ii)} \quad e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} \quad \text{for all } i \in S, \text{ and} \\ &\text{(iii)} \quad \tilde{u}_i(a, e_i(a) + x_i(b) - e_i(b)) > \tilde{u}_i(a, x_i(a)) \quad \text{for all } i \in S. \end{aligned}$$

If $S = \{i\}$, then the above definition reduces to individual incentive compatibility. A Pareto optimal allocation may be not coalitional incentive compatible and a contract which is individual incentive compatible may not be coalitional incentive compatible (see Glycopantis and Yannelis 2005; de Castro and Yannelis 2010).[12]

In this section, we will prove that any maximin core is incentive compatible. To this end, we need the following definition of maximin coalitional incentive compatibility, which is an extension of the Krasa and Yannelis (1994) definition to incorporate maximin preferences (see also de Castro and Yannelis 2008 for a related notion).

[11] Other notions of efficiency with maximin preferences can be found in Dana (2004), de Castro et al. (2010) and Ozsoylev and Werner (2009).

[12] The reader is also referred to Krasa and Yannelis (1994) and Koutsougeras and Yannelis (1993) for an extensive discussion of the Bayesian incentive compatibility in asymmetric information economies.

Definition 5.14 A feasible allocation x is said to be **maximin coalitional incentive compatible (MCIC)** if the following does not hold: there exist a coalition S and two states a and b such that

$$\begin{aligned}
&\text{(i)} && \mathcal{F}_i(a) = \mathcal{F}_i(b) \quad \text{for, all } i \notin S,\\
&\text{(ii)} && \tilde{u}_i(a,\cdot) = \tilde{u}_i(b,\cdot) \text{ and } \tilde{u}_i(a, x_i(a)) = \underline{u}_i(a, x_i) \quad \text{for all } i \notin S,\\
&\text{(iii)} && e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+} \quad \text{for all } i \in S, \ \textit{and}\\
&\text{(iv)} && \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i) \quad \text{for all } i \in S,
\end{aligned}$$

where for all $i \in S$,

$$(*) \qquad y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise.} \end{cases}$$

According to the above definition, an allocation is said to be maximin coalitional incentive compatible if it is not possible for a coalition to misreport the realized states of nature and have a distinct possibility of making its members better off in terms of maximin expected utility. Notice that in addition to Definition 5.13, we require that agents in the complementary coalition to have the same utility in states a and b that they cannot distinguish. Obviously, if $S = \{i\}$ then the above definition reduces to individual incentive compatibility.

Remark 5.15 Condition (ii) of Definition 5.14 does not necessarily mean that for all $i \notin S$ and $y \in R^{\ell}_{+}$, $\tilde{u}_i(\cdot, y)$ is $\mathcal{F}_i$-measurable. Indeed it may be the case that there exists $\omega \in \mathcal{F}_i(a) \setminus \{a, b\}$ such that $\tilde{u}_i(\omega,\cdot) \neq \tilde{u}_i(a,\cdot) = \tilde{u}_i(b,\cdot)$. Moreover, condition (ii) is not required for each state, but only for the realized state of nature which the members of S may misreport. Observe that Definition 5.14 implicity requires that the members of the coalition S are able to distinguish between a and b; i.e., $a \notin \mathcal{F}_i(b)$ for all $i \in S$. One could replace condition (i) by $\mathcal{F}_i(a) = \mathcal{F}_i(b)$ if and only if $i \notin S$.

It has been proved in the study by de Castro and Yannelis (2008) that any coalitional incentive compatible allocation is maximin CIC, but the converse may not be true.

5.2 Maximin efficiency implies maximin incentive compatibility

In this paper, we use a slightly different definition of incentive compatibility that the one used in the study by de Castro and Yannelis (2010) for type models. Because of that, we include a complete proof of the following result, which does not follow from that in the study by de Castro and Yannelis (2010).

Theorem 5.16 *Assume that for all $i \in I$ and for all $\omega \in \Omega$, $\tilde{u}_i(\omega,\cdot)$ is continuous and strongly monotone. Then, any maximin Pareto optimal allocation is maximin coalitional incentive compatible.*

Proof Let x be a maximin Pareto optimal allocation and assume on the contrary that it is not maximin CIC. This means that there exist a coalition S and two states a and b such that

(i) $\mathcal{F}_i(a) = \mathcal{F}_i(b)$ for all $i \notin S$,

(ii) $\tilde{u}_i(a, \cdot) = \tilde{u}_i(b, \cdot)$ and $\tilde{u}_i(a, x_i(a)) = \underline{u}_i(a, x_i)$ for all $i \notin S$,

(iii) $e_i(a) + x_i(b) - e_i(b) \in \mathbb{R}^{\ell}_{+}$ for all $i \in S$, and

(iv) $\underline{u}_i(a, y_i) > \underline{u}_i(a, x_i)$ for all $i \in S$,

where for all $i \in S$,

$$(*) \qquad y_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise.} \end{cases}$$

Define for all $i \in I$,

$$z_i(\omega) = \begin{cases} e_i(a) + x_i(b) - e_i(b) & \text{if } \omega = a \\ x_i(\omega) & \text{otherwise.} \end{cases}$$

Notice that for all $i \notin S$, from (i) it follows that $z_i(a) = x_i(b)$. Moreover condition (ii) implies that $\tilde{u}_i(a, x_i(b)) = \tilde{u}_i(b, x_i(b))$. Hence, for all $i \notin S$

$$\begin{aligned} \underline{u}_i(a, z_i) &= \min_{\omega \in \mathcal{F}_i(a)} \tilde{u}_i(\omega, z_i(\omega)) = \\ \min_{\omega \in \mathcal{F}_i(a) \setminus \{a\}} \tilde{u}_i(\omega, x_i(\omega)) &\geq \min_{\omega \in \mathcal{F}_i(a)} \tilde{u}_i(\omega, x_i(\omega)) \\ &= \tilde{u}_i(a, x_i(a)) = \underline{u}_i(a, x_i). \end{aligned}$$

On the other hand, for all $i \in S$, from (iv) it follows that

$$\underline{u}_i(a, z_i) = \underline{u}_i(a, y_i) > \underline{u}_i(a, x_i).$$

Since for all $i \in I$ and $\omega \in \Omega$, $\tilde{u}_i(\omega, \cdot)$ is continuous, there exists $\epsilon \in (0, 1)$ such that

$$\underline{u}_i(a, \epsilon z_i) > \underline{u}_i(a, x_i) \qquad \text{for all } i \in S.$$

Define for all $\omega \in \Omega$,

$$\tilde{z}_i(\omega) = \begin{cases} \epsilon z_i(\omega) & \text{if } i \in S \\ z_i(\omega) + \frac{1-\epsilon}{|I \setminus S|} \sum_{i \in S} z_i(\omega) & \text{if } i \notin S. \end{cases}$$

Notice that for all $i \in S$, $\underline{u}_i(a, \tilde{z}_i) > \underline{u}_i(a, x_i)$. Moreover, for all $i \notin S$ from (iii) and from the strong monotonicity of the utility function it follows that $\underline{u}_i(a, \tilde{z}_i) > \underline{u}_i(a, z_i) \geq \tilde{u}_i(a, x_i(a)) = \underline{u}_i(a, x_i)$.

Therefore, there exist $a \in \Omega$ and $\tilde{z}$ such that $\underline{u}_i(a, \tilde{z}_i) > \underline{u}_i(a, x_i)$ for all $i \in I$. Moreover, notice that condition (iv) and $(*)$ imply that for all $i \in S$, $\tilde{u}_i(a, x_i(a)) = \min_{\omega \in \mathcal{F}_i(a)} \tilde{u}_i(\omega, x_i(\omega)) = \underline{u}_i(a, x_i)$ (see de Castro et al. 2010). To get a contradiction, we just need to show that $\tilde{z}$ is feasible.

For any $\omega \neq a$, we have

$$\begin{aligned}
&\sum_{i\in I}\tilde{z}_i(\omega) = \sum_{i\in S}\epsilon z_i(\omega) + \sum_{i\notin S} z_i(\omega) + (1-\epsilon)\sum_{i\in S} z_i(\omega) = \\
&\sum_{i\in I} z_i(\omega) = \sum_{i\in I} x_i(\omega) \\
&\qquad\quad = \sum_{i\in I} e_i(\omega).
\end{aligned}$$

Finally, in state a, we have

$$\begin{aligned}
\sum_{i\in I}\tilde{z}_i(a) &= \sum_{i\in S}\epsilon z_i(a) + \sum_{i\notin S} z_i(a) + (1-\epsilon)\sum_{i\in S} z_i(a) = \\
\sum_{i\in S} z_i(a) + \sum_{i\notin S} z_i(a) &= \sum_{i\in S} e_i(a) + \sum_{i\in S}[x_i(b) - e_i(b)] \\
+\sum_{i\notin S} e_i(a) + \sum_{i\notin S}[x_i(b) - e_i(b)] &= \sum_{i\in I} e_i(a) + \sum_{i\in I}[x_i(b) - e_i(b)] \\
&= \sum_{i\in I} e_i(a).
\end{aligned}$$

This means that $\tilde{z}$ is feasible and hence we get a contradiction. □

The above theorem and Proposition 5.12 imply the following corollary.

Corollary 5.17 *Any maximin Walrasian equilibrium allocation is maximin coalitional incentive compatible and a fortiori individual incentive compatible.*

6 Concluding remarks and open questions

We examined the core and the Walrasian equilibrium in an asymmetric information economy where agents behave as non-expected utility maximizers, and obtained results on the existence, efficiency and incentive compatibility of these notions. The results contained in this paper may be summarized as follows:

- We provided a general framework for systems of ex ante, interim and ex post preferences.
- We introduced the following new concepts:
  1. General ex ante and interim core;
  2. General ex ante and interim Walrasian equilibrium;
  3. Maximin Walrasian equilibrium.
- We compared our concepts and some of the more important ones in the literature:
  1. ex ante core (private vs general);
  2. interim core (private vs general);
  3. interim Walrasian equilibrium (private vs general).

- We provided new existence results for:
  1. ex ante core with general preferences;
  2. interim core with general preferences;
  3. maximin interim Walrasian equilibria (for maximin preferences);
  4. maximin core (for maximin preferences).
- We also established some incentive compatibility results for maximin preferences:
  1. we proved that efficiency implies coalitional incentive compatibility;
  2. a maximin Walrasian Equilibrium is maximin coalitional incentive compatible.

The number of agents in our model is finite, and as a consequence at this stage, we have not obtained any equivalence results for the maximin core and the maximin Walrasian equilibrium. The rate of convergence of the maximin core seems to be a challenging question as the MEU may fail to be differentiable and the standard arguments may not be directly applicable. We hope to take up those details in subsequent work.

A Appendix

Proof of Proposition 3.4 Let x be an ex ante private core allocation and assume on the contrary that there exist a coalition S and an allocation y such that

(i) $y_i(\cdot)$ is $\mathcal{F}_i$-*measurable* for all $i \in S$

(ii) $v_i(y_i|\mathcal{F}_i)(\omega) > v_i(x_i|\mathcal{F}_i)(\omega)$ for all $i \in S$ and for all $\omega \in \Omega$

(iii) $\sum_{i \in S} y_i(\omega) = \sum_{i \in S} e_i(\omega)$ for all $\omega \in \Omega$.

Notice that for each agent i and each $t \in L$,

$$\begin{aligned}\sum_{\omega\in\Omega} v_i(t|\mathcal{F}_i)(\omega)\pi_i(\omega) &= \sum_{\omega\in\Omega}\left[\sum_{\omega'\in\mathcal{F}_i(\omega)} \tilde{u}_i(\omega', t(\omega'))\frac{\pi_i(\omega')}{\pi_i(\mathcal{F}_i(\omega))}\right]\pi_i(\omega)\\ &= \sum_{E\in\mathcal{F}_i}\left[\sum_{\omega'\in E} \tilde{u}_i(\omega', t(\omega'))\frac{\pi_i(\omega')}{\pi_i(E)}\right]\pi_i(E)\\ &= \sum_{\omega\in\Omega}\tilde{u}_i(\omega, t(\omega))\pi_i(\omega) = V_i(t).\end{aligned}$$

Thus, condition (ii) implies that $V_i(y_i) > V_i(x_i)$ for all $i \in S$, and hence x does not belong to the ex ante private core. This is a contradiction. We now prove that the converse may not be true. To this end, consider the following three agent differential information economy, i.e., $I = \{1, 2, 3\}$, with three equiprobable states of nature, i.e., $\Omega = \{a, b, c\}$ and whose primitives are given as follows:

$$\begin{array}{lll} e_1 = (5, 5, 0) & \mathcal{F}_1 = \{\{a, b\}; \{c\}\} & u_1(\cdot, x_1) = \sqrt{x_1}\\ e_2 = (5, 0, 5) & \mathcal{F}_2 = \{\{a, c\}; \{b\}\} & u_2(\cdot, x_2) = \sqrt{x_2}\\ e_3 = (0, 0, 0) & \mathcal{F}_3 = \{\{a\}; \{b, c\}\} & u_3(\cdot, x_3) = \sqrt{x_3}. \end{array}$$

It is easy to show that the initial endowment belongs to the weak interim private core, and therefore into the interim private core. On the other hand, it is privately blocked in the ex ante stage by the grand coalition I via the allocation x, where $x_1 = (4, 4, 1)$, $x_2 = (4, 1, 4)$ and $x_3 = (2, 0, 0)$. Indeed, first notice that the allocation $x_i(\cdot)$ is $\mathcal{F}_i$-measurable for all $i \in I$ and it is feasible. Moreover,

$$V_1(x_1) = \frac{5}{3} > \frac{2}{3}\sqrt{5} = V_1(e_1),$$
$$V_2(x_2) = \frac{5}{3} > \frac{2}{3}\sqrt{5} = V_2(e_2),$$
$$V_3(x_3) = \frac{1}{3}\sqrt{2} > 0 = V_3(e_3).$$

Thus, the interim private core contains properly the ex ante core which may not contain the weak interim private core. □

Proof of Proposition 3.7 Let x be an ex ante core allocation and assume on the contrary that there exist a coalition S and an allocation y such that

$$\text{(i)} \quad u_i(\omega, y_i) > u_i(\omega, x_i) \text{ for all } i \in S \text{ and } \omega \in \Omega,$$
$$\text{(ii)} \quad \sum_{i \in S} y_i(\omega) = \sum_{i \in S} e_i(\omega) \quad \text{for all } \omega \in \Omega.$$

Remember that for each $i \in I$,

$$U_i(\cdot) = \sum_{E \in \mathcal{F}_i} u_i(E, \cdot)\pi_i(E),$$

where $u_i(E; \cdot)$ can be also written (see (6) p. 9.) with $u_i(\omega, \cdot)$. Since $u_i(\omega, \cdot) = u_i(\bar{\omega}, \cdot)$ whenever $\mathcal{F}_i(\omega) = \mathcal{F}_i(\bar{\omega})$, it follows that

$$\sum_{E \in \mathcal{F}_i} u_i(E, \cdot)\pi_i(E) = \sum_{\omega \in \Omega} u_i(\omega, \cdot)\pi_i(\omega).$$

Since for each i, $U_i(\cdot) = \sum_{\omega \in \Omega} u_i(\omega, \cdot)\pi_i(\omega)$, condition (i) implies that $U_i(y_i) > U_i(x_i)$ for all $i \in S$. Therefore, x does not belong to the ex ante core, which is a contradiction. □

Proof of Theorem 3.8 The arguments are standard (see for example Scarf 1967). For the sake of completeness we provide the proof.
Define for each $i \in I$ the set,

$$L = \{x_i : \Omega \to \mathbb{R}^{\ell}_{+} \ : \ x_i(\omega) \in X \text{ for all } \omega \in \Omega\},$$

and let $L^n = \prod_{i \in I} L$. Notice that since for all i and ω, X is non-empty,[13] convex and compact, so is L.

[13] Notice that X is non-empty since it contains at least the initial endowment of each agent.

We want to show that the ex ante core is non-empty. To this end, define a game V as follows: for each $S \subseteq I$,

$$V(S) = \Big\{ v \in \mathbb{R}^n \ : \ \text{there exists } y \in L^S = \prod_{i \in S} L \text{ such that } U_i(y_i) \geq v_i \text{ for all } i \in S \text{ and } \sum_{i \in S} y_i(\omega) = \sum_{i \in S} e_i(\omega) \text{ for all } \omega \in \Omega \Big\}.$$

We just need to show that V satisfies all the proprieties of Scarf's Theorem. Clearly, by definition, each $V(S)$ is comprehensive from below,[14] bounded from above[15] and such that if $v_1 \in \mathbb{R}^n$, $v_2 \in V(S)$ and $v_{1i} = v_{2i}$ for all $i \in S$, then $v_1 \in V(S)$. Moreover for each S, $V(S)$ is closed. Indeed, let v_k be a sequence of $V(S)$ converging to v^*, we need to show that $v^* \in V(S)$. Since for each k, $v_k \in V(S)$, then there exists a sequence $y_k \in L^S$ such that

(i) $U_i(y_{ki}) \geq v_{ki}$ for all $i \in S$ and $k \in \mathbb{N}$

(ii) $\sum_{i \in S} y_{ki}(\omega) = \sum_{i \in S} e_i(\omega)$ for all $\omega \in \Omega$ and $k \in \mathbb{N}$.

Since L is compact, so is L^S. Thus, there exists a subsequence of y_k, still denoted by y_k, which converges to y^*. Clearly, $y^* \in L^S$ and from (ii), it follows that

$$\sum_{i \in S} y_i^*(\omega) = \sum_{i \in S} e_i(\omega) \text{ for all } \omega \in \Omega.$$

Moreover, the continuity of the utility functions implies that taking the limits in (i),

$$U_i(y_i^*) \geq v_i^* \text{ for all } i \in S.$$

Therefore, $v^* \in V(S)$, i.e., $V(S)$ is closed. To conclude the proof, we just need to verify that the game V is balanced.[16] Let $\mathcal{B}$ be a balanced family of coalitions with weights $\{\lambda_S \ : \ S \in \mathcal{B}\}$ and let v be an element of $\bigcap_{S \in \mathcal{B}} V(S)$. We must show that

[14] $V(S)$ is comprehensive from below if $v_1 \leq v_2$ and $v_2 \in V(S)$ imply $v_1 \in V(S)$.

[15] Each $V(S)$ is bounded from above if for each coalition S, there exists some $M_S > 0$ satisfying $v_i \leq M_S$ for all $v \in V(S)$ and for all $i \in S$.

[16] A game V is said to be balanced whenever every balanced family $\mathcal{B}$ of coalitions satisfies

$$\bigcap_{S \in \mathcal{B}} V(S) \subseteq V(I).$$

A non-empty family $\mathcal{B}$ of 2^I is said to be balanced whenever there exist non-negative weights $\{\lambda_S \ : \ S \in \mathcal{B}\}$ satisfying

$$\sum_{\substack{S \in \mathcal{B} \\ i \in S}} \lambda_S = 1 \text{ for all } i \in I.$$

$v \in V(I)$. For each $i \in I$, define $B_i = \{S \in \mathcal{B} : i \in S\}$. Since $v \in \bigcap_{S\in\mathcal{B}} V(S)$, then for each $S \in \mathcal{B}$ there exists $y^S \in L^S$ such that

$$
\begin{array}{ll}
\text{(i)} & U_i(y_i^S) \geq v_i \quad \text{for all } i \in S \\
\text{(ii)} & \sum_{i\in S} y_i^S(\omega) = \sum_{i\in S} e_i(\omega) \quad \text{for all } \omega \in \Omega.
\end{array}
$$

Define for each $i \in I$,

$$z_i = \sum_{S\in B_i} \lambda_S y_i^S, \quad \text{where } \sum_{S\in B_i} \lambda_S = 1,$$

and notice that the concavity assumption of the utility functions implies that for all $i \in I$,

$$U_i(z_i) \geq \sum_{S\in B_i} \lambda_S U_i(y_i^S) \geq \sum_{S\in B_i} \lambda_S v_i = v_i.$$

Moreover for all $\omega \in \Omega$,

$$
\begin{aligned}
\sum_{i\in I} z_i(\omega) &= \sum_{i\in I}\sum_{S\in B_i} \lambda_S y_i^S(\omega) = \sum_{S\in\mathcal{B}} \lambda_S \sum_{i\in S} y_i^S(\omega) \\
\sum_{S\in\mathcal{B}} \lambda_S \sum_{i\in S} e_i(\omega) &= \sum_{i\in I}\sum_{S\in B_i} \lambda_S e_i(\omega) = \sum_{i\in I} e_i(\omega).
\end{aligned}
$$

Thus, by Scarf's Theorem, the n-person game has a non-empty core. Pick $v \in Core(V) = V(I) \setminus \bigcup_{S\subseteq I} Int V(S)$ and since, in particular, $v \in V(I)$, let $x \in L^n$ be an allocation such that $U_i(x_i) \geq v_i$ for each $i \in I$ and $\sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega)$ for each $\omega \in \Omega$. To complete the proof, we just need to show that x is an ex ante core allocation. Clearly, x is feasible. Now, suppose on the contrary that there exist a coalition S and an allocation y such that

$$
\begin{array}{ll}
\text{(i)} & U_i(y_i) > U_i(x_i) \geq v_i \quad \text{for all } i \in S \text{ and} \\
\text{(ii)} & \sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega) \quad \text{for all } \omega \in \Omega.
\end{array}
$$

Therefore, conditions (i) and (ii) together with the continuity of $U_i(\cdot)$ imply that $v \in Int V(S)$, which contradicts the fact that $v \in Core(V)$. Hence, x is an ex ante core allocations. □

Proof of Proposition 3.10 Let x be an ex ante core allocation such that $x_i(\cdot)$ is $\mathcal{F}_i$-measurable for all $i \in I$. Assume on the contrary that there exist a coalition S

and an allocation y such that

$$\begin{aligned} &\text{(i)} \quad y_i(\cdot) \text{ is } \mathcal{F}_i-\text{measurable for all } i \in S \\ &\text{(ii)} \quad V_i(y_i) > V_i(x_i) \text{ for all } i \in S \text{ and} \\ &\text{(iii)} \quad \sum_{i \in S} y_i(\omega) = \sum_{i \in S} e_i(\omega) \quad \text{for all } \omega \in \Omega. \end{aligned}$$

Notice that since for all $i \in S$ and for all $t \in \mathbb{R}^{\ell}_{+}, \tilde{u}_i(\cdot, t)$ and $y_i(\cdot)$ are $\mathcal{F}_i$-measurable, it follows that

$$u_i(\omega, y_i) = \tilde{u}_i(\omega, y_i(\omega)) \quad \text{for all } \omega \in \Omega.$$

Hence, since for each i, $U_i(\cdot) = \sum_{\omega \in \Omega} u_i(\omega, \cdot)\pi_i(\omega)$, then for each i, $V_i(y_i) = U_i(y_i)$ and similarly $V_i(x_i) = U_i(x_i)$. Therefore, x is not in the ex ante core and this is a contradiction.

We now want to prove that the converse may not be true. To this end, consider a differential information economy with three equiprobable state of nature, i.e., $\Omega = \{a, b, c\}$ with $\pi_i(\omega) = \frac{1}{3}$ for each i and ω. There are two agents asymmetrically informed and only one good. Moreover, the primitives of the economy are given as follows:

$$\begin{aligned} e_1 &= (5, 5, 0) \quad \mathcal{F}_1 = \{\{a, b\}; \{c\}\} \quad u_1(\cdot, x_1) = \sqrt{x_1} \\ e_2 &= (5, 0, 5) \quad \mathcal{F}_2 = \{\{a, c\}; \{b\}\} \quad u_2(\cdot, x_2) = \sqrt{x_2}. \end{aligned}$$

It is easy to show that the initial endowment is an ex ante private core allocation. On the other hand, if we consider the MEU formulation,[17] the initial endowment is blocked by the grand coalition I via the feasible allocation $y_1 = (5, 4, 1)$ and $y_2 = (5, 1, 4)$.

Proof of Proposition 3.11 Let x be an interim core allocation such that $x_i(\cdot)$ is $\mathcal{F}_i$-measurable for all $i \in I$. Assume on the contrary that there exist a coalition S and an allocation y such that

$$\begin{aligned} &\text{(i)} \quad y_i(\cdot) \text{ is } \mathcal{F}_i\text{-measurable for all } i \in S \\ &\text{(ii)} \quad v_i(y_i|\mathcal{F}_i)(\omega) > v_i(x_i|\mathcal{F}_i)(\omega) \text{ for all } i \in S \text{ and } \omega \in \Omega, \\ &\text{(iii)} \quad \sum_{i \in S} y_i(\omega) = \sum_{i \in S} e_i(\omega) \quad \text{for all } \omega \in \Omega. \end{aligned}$$

Notice that from (i) it follows that for all $i \in S$ and $\omega \in \Omega, v_i(y_i|\mathcal{F}_i)(\omega) = \tilde{u}_i(\omega, y_i(\omega)) = u_i(\omega, y_i)$. Similarly $v_i(x_i|\mathcal{F}_i)(\omega) = \tilde{u}_i(\omega, x_i(\omega)) = u_i(\omega, x_i)$ for all $i \in S$. Hence, x is not in the interim core and this is a contradiction.

[17] In the MEU formulation, the ex ante maximin utility is:

$$U_i(x_i) = \sum_{\omega \in \Omega} \min_{\omega' \in \mathcal{F}_i(\omega)} \tilde{u}_i(\omega', x_i(\omega'))\pi_i(\omega).$$

We now want to prove that the converse may not be true. To this end, consider a differential information economy with two equiprobable states of nature, i.e., $\Omega = \{a, b\}$ with $\pi_i(\omega) = \frac{1}{2}$ for each i and ω. There are two agents asymmetrically informed and two goods. Moreover, the primitives of the economy are given as follows:

$$e_1(a,b) = ((6,4),(6,4)) \quad \mathcal{F}_1 = \{\{a,b\}\} \quad u_1(\cdot, x_1, y_1) = x_1 \cdot y_1$$
$$e_2(a,b) = ((0,1),(1,0)) \quad \mathcal{F}_2 = \{\{a\};\{b\}\} \quad u_2(a, x_2, y_2) = x_2 + \frac{1}{2}y_2 \quad u_2(b, x_2, y_2) = y_2 + \frac{1}{2}x_2.$$

It is easy to show that the initial endowment is an interim private core allocation. On the other hand, it is not in the interim core with MEU formulation.[18] Indeed, it is blocked by the grand coalition I via the feasible allocation $((5,5);(7,3.49))$ and $((1,0);(0,0.51))$. □

Proof of Proposition 5.2 Consider a differential information economy with two equiprobable states of nature, i.e., $\Omega = \{a, b\}$ with $\pi_i(\omega) = \frac{1}{2}$ for each i and ω. There are two agents asymmetrically informed and two goods. Moreover, the primitives of the economy are given as follows:

$$e_1(a,b) = ((6,4),(6,4)) \quad \mathcal{F}_1 = \{\{a,b\}\} \quad u_1(\cdot, x_1, y_1) = x_1 \cdot y_1$$
$$e_2(a,b) = ((1,2),(2,1)) \quad \mathcal{F}_2 = \{\{a\};\{b\}\} \quad u_2(a, x_2, y_2) = x_2 + \frac{1}{2}y_2 \quad u_2(b, x_2, y_2) = y_2 + \frac{1}{2}x_2.$$

One can show that the unique interim Walrasian expectations equilibrium (IWEE) is the initial endowment with $p(a) = (2p, p)$ and $p(b) = (p, 2p)$, but it is not a maximin interim Walrasian equilibrium (MIWE). Indeed, consider the following feasible allocation

$$(x_1(a), y_1(a)) = (5,6) \quad (x_1(b), y_1(b)) = (7, 7/2)$$
$$(x_2(a), y_2(a)) = (2,0) \quad (x_2(b), y_2(b)) = (1, 3/2),$$

and notice that such an allocation belongs to the budget set of each agent, but it is (maximin) preferred by agent 1, i.e.,

$$\underline{u}_1(a, x_1, y_1) = \min\{x_1(a)y_1(a), x_1(b)y_1(b)\} = \min\{30, 49/2\} = 49/2 > 24 = \underline{u}_1(a, e_1).$$

[18] In the MEU formulation, the interim maximin utility is:

$$u_i(\omega, x_i) = \min_{\omega' \in \mathcal{F}_i(\omega)} \tilde{u}_i(\omega', x_i(\omega')).$$

Hence, the initial endowment, which is the unique IWEE is not a MIWE. On the other hand, the following allocation $\tilde{x}$

$$(\tilde{x}_1(a), \tilde{y}_1(a)) = (9/2, 6) \quad (\tilde{x}_1(b), \tilde{y}_1(b)) = (7, 7/2)$$
$$(\tilde{x}_2(a), \tilde{y}_2(a)) = (5/2, 0) \quad (\tilde{x}_2(b), \tilde{y}_2(b)) = (1, 3/2),$$

with $p(a) = (4/3p, p)$ and $p(b) = (p, 2p)$ is a maximin interim Walrasian equilibrium but it is clearly not an IWEE. Therefore, the notions of IWEE and MIWE may not be comparable. □

Proof of Proposition 5.3 Let (p, x) be an ex post Walrasian equilibrium and assume, on the contrary that (p, x) is not a MIWE. First, notice that since for all $i \in I$ and $\omega \in \Omega$, $p(\omega) \cdot x_i(\omega) \leq p(\omega) \cdot e_i(\omega)$, then for all $i \in I$ and $\omega \in \Omega$, $x_i \in B_i(\omega, p)$. Thus, there exist an agent i, a state $\bar{\omega} \in \Omega$ and an allocation y_i such that

$$\begin{aligned} &\underline{u}_i(\bar{\omega}, y_i) > \underline{u}_i(\bar{\omega}, x_i) \text{ and} \\ &\qquad y_i \in B_i(\bar{\omega}, p), \text{ that is } p(\omega') \cdot y_i(\omega') \leq p(\omega') \cdot e_i(\omega') \text{ for all } \omega' \in \mathcal{F}_i(\bar{\omega}). \end{aligned} \tag{7}$$

Since Ω is finite, there exists a state $\omega' \in \mathcal{F}_i(\bar{\omega})$ such that

$$\underline{u}_i(\bar{\omega}, x_i) = \min_{\omega \in \mathcal{F}_i(\bar{\omega})} \tilde{u}_i(\omega, x_i(\omega)) = \tilde{u}_i(\omega', x_i(\omega')).$$

Thus,

$$\tilde{u}_i(\omega', y_i(\omega')) \geq \underline{u}_i(\bar{\omega}, y_i) > \underline{u}_i(\bar{\omega}, x_i) = \tilde{u}_i(\omega', x_i(\omega')),$$

which implies that

$$p(\omega') \cdot y_i(\omega') > p(\omega') \cdot e_i(\omega'), \tag{8}$$

because (p, x) is an ex post Walrasian equilibrium. Notice that (8) contradicts (7). Therefore, (p, x) is a maximin interim Walrasian equilibrium. □

Proof of Proposition 5.5 Let (p, x) be an ex post Walrasian equilibrium and assume by the way of contradiction that there exist a coalition S, a state $\bar{\omega}$ and an allocation y such that

$$\begin{aligned} &\text{(i)} \quad \underline{u}_i(\bar{\omega}, y_i) > \tilde{u}_i(\bar{\omega}, x_i(\bar{\omega})) \geq \underline{u}_i(\bar{\omega}, x_i) \quad \text{for all } i \in S, \\ &\text{(ii)} \quad \sum_{i \in I} y_i(\omega) = \sum_{i \in I} e_i(\omega) \quad \text{for all } \omega \in \Omega. \end{aligned}$$

From (i) it follows that $\tilde{u}_i(\bar{\omega}, y_i(\bar{\omega})) \geq \underline{u}_i(\bar{\omega}, y_i) > \tilde{u}_i(\bar{\omega}, x_i(\bar{\omega}))$ for all $i \in S$, and hence

$$p(\bar{\omega}) \cdot y_i(\bar{\omega}) > p(\bar{\omega}) \cdot e_i(\bar{\omega}) \quad \text{for all } i \in S.$$

Thus,

$$\sum_{1\in I} p(\bar{\omega}) \cdot y_i(\bar{\omega}) > \sum_{1\in I} p(\bar{\omega}) \cdot e_i(\bar{\omega}),$$

which contradicts (ii). □

Proof of Theorem 5.8 Let (p, x) be a maximin REE and assume by the way of contradiction that there exist a coalition S, a state $\bar{\omega} \in \Omega$ and an allocation y such that

$$\text{(i)} \quad \underline{u}_i(\bar{\omega}, y_i) > \tilde{u}_i(\bar{\omega}, x_i(\bar{\omega})) \geq \underline{u}_i(\bar{\omega}, x_i) \text{ for all } i \in S,$$
$$\text{(ii)} \quad \sum_{i\in S} y_i(\omega) = \sum_{i\in S} e_i(\omega) \text{ for all } \omega \in \Omega.$$

From condition (i), it follows that for all $i \in S$,

$$\begin{aligned} \underline{u}_i^{\mathrm{REE}}(\bar{\omega}, y_i) &\geq \underline{u}_i(\bar{\omega}, y_i) > \tilde{u}_i(\bar{\omega}, x_i(\bar{\omega})) \\ &\geq \underline{u}_i^{\mathrm{REE}}(\bar{\omega}, x_i) \geq \underline{u}_i(\bar{\omega}, x_i), \text{ i.e.,} \end{aligned}$$

$$\underline{u}_i^{\mathrm{REE}}(\bar{\omega}, y_i) > \underline{u}_i^{\mathrm{REE}}(\bar{\omega}, x_i). \tag{9}$$

Thus, from (9), it follows that for all $i \in S$, $y_i \notin B_i^{\mathrm{REE}}(\bar{\omega}, p)$, that is there exists a state $\omega_i \in \mathcal{G}_i(\bar{\omega})$ such that $p(\omega_i) \cdot y_i(\omega_i) > p(\omega_i) \cdot e_i(\omega_i)$. Consider, the coalition A defined as follows:

$$A = \{i \in S : \; p(\bar{\omega}) \cdot y_i(\bar{\omega}) \leq p(\bar{\omega}) \cdot e_i(\bar{\omega})\}.$$

If A is empty, then $p(\bar{\omega}) \cdot y_i(\bar{\omega}) > p(\bar{\omega}) \cdot e_i(\bar{\omega})$ for all $i \in S$ and hence

$$p(\bar{\omega}) \sum_{i\in S} y_i(\bar{\omega}) > p(\bar{\omega}) \sum_{i\in S} e_i(\bar{\omega}),$$

which contradicts condition (ii). On the other hand, if $A \neq \emptyset$, then for all $i \in A$, consider the constant allocation h_i such that $h_i(\omega) = y_i(\bar{\omega})$ for all $\omega \in \mathcal{G}_i(\bar{\omega})$. Since $p(\cdot)$ and $e_i(\cdot)$ are $\mathcal{G}_i$-measurable, it follows that for each $i \in A$, $h_i \in B_i^{\mathrm{REE}}(\bar{\omega}, p)$, and hence (9) implies that

$$\underline{u}_i^{\mathrm{REE}}(\bar{\omega}, h_i) \leq \underline{u}_i^{\mathrm{REE}}(\bar{\omega}, x_i) < \underline{u}_i^{\mathrm{REE}}(\bar{\omega}, y_i) \quad \text{for each } i \in A,$$

because (p, x) is a maximin REE. Moreover, since $\tilde{u}_i(\cdot, y)$ is $\mathcal{G}_i$-measurable, it follows that for each $i \in A$

$$\tilde{u}_i(\bar{\omega}, y_i(\bar{\omega})) = \tilde{u}_i(\omega, y_i(\bar{\omega})) = \underline{u}_i^{\mathrm{REE}}(\bar{\omega}, h_i) < \underline{u}_i^{\mathrm{REE}}(\bar{\omega}, y_i) \leq \tilde{u}_i(\bar{\omega}, y_i(\bar{\omega})),$$

which is clearly a contradiction. Thus, x belongs to the maximin core. □

Proof of Proposition 5.10 Let (p, x) be a maximin WE, thus x is a feasible allocation and p is a price vector. Moreover, since for each i and ω, $x_i \in B_i^*(\omega, p)$, it follows that $x_i \in B_i(\omega, p)$ for each i and ω. Assume on the contrary that (p, x) is not a maximin IWE. Therefore, there exist an agent i, a state $\bar{\omega}$ and an allocation $y_i \in L$ such that

$$\underline{u}_i(\bar{\omega}, y_i) > \underline{u}_i(\bar{\omega}, x_i), \tag{10}$$

and $y_i \in B_i(\bar{\omega}, p)$, that is

$$p(\omega) \cdot y_i(\omega) \leq p(\omega) \cdot e_i(\omega) \quad \text{for all } \omega \in \mathcal{F}_i(\bar{\omega}). \tag{11}$$

Clearly, from (10) it follows that $y_i \notin B_i^*(\bar{\omega}, p)$, that is $p(\bar{\omega}) \cdot y_i(\bar{\omega}) > p(\bar{\omega}) \cdot e_i(\bar{\omega})$, which contradict (11). □

Proof of Proposition 5.12 Let x be a MWE allocation and assume on the contrary that there exist a state $\bar{\omega}$ and an allocation $y \in L$ such that

$$\begin{array}{ll} \text{(i)} & \underline{u}_i(\bar{\omega}, y_i) > \tilde{u}_i(\bar{\omega}, x_i(\bar{\omega})) \geq \underline{u}_i(\bar{\omega}, x_i) \quad \text{for all } i \in I \text{ and} \\ \text{(ii)} & \displaystyle\sum_{i \in I} y_i(\omega) = \sum_{i \in I} e_i(\omega) \quad \text{for all } \omega \in \Omega. \end{array}$$

From (i), one can deduce that $p(\bar{\omega}) \cdot y_i(\bar{\omega}) > p(\bar{\omega}) \cdot e_i(\bar{\omega})$ for all $i \in S$, and hence

$$p(\bar{\omega}) \cdot \sum_{i \in I} y_i(\bar{\omega}) > p(\bar{\omega}) \cdot \sum_{i \in I} e_i(\bar{\omega}),$$

which contradicts (ii). □

References

Allouch, N., Florenzano, M.: Edgeworth and Walras equilibria of an arbitrage-free exchange economy. Econ Theory **23**(2), 353–370 (2004)

Angeloni, L., Martins-da Rocha, V.: Large economies with differential information and without free disposal. Econ Theory **38**(2), 263–286 (2009)

Balder, E., Yannelis, N.: Bayesian–Walrasian equilibria: beyond the rational expectations equilibrium. Econ Theory **38**(2), 385–397 (2009)

Cerreia, S., Maccheroni, F., Marinacci, M., Rustichini, A.: Axiomatic ECON. Discussion paper, Bocconi University (2011)

Condie, S., Ganguli, J.: Ambiguity and rational expectations equilibria. Rev Econ Stud (forthcoming) (2009)

Condie, S., Ganguli, J.: Informational efficiency with ambiguous information. Econ Theory (forthcoming) (2010)

Correia-da, S.J., Hervés-Beloso, J.C.: Prudent expectations equilibrium in economies with uncertain delivery. Econ Theory **39**(1), 67–92 (2009)

Dana, R.: Ambiguity, uncertainty aversion and equilibrium welfare. Econ Theory **23**(3), 569–587 (2004)

de Castro, L.I., Pesce, M., Yannelis, N.C.: A new perspective to rational expectations: maximin rational expectations equilibrium. Discussion paper, University of Illinois at Urbana-Champaign (2010)

de Castro, L.I., Yannelis, N.C.: Ambiguity aversion solves the conflict between efficiency and incentive compatibility. Discussion paper, University of Illinois at Urbana-Champaign (2008)

de Castro, L.I., Yannelis, N.C.: Ambiguity aversion solves the conflict between efficiency and incentive compatibility. Discussion paper, Northwestern University (2010)
Debreu, G.: Representation of a preference ordering by a numerical function. Decision Process 159–165 (1954)
Debreu, G.: Topological methods in cardinal utility theory. In: Mathematical Methods in the Social Sciences, chap. 2. Stanford: Stanford University Press (1960)
Drèze, J.H., Rustichini, A.: Moral hazard and conditional preferences. J Math Econ **31**(2), 159–181 (1999)
Drèze, J.H., Rustichini, A.:State-dependent utility and decision theory, vol. 2. In: Handbook of Utility Theory: Extensions, chap. 16. Université catholique de Louvain Center for Operations Research and Econometrics (CORE), pp. 839–892 (2004)
Ellsberg, D.: Risk, ambiguity, and the savage axioms. Q J Econ **75**(4), 643–669 (1961)
Florenzano, M.: On the non-emptiness of the core of a coalitional production economy without ordered preferences. J Math Anal Appl **141**(2), 484–490 (1989)
Gilboa, I., Schmeidler, D.: Maxmin expected utility with nonunique prior. J Math Econ **18**(2), 141–153 (1989)
Glycopantis, D., Yannelis, N.: Differential information economies, vol. 19. In: Studies in Economic Theory. Berlin: Springer (2005)
Hahn, G., Yannelis, N.: Efficiency and incentive compatibility in differential information economies. Econ Theory **10**(3), 383–411 (1997)
Hanany, E., Klibanoff, P.: Updating ambiguity averse preferences. BE J Theor Econ **9**(1), 37 (2009)
Karni, E., Safra, Z.: The hexagon condition and additive representation for two dimensions: an algebraic approach. J Math Psychol **42**(4), 393–399 (1998)
Koopmans, T.: Stationary ordinal utility and impatience. Econometrica **28**(2), 287–309 (1960)
Koutsougeras, L., Yannelis, N.: Incentive compatibility and information superiority of the core of an economy with differential information. Econ Theory **3**(2), 195–216 (1993)
Krasa, S., Yannelis, N.C.: The value allocation of an economy with differential information. Econometrica **62**, 881–900 (1994)
Luce, R.D., Krantz, D.H.: Conditional expected utility. Econometrica **39**(2), 253–271 (1971)
Machina, M.J.: Dynamic consistency and non-expected utility models of choice under uncertainty. J Econ Lit **27**(4), 1622–1668 (1989)
Ozsoylev, H., Werner, J.: Liquidity and asset prices in rational expectations equilibrium with ambiguous information. Econ Theory (forthcoming) (2009)
Radner, R.: Competitive equilibrium under uncertainty. Econometrica **36**(1), 31–58 (1968)
Scarf, H.E.: The core of an N person game. Econometrica **35**(1), 50–69 (1967)
Schmeidler, D.: Subjective probability and expected utility without additivity. Econometrica **57**(3), 571–587 (1989)
Yannelis, N.C.: The core of an economy with differential information. Econ Theory **1**(2), 183–197 (1991)