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SYMPOSIUM

Ambiguity aversion and trade

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Abstract What is the effect of ambiguity aversion on trade? Although in a Bewley's model, ambiguity aversion always leads to less trade; in other models, this is not always true. However, we show that if the endowments are unambiguous, then more ambiguity aversion implies less trade for a very general class of preferences. The reduction in trade caused by ambiguity aversion can be as severe as to lead to no trade. In an economy with MEU decision makers, we show that if the aggregate endowment is unanimously unambiguous, then every Pareto optima allocation is also unambiguous. We also characterize the situation in which every unanimously unambiguous allocation is Pareto optimal. Finally, we show how our results can be used to explain the home-bias effect. As a useful result for our methods, we also obtain an additivity theorem for CEU and MEU decision makers that does not require comonotonicity.

Keywords No-trade results · Ambiguity aversion · Pareto optimality

JEL Classification D51 · D6 · D8

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1 Introduction

What is the relationship between "uncertainty aversion" and trade? This question was considered by Frank Knight and John M. Keynes,[1] and was the main topic of seminal papers by Truman Bewley. Bewley had a clear intuition about the implications of uncertainty aversion to the propensity of trade: uncertainty aversion reduces trade. This can be explicitly noted in the initial phrases of Bewley (1989)'s sections 3 and 5, respectively: "Knightian decision theory yields an easy explanation of the infrequency of betting on events of unknown probability" (p. 8) and "An easy generalization of the results of Section 3 provides a simplistic explanation of the absence of markets for the insurance of uncertain events" (p. 12).

However, the Knightian decision theory introduced by Bewley (1986) (see also Bewley 2002) is not the only model of uncertainty or ambiguity aversion in economics. It seems natural to expect that Bewley's intuition would also hold for other models of ambiguity aversion, making all these models compatible in this aspect. Despite the impressive amount of knowledge accumulated about these models, so far the literature lacks proper parallel results to those of Bewley (1989). The first main result of this paper establishes sufficient conditions for Bewley's intuition that "*more uncertainty aversion implies less trade*" in the case of general preferences.

This result has some caveats, however. To see this, consider an Ellsberg urn (see Ellsberg 1961) with three balls (one red, and two blue or green) and two individuals. Individual one is given the endowment $e_1 = (e_1(R), e_1(B), e_1(G)) = (0, 1, 0)$, that is, she will receive one unit of the good (or \$1) if the ball is blue (B) and nothing otherwise. Individual two has the endowment $e_2 = (1, 0, 1)$.[2] If the individuals are risk neutral expected utility maximizers (with prior $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ say), then there is no opportunity for a Pareto improving trade. However, if there is more uncertainty aversion (like in the standard Ellsberg paradox), then the trade $z = (1, -1, 0)$ leading to allocations $x_1 = (1, 0, 0)$ and $x_2 = (0, 1, 1)$ is Pareto improving. In this example, more uncertainty aversion leads to *more* trade! How can this be possible, if we said above that "more uncertainty aversion implies less trade"? The answer lies in the assumption about the endowments: Bewley's intuition holds true (and hence our result), under the assumption that the endowments are *unambiguous*. If the endowments are ambiguous, then ambiguity aversion can lead to the opposite result, that is, *more* trade, as in this example.

We are agnostic whether Bewley's intuition is or not the right description of the effect of uncertainty aversion on trade. However, our first contribution is to identify a crucial assumption for Bewley's intuition to hold in general models of ambiguity aversion—that the endowments are unambiguous—and to show it fails otherwise.[3] We are unaware of papers that discuss this issue.

[1] See Keynes (1937), specially p. 216, and Knight (1921, p. 269).

[2] We are grateful to Mark Machina for suggesting us this simple example.

[3] It may be interesting to note that, *apparently*, Bewley obtained his results without any assumption similar to unambiguity of endowments. However, Bewley made extensive use of his "inertia assumption" and in his model this assumption seemed to play a parallel role to that of unambiguous endowments.

From this "comparative statics" result, a natural question is: Under what conditions will ambiguity aversion reduce trading opportunities so much that they will disappear? The absence of trading opportunities is the defining property of Pareto optimal allocations. Therefore, a rephrase of this question would be: Is it possible to give a characterization of Pareto optimal allocations when individuals' preferences have ambiguity aversion? It is not difficult to see how important these questions are for the understanding of the implications of uncertainty to trade.

To investigate these questions, we particularize our study to Schmeidler (1986)'s CEU and Gilboa and Schmeidler (1989)'s MEU models, the two most traditional ambiguity aversion models. We begin by following Nehring (1999) and define the set of unambiguous events and acts in these two settings. Then, we establish an additivity theorem (Theorem 2.1) that is of interest by itself, because it requires neither comonotonicity (as used since Schmeidler 1986) nor slice-comonotonicity (introduced by Ghirardato 1997). This additivity theorem requires instead that one of the acts is unambiguous.

From this, we investigate the effects of uncertainty aversion in two situations: betting and insurance. Betting corresponds to the situation where the event in consideration has no relevant implications for the endowment of the decision maker. Insurance, on the other hand, is related to situations where the individual's endowment may be affected by the realization of the event. It turns out that the implications for both cases are different.

In the betting case, we generalize a result by Dow and Werlang (1992). For the insurance case, we obtain a version of the main results of Billot et al. (2000) and Rigotti et al. (2008) for situations where the aggregate endowment is not constant. Our result shows the equivalence of a common conditional prior and that Pareto optimal allocations are unambiguous. We also show that if the individuals are risk neutral, then the fact that Pareto optimal allocations are unambiguous is equivalent to a common prior (not conditional).

The remaining sections are organized as follows. The related literature is discussed below in Sect. 1.1. Section 2 introduces notation and definitions and present the additivity theorem. Section 3 formalizes and proves that "more ambiguity aversion implies less trade". Section 4 presents our result for the betting situation. Our results regarding the characterization of Pareto optimal allocations and our no-trade theorems are presented in Sect. 5. Section 6 discusses some potential applications, specially to the home-bias effect. Section 7 is a conclusion. All proofs not included in the text are collected in the "Appendix".

1.1 Related literature

Many papers have established characterizations of full insurance Pareto optimal allocations, in which case there is no further opportunity for trade. See for instance, Dow and Werlang (1992), Epstein and Wang (1994), Tallon (1998), Billot et al. (2000), Chateauneuf et al. (2000), Mukerji and Tallon (2001), Dana (2002), Abouda and Chateauneuf (2002), Dana (2004), Rigotti and Shannon (2005), Asano (2006), Rigotti and Shannon (2007), Rigotti et al. (2008) and Strzalecki and Werner (2011).[4,5] We discuss in more detail three of these papers which seem closer related to our results.

Mukerji and Tallon (2001) explored the explanation of the incompleteness of markets through ambiguity aversion. They assume CEU-DMs and that there are two dimensions characterizing the world: an economic state and an idiosyncratic state. Their setting satisfies slice-comonotonicity, a property introduced by Ghirardato (1997) that is a generalization of comonotonicity. Ghirardato (1997) proved that slice-comonotonicity is sufficient to an additivity theorem that is fundamental for Mukerji and Tallon (2001) result. They show that some assets are not traded, leading to incomplete markets.

Rigotti and Shannon (2007) considered the correspondence from endowments to equilibria allocations and prices under variational preferences (as introduced by Maccheroni et al. 2006). They say that an economy with endowment e is determinate if the number of equilibria is finite and this correspondence is continuous. Then, they show that the set of endowments that lead to determinate economies is of full measure. This result highlights the importance about the assumptions about the endowments. If endowments are led completely undisciplined, their result implies that ambiguity aversion may have no discernible implication. However, when we focus our attention on some special class of endowments, then ambiguity aversion may lead to interesting phenomena, which are worth investigating.

Maybe the paper most closely related to ours is Strzalecki and Werner (2011), although our models are not completely comparable. Instead of focusing on unambiguous allocations, they consider conditional beliefs with respect to the partition induced by the aggregate endowment. They show that if there exists at least a common consistent conditional belief, then every interior Pareto optimal allocation is (essentially) measurable with respect to that partition (see details in their paper).

2 Preliminaries

This section establishes the notation, definitions, and a preliminary result (an additive theorem for CEU and MEU preferences) that will be useful later on.

2.1 Mathematical notation

Let Ω be the set of states of the world and let 2^{Ω} be the set of subsets of Ω. Ω is *not* assumed to be finite, unless explicitly stated otherwise. For $A \subset \Omega$, we denote $\Omega \backslash A$ by A^c. Let Σ be a sigma-algebra over Ω. Let Δ be the set of finitely additive probability measures $\pi : \Sigma \to [0, 1]$ over Ω.

[4] Stecher et al. (2011) discuss trade and ambiguity in a context of auctions. Condie and Ganguli (2010) and Ozsoylev and Werner (2009) study rational expectations equilibrium with ambiguous information.

[5] In some sense, this literature tries to accomplish something that was already advocated for by Machina (1989), who observed that "non-expected utility models of individual decision making can be used *to conduct analyses of standard economic decisions under uncertainty*, such as insurance, gambling, investment, or search." He urged for this kind of research on the following grounds: "unless and until economists are able to use these new models as engines of inquiry into basic economic questions, they—and the laboratory evidence that has inspired them—will remain on a shelf" (p. 1623).

The set of simple (finite-valued) acts $f : \Omega \to \mathbb{R}$ measurable with respect to Σ will be denoted by $B_0(\Sigma)$. Following the standard practice, we treat $x \in \mathbb{R}$ as an element of $B_0(\Sigma)$ simply by associating it with an act $x \in B_0(\Sigma)$ that is constant and has value x for each $\omega \in \Omega$.

We denote by $B(\Sigma)$ the closure of $B_0(\Sigma)$ in the sup-norm and by $B_\infty(\Sigma)$ the set of bounded Σ-measurable real-valued functions on Ω, endowed with the sup-norm $\|\cdot\|$ topology. $ba(\Sigma)$ denotes the space of bounded finitely additive measures on (Ω, Σ), which is (isometrically isometric to) the norm dual of $B_\infty(\Sigma)$. We will assume that $ba(\Sigma)$ is endowed with the weak* topology and will denote by Δ_Σ (or just Δ) the subset of finitely additive probabilities. We will write $B^+(\Sigma)$, $B_0^+(\Sigma)$ and $B_\infty^+(\Sigma)$ to refer to the subsets of $B(\Sigma)$, $B_0(\Sigma)$ and $B_\infty(\Sigma)$, respectively, which include only non-negative functions.

If $A \in \Sigma$, we denote by $1_A \in B_0(\Sigma)$ the indicator function of A, that is, $1_A(\omega) = 1$ if $\omega \in A$, and 0 otherwise. To simplify notation, we will denote the event $\{\omega \in \Omega : h(\omega) \geqslant t\}$ by $\{h \geqslant t\}$ and avoid the braces whenever this causes no confusion.

2.2 Notation and assumptions for economies

Let $N = \{1, 2, \ldots, n\}$ be the set of decision makers or consumers. Each consumer $i \in N$ has an endowment e_i, which is a function $e_i : \Omega \to \mathbb{R}_+$.[6] An economy $\mathcal{E}$ is a profile $(\succcurlyeq_i, e_i)_{i \in N}$ of preferences and endowments for each $i \in N$. For later use, let us denote by Σ_i the set of unambiguous events for consumer i.

Given the economy $\mathcal{E} = (\succcurlyeq_i, e_i)_{i \in N}$, an allocation $f = (f_1, \ldots, f_n) \in \left(B_\infty^+(\Sigma)\right)^n$ is feasible if $\sum_{i \in N} f_i \leqslant \sum_{i \in N} e_i$. A feasible trade is a vector $z = (z_i)_{i \in N}$ such that $\sum_{i \in N} z_i \leqslant 0$. *Unless otherwise stated, all allocations and trades in this paper will be assumed feasible and all allocations will be assumed non-negative.*

An allocation $f = (f_i)_{i \in N}$ is Pareto optimal if there is no feasible allocation $g = (g_i)_{i \in N}$ such that $g_i \succcurlyeq_i f_i$ for all i and $g_j \succ_j f_j$ for some j. The set of Pareto improving traded assets is the set $T(\mathcal{E})$ formed by the profiles $(z_i)_{i \in N} \neq 0$ that are Pareto improving (that is, $e_i + z_i \succcurlyeq_i e_i$ for all $i \in N$ and $\exists j \in N$ such that $e_j + z_j \succ_j e_j$). In other words, $T(\mathcal{E})$ is the set of trade profiles z that makes $e + z$ a Pareto improvement upon e. It is also useful to define the set of Pareto optimal trades $O(\mathcal{E}) \subset T(\mathcal{E})$, formed by those trade profiles z that make $e + z$ a Pareto optimal allocation. Similarly, let $O^u(\mathcal{E}) \subset T(\mathcal{E})$, denote the set of trade profiles z that make $e + z$ a Pareto optimal allocation, and for each $i \in N$, $e_i + z_i$ is unambiguous (see definition below).

2.3 Unambiguous events and acts

Epstein and Zhang (2001), Ghirardato and Marinacci (2002) and Zhang (2002) propose different definitions of unambiguous events for general preferences.[7] All of our

[6] Our results can be generalized for $l \geqslant 1$ goods. We work with $l = 1$ good for simplicity.

[7] See Amarante and Filiz (2007) and, more recently, Klibanoff et al. (2011) for a discussion and comparison of these definitions.

results that are stated for general preferences (e.g., those in Sect. 3) hold for any of these definitions. However, it will be useful to fix a simple definition in the MEU and the CEU case. (See the appendix for a definition of these preferences.) Dealing with CEU preferences, Nehring (1999) considers four alternative definitions of unambiguous events. In the CEU case, we will adopt the last of his definitions, since it has "nice" properties, as we explain below. We define unambiguous events as follows:

Definition 2.1 An event $A \in \Sigma$ is unambiguous if:

- In the MEU-paradigm: $\pi(A) = \pi'(A)$ for any $\pi, \pi' \in \mathcal{P}$.
- In the CEU paradigm: $\upsilon(B) = \upsilon(B \cap A) + \upsilon(B \cap A^c)$, $\forall B \in \Sigma$.

The set of unambiguous events $A \in \Sigma$ is denoted by Σ_u.[8]

See Nehring (1999) for a justification of the set of unambiguous events in the CEU paradigm. In the MEU paradigm, the definition seems the most natural one. The definition of unambiguous acts follows naturally:

Definition 2.2 An act $f \in B_\infty(\Sigma)$ is unambiguous if f is Σ_u-measurable in the following sense: $\{\omega \in \Omega : f(\omega) \geqslant t\} \in \Sigma_u$ holds for almost every t, with respect to the Lebesgue measure λ of $\mathbb{R}$.[9]

In Appendix A.2, we establish some facts about unambiguous events that may be of technical interest.

2.4 An additivity theorem for CEU and MEU preferences

The objective of this section is to establish an additivity theorem for CEU and MEU preferences, as introduced by Schmeidler (1986) and Gilboa and Schmeidler (1989), respectively. In the CEU pardagim, the additive property played a central role from the beginning. Indeed, Schmeidler (1986) proved the equivalence between the Choquet integral representation and the additivity property with respect to a special set of acts: the set of comonotonic acts.[10] Since then, additivity of the Choquet integral has been essentially restricted to the case of comonotonic acts. In fact, Ghirardato et al. (1998) established also a connection between comonotonicity and the additivity of a functional representing MEU preferences.[11] Another result related to additivity is presented by Ghirardato (1997), who defined the property of slice-comonotonicity, which is satisfied in the following simple setting: $\Omega = X \times Y$ and the functions $f, g \in B_0(\Sigma)$ can be described by $f(x, y) = x$ and $g(x, y) = y$. If υ satisfies a property that he calls Fubini property, and υ_x and υ_y are the marginals of υ with respect to X and Y, respectively, then $\int (f + g)\, d\upsilon = \int f d\upsilon_x + \int g d\upsilon_y$. This result was used by

[8] When we consider a set of DMs $N = \{1, \ldots, n\}$, the unambiguous set of events for individual $i \in N$ is denoted Σ_i.

[9] When dealing with economies, we say that an allocation $x = (x_1, \ldots, x_n)$ is unambiguous if x_i is unambiguous for each $i = 1, \ldots, n$.

[10] Two acts $g, h \in B(\Sigma)$ are comonotonic if $\left(g(\omega) - g(\omega')\right)\left(h(\omega) - h(\omega')\right) \geqslant 0$, $\forall \omega, \omega' \in \Omega$.

[11] They characterize additivity of pair of functions through comonotonicity and affine transformations.

Mukerji and Tallon (2001) to prove a result on the incompleteness of financial markets with ambiguity aversion. This suggests that additivity is not only a mathematical curiosity but an important property for studying economic aspects of ambiguity.

Theorem 2.1 (Additivity Theorem) *Consider CEU or MEU maximizers. Let* $f \in B_\infty(\Sigma)$ *be unambiguous.*[12] *Then, for all* $g \in B_\infty(\Sigma)$,

$$I(f+g) = I(f) + I(g).$$

The proof of Theorem 2.1, given in the appendix, depends crucially on the definition of unambiguous events and acts (Definitions 2.1 and 2.2). That is, other definitions of unambiguous events will not yield the above result. For the case of CEU decision makers with a *convex* capacity, there is a short proof, as follows.

Proof of Theorem 2.1 for the case of convex υ. From the study by Schmeidler (1986, Proposition 3, p. 260), we know that $I_\upsilon(f+g) \geqslant I_\upsilon(f) + I_\upsilon(g)$ and $I_\upsilon(g) = \min_{\pi \in core(\upsilon,\Sigma)} I_\pi(g)$. Let $\pi \in core(\upsilon, \Sigma)$ be such that $I_\upsilon(g) = I_\pi(g)$. We intend to show that $I_\upsilon(f) = I_\pi(f)$. Recall from (3) that $I_\upsilon(f) = \int_{-\infty}^{0} [\upsilon(h \geqslant t) - 1]\, dt + \int_0^\infty \upsilon(h \geqslant t)\, dt$. Since f is unambiguous, $\{f \geqslant t\} \in \Sigma_u$, for almost all $t \in \mathbb{R}$. Taking $A = \{f \geqslant t\}$ and $B = \Omega$ in item (ii) of Definition 2.1, we obtain $\upsilon(h \geqslant t) + \upsilon(h < t) = 1$ for almost all t. Since $\pi \in core(\upsilon, \Sigma)$, this implies $\pi(h \geqslant t) = \upsilon(h \geqslant t)$, for almost all $t \in \mathbb{R}$ and, consequently, $I_\upsilon(f) = I_\pi(f)$. Thus, $I_\upsilon(f) + I_\upsilon(g) = I_\pi(f) + I_\pi(g) \geqslant \min_{p \in core(\upsilon,\Sigma)} I_p(f+g) = I_\upsilon(f+g)$. This completes the
proof. □

Unfortunately, the converse of the Theorem 2.1 does not hold in general for the MEU case, as the following example shows.

Example 2.2 Suppose that $\Omega = \{a, b, c, d\}$ and $\mathcal{P} = co\{\pi^1, \pi^2\}$ defined by $\pi^1 = (\pi(\{a\}), \pi(\{b\}), \pi(\{c\}), \pi(\{d\})) = (\frac{1}{2}, 0, 0, \frac{1}{2})$ and $\pi^2 = (0, \frac{1}{2}, \frac{1}{2}, 0)$. It is easy to see that $\mathcal{P} = \{\frac{\alpha}{2}, \frac{1-\alpha}{2}, \frac{1-\alpha}{2}, \frac{\alpha}{2} : \alpha \in [0,1]\}$ and $\Sigma_u = \{\{a, b\}, \{c, d\}, \{a, c\}, \{b, d\}\}$. Consider the act f defined by $(f(a), f(b), f(c), f(d)) = (4, 3, 2, 1)$. This is obviously not Σ_u-measurable, but $\forall \pi \in \mathcal{P}$,

$$I_\pi(f) = 4 \cdot \frac{\alpha}{2} + 3 \cdot \frac{1-\alpha}{2} + 2 \cdot \frac{1-\alpha}{2} + 1 \cdot \frac{\alpha}{2} = \frac{5}{2}.$$

Thus, $I_\pi(f) = I_{\mathcal{P}}(f)\ \forall \pi \in \mathcal{P}$. Following the proof of Theorem 2.1 in the appendix, it is not difficult to see that this is sufficient for additivity.

It is, however, remarkable that for the CEU paradigm in the case of ambiguity aversion, i.e., for υ convex, the converse of Theorem 2.1 holds true. Such a result, Theorem 2.3 below, reinforces the meaningfulness of what we called unambiguous acts.[13]

12 See Sect. 2.3 for a definition of unambiguous acts in the CEU and MEU frameworks.

13 A similar result can be found in the study by Bassanezi and Greco (1984)'s Proposition 4.5 for non-negative functions under more complex measurability conditions. Our proof is more direct than theirs.

Theorem 2.3 *Let $f \in B^{+}_{\infty}(\Sigma)$. For CEU ambiguity averse decision makers, i.e., when υ is convex, the following assertions are equivalent:*

(i) $I(f+g) = I(f) + I(g)$, *for all* $g \in B^{+}_{\infty}(\Sigma)$;[14]
(ii) $I(f) + I(-f) = 0$;
(iii) f *is unambiguous.*

3 More ambiguity aversion implies less trade

The purpose of this section is to formalize and prove the following claim: "more ambiguity aversion implies less trade". This result is valid for any rational (complete and transitive) preference such that the endowment of each consumer is unambiguous for that consumer. Before describing the results, we need to clarify what we mean by "more ambiguity aversion" and "less trade."

3.1 Definition of "more ambiguity aversion"

We consider two economies $\mathcal{E}^k = \left(\succcurlyeq^k_i, e_i\right)_{i \in N}$ where $k = 1, 2$ represent different situations of ambiguity aversion. The difference between the two situations might be due to the individuals' personal perception of the economic environment or to the evolution of the objective information about the environment.

For this, we need to formalize the notion of "more ambiguity aversion". Different formalizations were proposed by Epstein (1999) and by Ghirardato and Marinacci (2002), although their main idea agrees with the following:[15]

Definition 3.1 The preference $\succcurlyeq^2$ is *more uncertainty averse than* $\succcurlyeq^1$ if for every unambiguous act h and every $f \in B_{\infty}(\Sigma)$,[16]

$$h \succcurlyeq^1 \left(\succ^1\right) f \Rightarrow h \succcurlyeq^2 \left(\succ^2\right) f. \tag{1}$$

It should be understood in (1) that h is unambiguous for both $\succcurlyeq^1$ and $\succcurlyeq^2$.

That is, if $\succcurlyeq^1$ prefers an unambiguous act to f, then $\succcurlyeq^2$ also prefers. Ghirardato and Marinacci (2002) show that in the case of MEU, this corresponds to having the same utility function (up to affine transformations) and having a bigger set of priors, that is, $\mathcal{P}^2 \supset \mathcal{P}^1$. At any rate, what we need is that (1) holds.

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[14] It is worth noticing the close relationship between $f \in B_{\infty}(\Sigma)$ satisfying (i) of Theorem 2.3, and what Ghirardato et al. (2004) called a crisp act, interpreting that such acts cannot be used for hedging other acts.

[15] Ghirardato and Marinacci (2002) make a distinction between "more uncertainty averse than" and "more ambiguity averse than", which for simplicity we do not. Their notion of "more ambiguity averse than" builds upon the less restrictive "more uncertainty averse than". The difference between the two notions is, loosely speaking, that "uncertainty" includes both "ambiguity" and "risk". To talk about "more ambiguity averse than", one has to take out the "risk" part.

[16] Any definition of unambiguous acts will work for the results in this section. See, however, Sect. 2.3 for a specific definition of unambiguous acts for MEU and CEU preferences.

The above definition is inspired by the standard notion of "more risk averse than". Recall that a preference $\succcurlyeq^2$ is more risk averse than $\succcurlyeq^1$ if for every risk-free act h and every $f \in B_\infty(\Sigma)$, $h \succcurlyeq^1 \left(\succ^1\right) f \Rightarrow h \succcurlyeq^2 \left(\succ^2\right) f$. Thus, this definition is just an adaptation of this to the ambiguity aversion case. Note also that (1) requires consideration of a class of unambiguous acts, and this is taken as primitive by Epstein, while endogenously defined by Ghirardato and Marinacci.

Definition 3.2 We say that an economy $\mathcal{E}^2 = \left(\succcurlyeq_i^2, e_i^2\right)_{i\in N}$ *has more ambiguity aversion than economy* $\mathcal{E}^1 = \left(\succcurlyeq_i^1, e_i^1\right)_{i\in N}$ if:

- $\succcurlyeq_i^2$ is more uncertainty averse than $\succcurlyeq_i^1$;
- $e_i^2 = e_i^1$ is unambiguous in both situations, for all $i \in N$.

3.2 More ambiguity aversion implies less trade opportunities

For stating the result that "more ambiguity aversion implies less trade", we need also to formalize the notion of "less trade". There are at least two alternative definitions for this: having a smaller set of Pareto improving trades $T(\mathcal{E})$ (see the formal definition of this set in Sect. 2.2) or having a smaller set of Pareto optimal trades. We begin with the first notion and discuss the second notion afterward. The following theorem proves that more ambiguity aversion leads to *less* trade in the sense of having a smaller set of Pareto improving trade opportunities.

Theorem 3.1 *If $\mathcal{E}^m$ has more ambiguity aversion than $\mathcal{E}^l$, then the set of Pareto improving trades in economy $\mathcal{E}^l$ is bigger than the same set in economy $\mathcal{E}^m$, that is, $T(\mathcal{E}^m) \subset T\left(\mathcal{E}^l\right)$.*

The following example shows that the inclusion $T(\mathcal{E}^m) \subset T(\mathcal{E}^l)$ can be strict.

Example 3.2 Let $\Omega = \{a, b, c\}$ and $\Sigma = 2^\Omega$. A probability over Ω is defined by a vector $\pi = (\pi_a, \pi_b, \pi_c) \in [0, 1]^3$, with the natural meaning that $\pi_i = \pi(\{i\})$, for $i = a, b, c$. There are two parametrized economies: more, m, and less, l, ambiguous averse, with two MEU-DMs ($N = \{1, 2\}$) such that:

- $e_1 = (7, 1, 1)$, $u_1(t) = \sqrt{t}$ and $\mathcal{P}_1^l = \{p^0\}$; $\mathcal{P}_1^m = \{p^0, q^\varepsilon\}$,
- $e_2 = (1, 7, 9)$, $u_2(t) = \sqrt{t}$ and $\mathcal{P}_2^l = \mathcal{P}_2^m = \{p^\alpha\}$, for

where $\varepsilon \in [0, \frac{3}{4}]$ and $\alpha \in [0, \frac{1}{4}]$ are parameters for:

$$p^\alpha = \left(\frac{1}{4} - \alpha, \frac{1}{2} + 2\alpha, \frac{1}{4} - \alpha\right);$$
$$q^\varepsilon = \left(\frac{1}{4}, \frac{3}{4} - \varepsilon, \varepsilon\right).$$

Thus, more than one example, this presents a set of (parametrized) examples. (The reason for offering a set of examples will become clear momentarily.) In the appendix, we observe that this example satisfies the assumptions of Theorem 3.1. There, we also show that $z = (z_1, z_2)$, where $z_1 = (-3, 0, 3)$ and $z_2 = (3, 0, -3)$, is a Pareto improving trade for economy $\mathcal{E}^l$ for sufficiently small $\alpha \geqslant 0$, that is, $z \in T(\mathcal{E}^l)$. However, we also show that $z \notin T(\mathcal{E}^m)$, for sufficiently small $\varepsilon \geqslant 0$, that is, $T(\mathcal{E}^l) \not\subset T(\mathcal{E}^m)$. Actually, varying the parameter α, we see that $T(\mathcal{E}^l) \not\subset T(\mathcal{E}^m)$ irrespective of whether a common prior exists ($\cap_{i\in N}\mathcal{P}_i^k \neq \varnothing \Leftrightarrow \alpha = 0$) or not ($\cap_{i\in N}\mathcal{P}_i^k = \varnothing \Leftrightarrow \alpha > 0$), for $k = m, l$.

One may argue that the previous example is not totally convincing since the considered possible trade z for $\mathcal{E}^l$ will not be observed since it can be Pareto improved. Example 3.3 below aims to show that even some individually rational Pareto optimal allocations of $\mathcal{E}^l$ may not correspond to a possible trade for economy $\mathcal{E}^m$.

Example 3.3 Consider the above economy with parameters $\alpha = \varepsilon = 0$. Notice that in economy l, the two decision makers are expected-utility maximizers with the same prior p^0 and the same utility function, which exhibits a constant coefficient of relative risk aversion $r = \frac{1}{2} = -t\frac{u''(t)}{u'(t)}$. In such a case, it is known that Pareto optimal allocations f_1, f_2 satisfy $f_1 = \lambda e$, $f_2 = (1-\lambda)e$, where $\lambda \in (0, 1)$ and $e = (8, 8, 10)$ is the aggregate endowment. Simple computations show that (f_1, f_2) is strictly individually rational if $0.49 \leqslant \sqrt{\lambda} \leqslant 0.6$. We show in the appendix that if $\sqrt{\lambda} = 0.49$ then $e_1 \succ_1^m f_1 = \lambda e$. This implies that some of the individually rational Pareto optimal for $\mathcal{E}^l$ are not Pareto improving trades for $\mathcal{E}^m$.

As we observed before, it is possible to consider, however, another notion of "less trade", which considers only the trades that are Pareto optimal, $O\,(\mathcal{E})$. We do not have a result about this case, but we are able to provide a similar result to the previous one if we restrict our attention to unambiguous Pareto optimal allocations. For this, recall from Sect. 2.2 that $O^u\,(\mathcal{E}) \subset T\,(\mathcal{E})$ denotes the set of trade profiles z such that $e + z$ is a Pareto optimal allocation and, for each i, $e_i + z_i$ is unambiguous. Then, we have:

Theorem 3.4 *If $\mathcal{E}^m$ has more ambiguity aversion than $\mathcal{E}^l$, then $O^u\,(\mathcal{E}^m) \subset O^u\left(\mathcal{E}^l\right)$.*

It is important to observe that we *did not* restrict the preferences in Theorems 3.1 and 3.4 to be CEU or MEU preferences. The proof uses only the rationality of preferences $\succcurlyeq_i$ for all $i \in N$. Thus, as long as one has a notion of unambiguous events that is consistent with (1) and assumes that endowments are unambiguous, these results hold.

Finally, we observe that it is possible to prove a converse of the Theorem 3.1, as follows.

Theorem 3.5 *Let $\left(\succcurlyeq_i^1\right)_{i\in N}$ and $\left(\succcurlyeq_i^2\right)_{i\in N}$ be two profiles of preferences. If*

$$T((\succcurlyeq_i^2, e_i)_{i\in N}) \subset T\left(\left(\succcurlyeq_i^1, e_i\right)_{i\in N}\right)$$

for all unambiguous allocations $(e_i)_{i\in N}$, then $\succcurlyeq_i^2$ is more ambiguity averse than $\succcurlyeq_i^1$, for all i.

Theorem 3.1 shows that ambiguity aversion may lead to a reduction in trade, but it does not describe how much. An interesting case occurs when there is no trade at all. This situation corresponds to the case where the initial endowment is itself Pareto optimal. Thus, a characterization of no trade situations is equivalent to a characterization of Pareto optimal allocations. This is the subject of Sect. 4 in the case of betting and Sect. 5 in the case of insurance.

4 Betting and ambiguity aversion

In this section, we will characterize *betting* situations when two persons may not trade because of ambiguity. Dow and Werlang (1992) were the first to consider the effect of ambiguity in a betting situation, for a single decision maker. Their result was later generalized by Abouda and Chateauneuf (2002) and Asano (2006). Chateauneuf et al. (2000) go a step further, by considering an economy where there are multiple CEU-DMs agents. Also, Billot et al. (2000) consider an economy with MEU-DMs.

We call "betting" a situation where the outcome does not affect the individual's endowment (so there is no opportunity for insurance) or the stakes are sufficiently small so that the individual is risk neutral. Both cases are summarized by the following assumption:

Assumption 4.1 (*Betting*) The consumers are CEU or MEU maximizers and that $I_i\left(u_i\left(e_i\right)\right) = u_i\left(I_i\left(e_i\right)\right)$.

Indeed, Assumption 4.1 holds in two important cases: (1) when the endowments are constant; (2) the endowments are unambiguous and the individuals are risk neutral (utilities are linear). The betting assumption leads us to the following:

Theorem 4.2 *Consider an economy $\mathcal{E} = (\succsim_i, e_i)_{i\in N}$ such that Assumption 4.1 holds. Moreover, assume:*

(i) *for the MEU-paradigm, $\cap_{i\in N}\mathcal{P}_i \neq \varnothing$;*
(ii) *for the CEU paradigm, there are two consumers and $\upsilon_1 \leqslant \overline{\upsilon}_2$.*

Then $T\left(\mathcal{E}\right) = \varnothing$.

Notice that this result is related to one implication of Theorem 1 of Billot et al. (2000), which shows, under the MEU setting, that the existence of a common prior ($\cap_{i\in N}\mathcal{P}_i \neq \varnothing$) implies that any full insurance allocation is Pareto optimal. For this, they assumed that the aggregate endowment is constant. Theorem 4.2 allows for the case where the aggregate endowment is not constant. If the allocation is unambiguous, then it will be Pareto optimal if the individuals are risk neutral. Note that the assumption that individuals are risk neutral may be reasonable in some situations, especially when the stakes involved are small, which is sometimes the case in *betting* situations.

5 Insurance and ambiguity aversion

This section offers characterizations of Pareto optimal allocations under different conditions. Since Pareto optimal allocations are, by definition, exactly those where trade cannot benefit consumers, one can understand most results of this section as no-trade theorems. Section 5.2 presents a general characterization that will be useful for the subsequent results. The main result regarding non-aggregate endowments is presented in Sect. 5.3. This offers a new version of the main result of Billot et al. (2000) and Rigotti et al. (2008) for the case of MEU-DMs and non-constant aggregate endowments. Interestingly, one of the implications in the main result of these papers is not part of our Theorem 5.2. This difference is an essential characteristic of the more general setting that we consider, as we explain in Sect. 5.4. Section 5.5 concludes with a result linking the set of unambiguous allocations in the MEU case with the set of Pareto optimal allocations in the expected utility (EU) case with a common prior. In this section, we will assume that the economy is "well-behaved", as we describe next.

5.1 Well-behaved economies

Recall that Σ denotes a σ-algebra of subsets of Ω and that $\Sigma_i \subset \Sigma$ represents the class of all unambiguous events for individual i. Let Σ^{uu} represents the unanimously unambiguous σ-algebra, that is, $\Sigma^{uu} \equiv \vee_{i \in N} \Sigma_i$ is the finest common coarsening of the Σ_i. In particular, if $A \in \Sigma^{uu}$, then $A \in \Sigma_i$ for all $i \in N$ and if f is Σ^{uu}-measurable, then it is Σ_i-measurable for all $i \in N$. Let e_N denotes the total endowment of the economy, that is, $e_N(\omega) \equiv \sum_{i \in N} e_i(\omega)$, $\forall \omega \in \Omega$. We say that e_N is *unanimously unambiguous* if e_N is Σ^{uu}-measurable. We say that an allocation $x = (x_i)_{i \in N}$ is *unanimously unambiguous* if x_i is unanimously unambiguous for each $i \in N$.

Given a probability $P \in \Delta(\Omega)$ and a σ-field $\mathcal{F}$, a conditional probability $P(\cdot|\mathcal{F})$ is a function $Q : \Omega \times \Sigma \to \mathbb{R}_+$ satisfying:

(a) $\omega \mapsto Q(\omega, A)$ is $\mathcal{F}$-measurable, for any $A \in \Sigma$;
(b) for every $B \in \mathcal{F}$ and $A \in \Sigma$,

$$\int_B Q(\omega, A)\, dP(\omega) = P(A \cap B).$$

In this case, we define the P-conditional probability of $A \in \Sigma$ at ω, denoted $P(A|\mathcal{F})_\omega$ as $Q(\omega, A)$. Following the usual practice, ω will be omitted.

Finally, we denote the set $\{\pi(\cdot|\Sigma^{uu}) : \pi \in \mathcal{P}_i\}$ by $\mathcal{P}_i^{uu}$. This set corresponds to the set of all conditional probabilities with respect to the unanimously unambiguous σ-algebra. Thus, the condition $\cap_{i \in N} \mathcal{P}_i^{uu} \neq \emptyset$ below means that there is at least one conditional probability that is common to all individuals.

Definition 5.1 (*Well-behaved economy*) We say that an economy $\mathcal{E} = (\succcurlyeq_i, e_i)_{i \in N}$ is well behaved if:

- each $i \in N$ is MEU-DM with a C^2 utility function $u_i : \mathbb{R}_+ \to \mathbb{R}$, satisfying $u_i' > 0$ and $u_i'' < 0$ and the $\lim_{x \to 0^+} u_i'(x) = +\infty$;[17]
- for each $i \in N$, e_i is norm-interior, that is, $\inf_{\omega \in \Omega} e_i(\omega) > 0$;
- for every $i \in N$ and $\pi \in \mathcal{P}_i$, π is countably additive;

[17] Strict concavity of u_i is not necessary for most of the results below.

- for every $i, j \in N$, and every $\pi_i \in \mathcal{P}_i$ and $\pi_j \in \mathcal{P}_j$, π_i and π_j are mutually absolutely continuous.

5.2 Characterization of Pareto optimal allocations

We now consider n MEU-DM, where u_i is of class C^1, and $u_i'(\cdot) > 0$ for all i. We also assume that the endowments e_i belong to $B_\infty^+(\Sigma)$. The corresponding economy is $\mathcal{E} = (\succcurlyeq_i, e_i)_{i \in N}$.

If $\pi \in \Delta_\Sigma$ and $f \in B_\infty^+(\Sigma)$, let $f\pi$ denote the (finitely additive) measure given by $(f\pi)(A) = \int_A f \mathrm{d}\pi$, for all $A \in \Sigma$. For $f \in B_\infty^+(\Sigma)$, define

$$\mathcal{P}_i(f) \equiv \left\{ \frac{u_i'(f(\cdot))}{\int_\Omega u_i'(f)\,\mathrm{d}\pi}\pi : \pi \in \arg\min_{p \in \mathcal{P}_i} \int_\Omega u_i(f(\cdot))\,\mathrm{d}p \right\}. \tag{2}$$

The theorem below characterizes Pareto optimal allocations through the sets defined in (2). This result parallels, but for an infinite state space, Rigotti and Shannon (2005)'s Theorem 3, derived for Bewley (1986, 2002)'s model. It extends the main result of Billot et al. (2000) to the situation of aggregate uncertainty, allowing for the MEU-model to characterize full insurance Pareto optimal allocation under weaker conditions. We proved this result in the first version of this paper, before we became aware of a more general result, by Rigotti et al. (2008) (see their Proposition 7). According to these authors, their main contribution is to recognize that this theorem and other results that they obtain are easy consequences of the Second Welfare Theorem.[18] However, another contribution of this theorem is to recognize that one should use a modified set of priors, which, in our setup, is the set defined by (2).

Theorem 5.1 *Consider a well-behaved economy $\mathcal{E} = (\succcurlyeq_i, e_i)_{i \in N}$ of MEU-DM and let $x = (x_i)_{i \in N}$ be norm-interior. Then, the following are equivalent:*

(i) $(x_i)_{i \in N}$ *is Pareto optimal;*
(ii) $\cap_{i \in N} \mathcal{P}_i(x_i) \neq \varnothing$.

This theorem will be instrumental for our next results below.[19]

[18] Our original proof (omitted in this version) used a separation of convex sets as an essential step. Rigotti et al. (2008) were able to avoid separation arguments, since the proof of the Second Welfare Theorem is already based in such kind of arguments.

[19] Despite its generality and usefulness, Theorem 5.1 is not completely satisfactory from a characterization point of view, because the probabilities in the sets $\mathcal{P}_i(x_i)$ are distorted by the marginal utilities of consumers. Therefore, those probabilities do not depend exclusively on the agents' beliefs.

5.3 Unanimously unambiguous aggregate endowments

The following holds for well-behaved economies:

Theorem 5.2 *Consider a well-behaved economy $\mathcal{E} = (\succcurlyeq_i, e_i)_{i \in N}$ and assume that the aggregate endowment $e_N \equiv \sum_{i \in N} e_i$ is unanimously unambiguous. The following assertions are equivalent:*

(i) *There exists a norm-interior unanimously unambiguous Pareto optimal allocation.*
(ii) *Any Pareto optimal allocation is an unanimously unambiguous allocation.*
(iii) $\cap_{i \in N} \mathcal{P}_i^{uu} \neq \emptyset$.

It is worth comparing the previous result with the main result of Billot et al. (2000) and Rigotti et al. (2008):

Theorem (Billot et al. 2000; Rigotti et al. 2008) Assume that the economy is well behaved and that the aggregate endowment is constant. Then, the following statements are equivalent:

(i) There exists a norm-interior full insurance Pareto optimal allocation.
(ii) Any Pareto optimal allocation is a full insurance allocation.
(iii) Every full insurance allocation is Pareto optimal.
(iv) $\cap_{i \in N} \mathcal{P}_i \neq \emptyset$.

The main differences are the following: (1) they assume that the aggregate endowment is constant, while we assume that it is unanimously unambiguous; (2) their common prior condition is in terms of ex-ante beliefs, while ours is in terms of conditional beliefs; and (3) we do not include a parallel statement for "every unanimously unambiguous allocation is Pareto optimal". The reason for the third difference will be discussed in the next subsection.

5.4 Pareto optimality for every unambiguous allocation

Let us explain why we cannot include a statement like "every unanimously unambiguous allocation is Pareto optimal" in Theorem 5.2. Recall that $e_N(\omega) = \sum_{i \in N} e_i(\omega)$ and let $\mathcal{B}$ denote $\sigma(e_N)$, that is, the smallest σ-algebra with respect to which e_N is measurable. An allocation $x = (x_i)_{i \in N}$ is $\mathcal{B}$-unambiguous if each x_i is $\mathcal{B}$-measurable for every $i \in N$. Note that if e_N is unanimously unambiguous, the requirement that x is $\mathcal{B}$-unambiguous is stronger than to require that x is unanimously unambiguous. Also, it is not difficult to construct examples where the aggregate endowment e_N is unanimously unambiguous but the endowment $e = (e_i)_{i \in N}$ is not $\mathcal{B}$-unambiguous nor unanimously unambiguous.

Theorem 5.3 *Consider a well-behaved economy $\mathcal{E} = (\succcurlyeq_i, e_i)_{i \in N}$ and assume that the aggregate endowment e_N is unanimously unambiguous; Ω is finite and $n \geqslant 3$. Then, the following are equivalent:*

(*i*) *Every $\mathcal{B}$-unambiguous allocation is Pareto optimal.*[20]
(*ii*) $\cap_i \mathcal{P}_i \neq \emptyset$ *and* e_N *is constant* (*which implies* $\mathcal{B} = \{\emptyset, \Omega\}$).

In this case, we also have the existence of a norm-interior full insurance Pareto optimal allocation and that any Pareto optimal allocation is full insurance.

This theorem explains why the statement "every unanimously unambiguous allocation is Pareto optimal" cannot be among the equivalent statements of Theorem 5.2. Namely, that statement is stronger than any of the statements in Theorem 5.2. In fact, Theorem 5.3 shows that even a weaker statement like "every $\mathcal{B}$-unambiguous allocation is Pareto optimal" is sufficient to imply that the aggregate endowment is constant.

Finally, we state a result that is used in the proof of the above theorems and may be of interest by its own. It is related to Theorem 2 of Rothschild and Stiglitz (1970).

Proposition 5.4 *Let* $(\Omega, \mathcal{A}, P)$ *be a probabilistic space. Let* $\mathcal{B}$ *be a sub-*σ*-algebra of* $\mathcal{A}$*,* X *a bounded* $\mathcal{A}$*-measurable random variable which is not* $\mathcal{B}$*-measurable and* $Y \equiv E[X|\mathcal{B}]$. *Then, for any concave, increasing function* $u : \mathbb{R} \to \mathbb{R}$, $E[u(X)] \leqslant E[u(Y)]$. *If* u *is strictly concave, the inequality is strict.*

5.5 Pareto optimal allocations in MEU and EU

Now, we obtain a characterization of unambiguous Pareto optimal allocations with respect to Pareto optimal allocations in an economy with expected utility (EU) DMs. This is related to a result by Chateauneuf et al. (2000) for the CEU case. For this, let us introduce the following notation:

- O^u_{MEU} denotes the set of unambiguous Pareto optimal allocations of a MEU economy;
- A^u_{MEU} is the set of unambiguous allocations for the MEU economy;
- $O_{EU(\pi)}$ denotes the set of Pareto optimal allocations of the economy with expected utility (EU) DMs with common prior π.

Theorem 5.5 *Assume that* $\pi \in \cap_{i \in N} \mathcal{P}_i$. *Then,* $O^u_{MEU} = O_{EU(\pi)} \cap A^u_{MEU}$.

This result is useful because it establishes a clear connection between the set of Pareto optimal allocations in the economy with ambiguity aversion and each of the related economies with standard expected utility DMs.

6 Application: home bias

In this section, we show how our results could be used to explain phenomena in real-world economies. The ideas developed here are not completely new. They go back at

[20] If e_N is unanimously unambiguous, then every $\mathcal{B}$-unambiguous allocation is also unanimously unambiguous. This shows that, under the hypotheses of the Theorem, item (i) is a *weaker* statement than "every unanimously unambiguous allocation is Pareto optimal."

least to Bewley (1989). In fact, many authors have tried to use ambiguity aversion to explain the phenomena below, most notably Epstein and Miao (2003).[21]

Home bias has been observed in two related forms: "equity home bias," which was observed in the finance literature as the tendency to hold few foreign assets; and "consumption home bias", reported in the international trade literature for the fact that there is less trade between countries than reasonable transportation costs would be able to explain.[22] Our results can explain these phenomea, since the key feature of our no-trade result is well fitted to the international trade situation: the consumer knows the probability governing his own endowments, but has uncertainty regarding the endowments in the foreign country, because of lack of knowledge about that country. Our theory also has a testable prediction: if the ambiguity aversion diminishes (for instance, with better knowledge), then trade should increase.

Let us consider an economy with one good, two periods, and two countries. In the first period, there is trade of contingent contracts for the second period. In country H (home), the set of consumers is $N^H = \{1, \ldots, n^H\}$, and in country F (foreign), it is $N^F = \{n^H + 1, \ldots, n^H + n^F\}$. The set of all consumers, assumed to be MEU-DMs with strictly concave u_i, is $N = N^H \cup N^F$. The set of states of the world is Ω, assumed to be finite with m different states. We assume that for each country C, $C = H, F$, all consumers have the same set of prior $\mathcal{P}_C$ and that the set of unambiguous events for all consumers in country C is an algebra Σ_C.[23] We again assume that each consumer has unambiguous endowments, that is, a consumer i who lives in country C, for $C = H, F$, has endowment $e_i : \Omega \to \mathbb{R}$, which is measurable with respect to Σ_C.[24] Thus, endowments are unambiguous inside each country, but may be ambiguous across countries. This is very close to the standard assumption of expected utility maximizers and seems reasonable.

Now, we define Pareto optimal allocations for two settings: closed and open economies. In closed economies, the flow of goods between the countries is forbidden, and Pareto improvement is considered only if trade is restricted to be among individuals in the same country. In the open economy, goods are free to circulate between the countries.

Definition 6.1 An allocation $(x_i)_{i\in N}$ is closed-Pareto optimal if there is no vector $(z_i)_{i\in N}$ such that

(i) for all $i \in N$, $x_i + z_i \succcurlyeq_i x_i$ and there exists $j \in N$ such that $x_j + z_j \succ_j x_j$.
(ii) $\sum_{i\in N^C} z^i = 0$, for $C = H, F$.

[21] See also Alonso (2004).

[22] Trefler (1995) coined the term "missing trade" to depict the extent to which measured trade is still negligible compared with the prediction of pure theory. He relates the home bias effect to other puzzles in international trade.

[23] In fact, it is sufficient that the consumers in each country share at least a common prior. Also, as discussed in Sect. 2.3, the assumption that Σ_C is an algebra will be without loss of generality in the particular case of ambiguity averse CEU-DMs.

[24] It is sufficient to assume that only the aggregate endowment of each country is unambiguous to the consumers in that country.

Definition 6.2 An allocation $(x_i)_{i\in N}$ is open-Pareto optimal if it is Pareto optimal for the whole economy $\mathcal{E} = (\succsim_i, e_i)_{i\in N}$, that is , where it is valid (i) of the previous definition and:

(ii)' $\sum_{i\in N} z^i = 0$.

We first establish a useful result that may be of interest by itself.

Corollary 6.1 *If $(x_i)_{i\in N}$ is a closed-Pareto optimal allocation, then it is unambiguous for each consumer.*

Recall that $\mathcal{P}_H$ and $\mathcal{P}_F$ characterize the set of priors in the home and in the foreign countries, respectively. Let us denote by $\mathcal{P}_C^{uu}$ the set of conditional priors obtained from $\mathcal{P}_C$ by conditioning the prior to the set of unambiguous events Σ_C, for $C = H, F$. The following theorem establishes conditions for the non-existence of trade between countries in the open economy.

Corollary 6.2 *A closed-Pareto optimal allocation $(x_i)_{i\in N}$ is open-Pareto optimal if and only if $\mathcal{P}_H^{uu} \cap \mathcal{P}_F^{uu} \neq \varnothing$.*

The interpretation of these results is that if the individuals in different countries have sufficiently broad set of priors, they will not trade. In particular, if in each country, they believe that the individuals in the foreign country know more than themselves about that country; they will be willing to include the priors of these individuals in their set of possible priors. This will lead to no trade. It is also useful to observe that a much weaker condition is sufficient for both results above, since Theorem 5.2 requires only that individuals share a common conditional prior.

7 Conclusion

This paper presents results that do not depend on the standard assumptions in MEU and CEU paradigms, constant endowments, no aggregate uncertainty or comonotonicity of endowments, as most of the available papers assume. In doing this, we provide no-trade theorems that can be used to explain and understand many phenomena of the real world economy.

Some of the results of this paper are established only for CEU and MEU preferences. It is an open question whether these results hold for more general preferences; for instance, the MBC and MBA preferences characterized by Ghirardato and Siniscalchi (2010) and Cerreia-Vioglio et al. (2010).

A Appendix

A.1 CEU and MEU decision makers

Except for the results in Sect. 3, we assume that consumers have preferences following the Choquet expected utility (CEU) or the maximin expected utility (MEU) paradigms, as we define now.[25]

[25] For axiomatizations of CEU preferences, see Gilboa (1987), Schmeidler (1989) and Chew and Karni (1994). For axiomatizations of MEU preferences, Gilboa and Schmeidler (1989), Casadesus-Masanell et al. (2000) and Ghirardato et al. (2003).

In the CEU paradigm, there is a capacity $\upsilon : \Sigma \to [0,1]$, which is a set function satisfying $\upsilon(\emptyset) = 0$, $\upsilon(\Omega) = 1$ and monotonicity, i.e., for all $A, B \in \Sigma$, $A \subset B \Rightarrow \upsilon(A) \leqslant \upsilon(B)$. Although this is not necessary for most results, we will assume that all capacities in our paper are continuous; that is, for any increasing sequence of sets A_n, $A_n \uparrow \Omega$, we have $\upsilon(A_n) \uparrow 1$.

For a function $h \in B_\infty(\Sigma)$, the Choquet integral of h with respect to υ is denoted by $\int h \mathrm{d}\upsilon$ or $I_\upsilon(h)$ and defined as:

$$I_\upsilon(h) = \int h \mathrm{d}\upsilon \equiv \int_{-\infty}^{0} [\upsilon(h \geqslant t) - 1]\,\mathrm{d}t + \int_{0}^{\infty} \upsilon(h \geqslant t)\,\mathrm{d}t. \tag{3}$$

We assume that a CEU consumer or decision maker (DM) has a preference over $B_\infty(\Sigma)$ defined by a utility function $U : B_\infty(\Sigma) \to \mathbb{R}$ given by

$$U(f) = I_\upsilon(u \circ f) = \int u(f)\,\mathrm{d}\upsilon,$$

for some continuous function $u : \mathbb{R} \to \mathbb{R}$.

It is useful to introduce some standard definitions. We say that υ is convex if $\forall A, B \in \Sigma$, $\upsilon(A \cup B) + \upsilon(A \cap B) \geqslant \upsilon(A) + \upsilon(B)$. The core of υ is the set of probability measures defined over Σ which are never below υ, that is, the set:

$$core(\upsilon, \Sigma) = \{\pi \in \Delta(\Omega, \Sigma) : \pi(A) \geqslant \upsilon(A), \quad \forall A \in \Sigma\}.$$

If Σ is clear, we may write $core(\upsilon)$ instead of $core(\upsilon, \Sigma)$. The dual capacity of υ, denoted by $\overline{\upsilon}$, is defined as $\overline{\upsilon}(A) = 1 - \upsilon(A^c)$, $\forall A \in \Sigma$.

In the MEU paradigm, we assume that there is a weak* compact set $\mathcal{P}$ of probability measures over Ω. The MEU decision maker (MEU-DM) has a preference over $B_\infty(\Sigma)$ which can be represented by the utility function $U : B_\infty(\Sigma) \to \mathbb{R}$ defined by

$$U(f) = \min_{p \in \mathcal{P}} I_p(u \circ f) = \min_{p \in \mathcal{P}} \int u(f)\,\mathrm{d}p,$$

again for some continuous function $u : \mathbb{R} \to \mathbb{R}$. For convenience, if $h \in B_\infty(\Sigma)$, we will denote $\int hdp$ by $I_p(h)$ and $\min_{p \in \mathcal{P}} \int hdp$ by $I_{\mathcal{P}}(h)$. When we want to refer both to CEU-DMs and MEU-DMs, we will write just $I(h)$ instead of $I_\upsilon(h)$ and $I_{\mathcal{P}}(h)$. When we consider a set of consumers indexed by i, we will use subscripts in all the notation given above, that is, we will write $I_i(h)$, υ_i , $\mathcal{P}_i$, u_i and U_i.

It is well known that the CEU and MEU paradigms are identical in the particular case where $\mathcal{P} = core(\upsilon, \Sigma)$ and υ is convex. In the general case, we will maintain the following assumption for both paradigms:

Assumption 7.1 u is strictly increasing, weakly concave, and continuous.

Let $\uparrow_u$ and $\downarrow_u$ denote monotonic uniform convergence. The following well-known result will be useful.

Lemma 7.2 *Assume that* $\{b_n\}_{n\in\mathbb{N}}$ *is a sequence in* $B_\infty(\Sigma)$ *and* $b \in B_\infty(\Sigma)$. *If* $b_n \uparrow_u b$, *then,* $I(b_n) \uparrow_u I(b)$. *Similarly, if* $b_n \downarrow_u b$, *then* $I(b_n) \downarrow_u I(b)$.

Proof Omitted. □

A.2 Some results about unambiguous acts and events

As Zhang (2002) argues, the set of unambiguous events should be a λ-system. A set $\Lambda \subset \Sigma$ is a λ-system if the following holds:

(i) $\Omega \in \Lambda$;
(ii) $A \in \Lambda \Rightarrow A^c \in \Lambda$;
(iii) If $A_1, A_2, \ldots \in \Lambda$ are pairwise disjoint, then $\cup_{k\in\mathbb{N}} A_k \in \Lambda$.

For completeness, we observe the following:

Proposition 7.3 Σ_u *is a* λ *-system.*

Proof of Proposition 7.3. Items (i) and (ii) of the definition are trivial. Consider now $A, B \in \Sigma_u$, $A \cap B = \varnothing$ and let $C \in \Sigma$. In the CEU paradigm,

$$\begin{aligned}
\upsilon(C) &= \upsilon(C\cap A) + \upsilon\left(C\cap A^c\right)\\
&= \upsilon(C\cap A) + \upsilon\left(C\cap A^c\cap B\right) + \upsilon\left(C\cap A^c\cap B^c\right)\\
&= \upsilon(C\cap(A\cup B)\cap A) + \upsilon\left(C\cap(A\cup B)\cap A^c\right) + \upsilon\left(C\cap\left(A^c\cap B^c\right)\right)\\
&= \upsilon(C\cap(A\cup B)) + \upsilon\left(C\cap(A\cup B)^c\right).
\end{aligned}$$

Thus, $A\cup B \in \Sigma_u$. In MEU-paradigm, $\pi(A\cup B) = \pi(A) + \pi(B)$. If $\pi(A) = \pi'(A)$ and $\pi(B) = \pi'(B)$ $\forall \pi, \pi' \in \mathcal{P}$, then

$$\pi(A\cup B) = \pi(A) + \pi(B) = \pi'(A) + \pi'(B) = \pi'(A\cup B),$$

$\forall \pi, \pi' \in \mathcal{P}$ and we conclude that $A\cup B \in \Sigma_u$. This shows that Σ_u is closed to finite unions. Property (iii) now follows from the Monotone Uniform Convergence Theorem (Lemma 7.2). □

Although Σ_u may fail to be an algebra in the MEU paradigm, we have the following:

Proposition 7.4 *In the CEU paradigm,* Σ_u *is an algebra.*

Proof of Proposition 7.4. Let $A_1, A_2 \in \Sigma_u$. Because of Proposition 7.3, it is sufficient to prove that $A_1 \cap A_2$ belongs to Σ_u. So let $B \in \Sigma$. From $A_1 \in \Sigma_u$, we obtain:

$$\upsilon(A_1\cap A_2\cap B) = \upsilon(A_2\cap B) - \upsilon\left(A_1^c\cap A_2\cap B\right). \tag{4}$$

From $A_2 \in \Sigma_u$, we obtain:

$$\upsilon\left(\left(A_1^c\cup A_2^c\right)\cap B\right) = \upsilon\left(A_2\cap\left(A_1^c\cup A_2^c\right)\cap B\right) + \upsilon\left(A_2^c\cap\left(A_1^c\cup A_2^c\right)\cap B\right),$$

which simplifies to:

$$\upsilon\left((A_1 \cap A_2)^c \cap B\right) = \upsilon\left(A_1^c \cap A_2 \cap B\right) + \upsilon\left(A_2^c \cap B\right). \tag{5}$$

Adding (4) and (5) , we have:

$$\upsilon\left((A_1 \cap A_2) \cap B\right) + \upsilon\left((A_1 \cap A_2)^c \cap B\right) = \upsilon\left(A_2 \cap B\right) + \upsilon\left(A_2^c \cap B\right) = \upsilon(B),$$

where the last equality holds because $A_2 \in \Sigma_u$. This concludes the proof. $\square$

The following Proposition 7.5 enlightens some properties of unambiguous events in the CEU paradigm with ambiguity aversion, that is, when the decision makers have a convex capacity υ. This result can be found in Nehring (1999), but the following proof is direct.

Proposition 7.5 *For CEU-DM with convex* υ*,*

$$\Sigma_u = \left\{A \in \Sigma : \upsilon(A) + \upsilon\left(A^c\right) = 1\right\}.$$

Proof $A \in \Sigma_u \Rightarrow \upsilon(A) + \upsilon(A^c) = 1$ follows by taking $B = \Omega$. We want to show that if $A \in \Sigma$ such that $\upsilon(A) + \upsilon(A^c) = 1$, then for any $B \in \Sigma$, $\upsilon(B) = \upsilon(B \cap A) + \upsilon(B \cap A^c)$. Since υ is convex and $A \cap B \subset B \subset A^c \cup B$, there exists $\pi \in core(\upsilon)$ such that $\pi(A \cap B) = \upsilon(A \cap B)$, $\pi(B) = \upsilon(B)$ and $\pi(A^c \cup B) = \upsilon(A^c \cup B)$. From this, $\upsilon(B) = \pi(B) = \pi(A \cap B) + \pi(A^c \cap B) = \upsilon(A \cap B) + \pi(A^c \cap B)$. Thus, it is sufficient to prove that $\pi(A^c \cap B) = \upsilon(A^c \cap B)$. Since $\pi \in core(\upsilon)$, then $\pi(A^c \cap B) \geqslant \upsilon(A^c \cap B)$ and $\pi(A) \geqslant \upsilon(A)$ and $\pi(A^c) \geqslant \upsilon(A^c)$. Since $\upsilon(A) + \upsilon(A^c) = 1$, then $\pi(A^c) = \upsilon(A^c)$. Then,

$$\begin{aligned}\upsilon\left(A^c \cap B\right) &\leqslant \pi\left(A^c \cap B\right)\\ &= \pi(B) + \pi\left(A^c\right) - \pi\left(A^c \cup B\right) = \upsilon(B) + \upsilon\left(A^c\right) - \upsilon\left(A^c \cup B\right).\end{aligned}$$

Since υ is convex, $\upsilon(A^c \cap B) + \upsilon(A^c \cup B) \geqslant \upsilon(B) + \upsilon(A^c)$, which proves that the inequality above is, in fact, an equality, as we wanted to show. $\square$

Notice also that for CEU ambiguity averse DMs, Σ_u consistently coincides with the definition given in the MEU-paradigm, that is, we have:

Lemma 7.6 *If* υ *is a convex capacity, then*

$$\Sigma_u = \left\{A \in \Sigma : \pi(A) = \pi'(A),\ \textit{for any}\ \pi, \pi' \in core(\upsilon)\right\}.$$

Proof If $A \in \Sigma_u$, by Proposition 7.5, $\upsilon(A) + \upsilon(A^c) = 1$ which implies $\pi(A) = \upsilon(A)$, $\forall \pi \in core(\upsilon)$, because $\pi(A) \geqslant \upsilon(A)$ and $\pi(A^c) \geqslant \upsilon(A^c)$. Conversely, let $A \in \Sigma$ be such that $\pi(A) = \pi'(A)$ for any $\pi, \pi' \in core(\upsilon)$. Since $\pi(A^c) = 1 - \pi(A)$, then $\pi(A^c) = \pi'(A^c)$ for any $\pi, \pi' \in core(\upsilon)$. There exist $\pi, \pi' \in core(\upsilon)$ such that $\pi(A) = \upsilon(A)$ and $\pi'(A^c) = \upsilon(A^c)$. Thus, $1 = \pi(A) + \pi(A^c) = \pi(A) + \pi'(A^c) = \upsilon(A) + \upsilon(A^c)$. By Proposition 7.5, $A \in \Sigma_u$. $\square$

The following observation may be useful.

Lemma 7.7 *Let $f : \Omega \to \mathbb{R}_+$ be a simple act given $f = \sum_{k=1}^{m} x_k 1_{A_k}$, where $x_k < x_{k+1}$, for all $k \in \{1, \ldots, m-1\}$, $A_k \in \Sigma$ and $\{A_k : 1 \leqslant k \leqslant m\}$ is a partition of Ω. If each A_k is unambiguous then f is unambiguous. In the CEU paradigm, the converse is also true.*

Proof Observe that the events $\{f \geqslant t\}$ are of the form Ω, $A_i \cup \cdots \cup A_m$, for $i = 1, \ldots, m$ or $\varnothing$. Since Σ_u is a λ-system by Proposition 7.3, these events are in Σ_u, which means that f is unambiguous.

For the converse in the CEU paradigm, we write f as

$$f = x_1 1_\Omega + (x_2 - x_1)\, 1_{A_2 \cup \cdots \cup A_n} + \cdots + (x_m - x_{m-1})\, 1_{A_n}.$$

Since $\{f \geqslant t\} \in \Sigma_u$ for almost all t, there exists $t \in (x_k, x_{k+1})$ such that $\{f \geqslant t\} = A_{k+1} \cup \cdots \cup A_m \in \Sigma_u$. Since Σ_u is an algebra in the CEU paradigm, then the A_k are unambiguous. □

A.3 Proofs

Proof of Theorem 2.1. Let us first consider the MEU case. We know that for any $p \in \mathcal{P}$,

$$I_p(f) = \int_{-\infty}^{0} [p(f \geqslant t) - 1]\, \mathrm{d}t + \int_{0}^{\infty} p(f \geqslant t)\, \mathrm{d}t.$$

Since f is unambiguous, $\{f \geqslant t\} \in \Sigma_u$ for all $t \in \mathbb{R}$, which means that $p(f \geqslant t) = \pi(f \geqslant t)$ for all $p, \pi \in \mathcal{P}$. This implies that $I_p(f)$ is constant across all $p \in \mathcal{P}$. In particular, $I_p(f) = I_{\mathcal{P}}(f)$, for all $p \in \mathcal{P}$. This allows us to obtain:

$$\begin{aligned}
I_{\mathcal{P}}(f+g) &= \min_{p \in \mathcal{P}} \int (f+g)\, \mathrm{d}p \\
&= \min_{p \in \mathcal{P}} \left(\int f \mathrm{d}p + \int g \mathrm{d}p \right) \\
&= \min_{p \in \mathcal{P}} \left[I_{\mathcal{P}}(f) + I_p(g) \right] \\
&= I_{\mathcal{P}}(f) + \min_{p \in \mathcal{P}} I_p(g) \\
&= I_{\mathcal{P}}(f) + I_{\mathcal{P}}(g),
\end{aligned}$$

as we wanted to show.

Let us turn now to the CEU case. Let $f, g \in B_\infty(\Sigma)$, with f unambiguous. We want to prove that

$$I_\upsilon(f+g) = I_\upsilon(f) + I_\upsilon(g). \tag{6}$$

First, we claim that it is enough to prove (6) for $f \geqslant 0$ and $g \geqslant 0$. In fact, since $f, g \in B_\infty^+(\Sigma)$, there exist $a, b \in \mathbb{R}$, $a \geqslant 0$ and $b \geqslant 0$, such that $f + a \geqslant 0$ and $g + b \geqslant 0$. It is clear that $f + a \in B_\infty^+(\Sigma)$ and $g + b \in B_\infty^+(\Sigma)$ and $f + a$ is unambiguous, that is, $\{f + a \geqslant t\} \in \Sigma_u$, for all $t \in \mathbb{R}$. Thus, if (6) is valid for non-negative functions, $I_\upsilon(f + a + g + b) = I_\upsilon(f + a) + I_\upsilon(g + b)$. Since the Choquet integral is additive for constants, we conclude that $I_\upsilon(f + g) = I_\upsilon(f) + I_\upsilon(g)$.

Now, let us define $\int_A f \, \mathrm{d}\upsilon$ as $I_\upsilon(f 1_A)$ and prove that

$$I_\upsilon(h) = \int\limits_A h \, \mathrm{d}\upsilon + \int\limits_{A^c} h \, \mathrm{d}\upsilon, \tag{7}$$

for all $A \in \Sigma_u$ and $h \in B_\infty^+(\Sigma)$. In fact, since A is unambiguous, $\upsilon(h \geqslant t) = \upsilon(\{h \geqslant t\} \cap A) + \upsilon(\{h \geqslant t\} \cap A^c)$. Thus, since $h \geqslant 0$,

$$\begin{aligned} I_\upsilon(h) &= \int\limits_0^\infty \upsilon(h \geqslant t) \, \mathrm{d}t \\ &= \int\limits_0^\infty \upsilon(\{h \geqslant t\} \cap A) \, \mathrm{d}t + \int\limits_0^\infty \upsilon\left(\{h \geqslant t\} \cap A^c\right) \mathrm{d}t \\ &= \int\limits_A h \, \mathrm{d}\upsilon + \int\limits_{A^c} h \, \mathrm{d}\upsilon. \end{aligned}$$

Now, suppose that $h = \alpha 1_A + g$, for some $g \in B_\infty^+(\Sigma)$, $\alpha > 0$ and $A \in \Sigma_u$. From (7), we have:

$$\begin{aligned} I_\upsilon(\alpha 1_A + g) &= \int\limits_A (\alpha 1_A + g) \, \mathrm{d}\upsilon + \int\limits_{A^c} (\alpha 1_A + g) \, \mathrm{d}\upsilon \\ &= \int\limits_A (\alpha + g) \, \mathrm{d}\upsilon + \int\limits_{A^c} g \mathrm{d}\upsilon. \end{aligned}$$

Since α is a constant (and hence comonotonic with g), we have $\int_A (\alpha + g) \, \mathrm{d}\upsilon = \int_A \alpha \mathrm{d}\upsilon + \int_A g \mathrm{d}\upsilon$ (from Schmeidler 1986's Theorem). Also, $\int_A \alpha \mathrm{d}\upsilon = \alpha\upsilon(A) = I_\upsilon(\alpha 1_A)$. Again by (7), $\int_A g \mathrm{d}\upsilon + \int_{A^c} g \mathrm{d}\upsilon = I_\upsilon(g)$. Thus,

$$I_\upsilon(\alpha 1_A + g) = \alpha\upsilon(A) + I_\upsilon(g) = I_\upsilon(\alpha 1_A) + I_\upsilon(g), \tag{8}$$

which is (6) for the case $f = \alpha 1_A$.

Now, consider $f \in B_0^+(\Sigma)$ and $g \in B_\infty^+(\Sigma)$. Since $f \in B_0^+(\Sigma)$, there exist $n \in \mathbb{N}$ and, for each $i = 1, \ldots, n$, $x_i \in \mathbb{R}_+$, $0 \leqslant x_1 < x_2 < \cdots < x_n$, such that:

$$f = x_1 1_\Omega + (x_2 - x_1) 1_{\{f \geqslant x_2\}} + \cdots + (x_n - x_{n-1}) 1_{\{f \geqslant x_n\}}.$$

Define the following functions: $g_n = g + (x_n - x_{n-1}) 1_{\{f \geqslant x_n\}}$, $g_{n-1} = g_n + (x_{n-1} - x_{n-2}) 1_{\{f \geqslant x_{n-1}\}}, \ldots, g_2 = g_3 + (x_2 - x_1) 1_{\{f \geqslant x_2\}}$. Thus, $f + g = x_1 1_\Omega + g_2$. Using (8) repeatedly, we have:

$$\begin{aligned} I_\upsilon (f+g) &= I_\upsilon (x_1 1_\Omega) + I_\upsilon (g_2) \\ &= x_1 + (x_2 - x_1)\,\upsilon (f \geqslant x_2) + I_\upsilon (g_3) \\ &= \ldots \\ &= x_1 + (x_2 - x_1)\,\upsilon (f \geqslant x_2) + \cdots + (x_n - x_{n-1})\,\upsilon (f \geqslant x_n) + I_\upsilon (g) \\ &= I_\upsilon (f) + I_\upsilon (g)\,. \end{aligned}$$

It remains to prove (6) for any unambiguous $f \in B^+_\infty (\Sigma)$. We use the standard approximation of f from below:

$$f_n = \sum_{i=0}^{n2^n} \frac{i}{2^n} 1_{\left\{\frac{i}{2^n} \leqslant f < \frac{i+1}{2^n}\right\}},$$

It is easy to see that f unambiguous implies that $f_n \in B^+_\infty (\Sigma)$ is unambiguous. Hence,

$$I_\upsilon (f_n + g) = I_\upsilon (f_n) + I_\upsilon (g)\,,$$

for all n. It is also clear that $f_n + g \uparrow_n f + g$ and $f_n \uparrow_n f$. By Lemma 7.2, we have $I_\upsilon (f_n + g) \uparrow_n I_\upsilon (f + g)$ and $I_\upsilon (f_n) \uparrow_n I_\upsilon (f)$, which concludes the proof. □

Proof of Theorem 2.3. Since $(iii) \Rightarrow (i)$ was already proved and $(i) \Rightarrow (ii)$ is trivial (take $g = -f$), it is sufficient to prove that $(ii) \Rightarrow (iii)$. So take $f \in B_\infty (\Sigma)$ such that $I(f) + I(-f) = 0$. We know that the Choquet integral can also be written with strict inequalities:

$$I(-f) = \int_{-\infty}^{0} [\upsilon (-f > t) - 1]\,\mathrm{d}t + \int_{0}^{\infty} \upsilon (-f > t)\,\mathrm{d}t.$$

Changing the variable of integration to $t = -\alpha$, we have

$$I(-f) = \int_{0}^{\infty} [\upsilon (f < t) - 1]\,\mathrm{d}t + \int_{-\infty}^{0} \upsilon (f < t)\,\mathrm{d}t.$$

Since

$$I(f) = \int_{-\infty}^{0} [\upsilon (f \geqslant t) - 1]\,\mathrm{d}t + \int_{0}^{\infty} \upsilon (f \geqslant t)\,\mathrm{d}t,$$

we have

$$\int_{-\infty}^{0} [\upsilon (f \geqslant t) + \upsilon (f < t) - 1] \, \mathrm{d}t + \int_{0}^{\infty} [\upsilon (f \geqslant t) + \upsilon (f < t) - 1] \, \mathrm{d}t = 0. \quad (9)$$

Since f is bounded, the interval of integration of the integrand $x(t) = \upsilon (f \geqslant t) + \upsilon (f < t) - 1$ is a compact interval K. Moreover, $\upsilon (f \geqslant t)$ and $\upsilon (f < t)$ are monotone and so continuous, except on a countable (finite or denumerable) set of points of K. As υ is convex, $\upsilon (f \geqslant t) + \upsilon (f < t) \leqslant 1$, that is, $x(t) \leqslant 0$. From (9), $x(t) < 0$ on K in a set of zero measure. Thus, $\upsilon (f \geq t) + \upsilon (f < t) = 1$ almost everywhere. Since

$$\Sigma_u = \left\{A \in \Sigma : \upsilon (A) + \upsilon \left(A^c\right) = 1\right\}$$

by Proposition 7.5, then $\{f \geqslant t\} \in \Sigma_u$ almost everywhere, i.e., f is unambiguous. □

Proof of Theorem 3.1. Let $(z_i)_{i \in N} \in T\left(\mathcal{E}^2\right)$, that is, $\sum_{i \in N} z_i = 0$, $(z_i)_{i \in N} \neq 0$, $e_i + z_i \succcurlyeq_i^2 e_i$ for all $i \in N$ and there exists j such that $e_j + z_j \succ_j^2 e_j$. We claim that the same $(z_i)_{i \in N}$ satisfies $e_i + z_i \succcurlyeq_i^1 e_i$ for all $i \neq j$ and $e_j + z_j \succ_j^1 e_j$, which will establish the wanted fact that $(z_i)_{i \in N} \in T\left(\mathcal{E}^1\right)$. To see that the claim is true, suppose by contradiction that $e_i \succ_i^1 e_i + z_i$ for some $i \neq j$. Since $\succcurlyeq_i^2$ is more uncertainty averse than $\succcurlyeq_i^1$ and e_i is unambiguous for individual i, this implies that $e_i \succ_i^2 e_i + z_i$, a contradiction. In the same fashion, if $e_j \succcurlyeq_j^1 e_j + z_j$ then $e_j \succcurlyeq_j^2 e_j + z_j$, again a contradiction. □

Proof of claims in Example 3.2. It is not difficult to see that e_1 is unambiguous for both $\succcurlyeq_1^l$ and $\succcurlyeq_1^m$ and e_2 is unambiguous for $\succcurlyeq_2^l$ and $\succcurlyeq_2^m$. Moreover, clearly, $\succcurlyeq_i^m$ is more ambiguous averse than $\succcurlyeq_i^l$, since $\mathcal{P}_i^m \supset \mathcal{P}_i^l$, for $i = 1, 2$.

Notice that $z = (z_1, z_2)$, where $z_1 = (-3, 0, 3)$ and $z_2 = (3, 0, -3)$, is a Pareto improving trade for economy $\mathcal{E}^l$ if $\alpha = 0$. In fact, let $x_1 = e_1 + z_1 = (4, 1, 4)$ and $x_2 = e_2 + z_2 = (4, 7, 6)$. Then,

$$I_{\mathcal{P}_1^l}(u(x_1)) = \frac{1}{4}(2+2+2) > \frac{1}{4}(\sqrt{7}+2+1) = I_{\mathcal{P}_1^l}(u(e_1)); \text{ and}$$
$$I_{\mathcal{P}_2^l}(u(x_2)) = \frac{1}{4}(2+2\sqrt{7}+\sqrt{6}) > \frac{1}{4}(1+2\sqrt{7}+3) = I_{\mathcal{P}_2^l}(u(e_2)).$$

Since $\lim_{\alpha \to 0} p^\alpha = p^0$, note that by continuity, there exists $\alpha_0 \in \left[0, \frac{1}{4}\right)$ such that z remains a feasible, Pareto improving trade for economy $\mathcal{E}^l$ for any $\alpha \in [0, \alpha_0]$. In what follows we assume that such an α has been chosen; this is to emphasize the fact that $T(\mathcal{E}^m) \not\subset T(\mathcal{E}^l)$ is independent of the existence or non-existence of a common prior for the individuals, i.e., it is independent of $\cap_{i \in N} \mathcal{P}_i \neq \varnothing$ ($\Longleftrightarrow$ $\alpha = 0$) or $\cap_{i \in N} \mathcal{P}_i = \varnothing$ ($\Longleftrightarrow$ $\alpha \neq 0$). It is enough to see that for a suitable $\varepsilon \in \left[0, \frac{3}{4}\right]$,

$$\min_{\pi \in \mathcal{P}_1^m} I_\pi(u(x_1)) < \min_{\pi \in \mathcal{P}_1^m} I_\pi(u(e_1)),$$

i.e., that $\frac{1}{4} \cdot 2 + (\frac{3}{4} - \varepsilon) \cdot 1 + \varepsilon \cdot 2 < \frac{1}{4}\sqrt{7} + \frac{3}{4}$. Since this inequality is true for $\varepsilon = 0$, this completes the proof. □

Proof of claim in Example 3.3. We want to show that $e_1 \succ_1^m f_1 = \lambda e$, that is:

$$\begin{aligned}
\min_{\pi \in \mathcal{P}_1^m} I_\pi(u(f_1)) &= \min\left\{\frac{1}{4}\sqrt{\lambda 8} + \frac{1}{2}\sqrt{\lambda 8} + \frac{1}{4}\sqrt{\lambda 10}, \frac{1}{4}\sqrt{\lambda 8} + \frac{3}{4}\sqrt{\lambda 8}\right\} \\
&= 0.49\left(\frac{1}{4}\sqrt{8} + \frac{3}{4}\sqrt{8}\right) < 1.4 \\
&< \frac{1}{4}(\sqrt{7} + 3) \\
&= \min\left\{\frac{1}{4}\sqrt{7} + \frac{1}{2}\sqrt{1} + \frac{1}{4}\sqrt{1}, \frac{1}{4}\sqrt{7} + \frac{3}{4}\sqrt{1}\right\} \\
&= \min_{\pi \in \mathcal{P}_1^m} I_\pi(u(e_1)).
\end{aligned}$$

□

Proof of Theorem 3.4. Let $(z_i)_{i\in N} \in O\left(\mathcal{E}^2\right) \setminus O\left(\mathcal{E}^1\right)$. This implies that there is $\left(z_i'\right)_{i\in N}$ such that $e_i + z_i' \succcurlyeq_i^1 e_i + z_i$ for all $i \in N$ and there exists j such that $e_j + z_j' \succ_j^1 e_j + z_j$. Using the same argument of the proof of Theorem 3.1, we reach a contradiction. □

Proof of Theorem 3.5. Since $(z_i)_{i\in N} \in T\left(\left(\succcurlyeq_i^2, e_i\right)_{i\in N}\right) \Rightarrow (z_i)_{i\in N} \in T\left(\left(\succcurlyeq_i^1, e_i\right)_{i\in N}\right)$, we have $e_i + z_i \succcurlyeq_i^2 \left(\succ_i^2\right) e_i \Rightarrow e_i + z_i \succcurlyeq_i^1 \left(\succ_i^1\right) e_i$ or, equivalently, $e_i \succcurlyeq_i^1 \left(\succ_i^1\right) e_i + z_i \Rightarrow e_i \succcurlyeq_i^2 \left(\succ_i^2\right) e_i + z_i$. Since this holds for all unambiguous endowments, $\succcurlyeq_i^2$ is more uncertainty averse than $\succcurlyeq_i^1$ by the definition. □

The following result is useful for the proof of Theorem 4.2.

Lemma 7.8 (i) *In the MEU-paradigm, $\cap_{i\in N}\mathcal{P}_i \neq \varnothing$ if and only if $\sum_{i\in N} z_i = 0 \Rightarrow \sum_{i\in N} I_i(z_i) \leqslant 0$;*
(ii) *In the CEU paradigm, if $\upsilon_1 \leqslant \overline{\upsilon}_2$, then $I_1(z) + I_2(-z) \leqslant 0$, for all $z \in B_\infty(\Sigma)$.*

Proof (i) Let $\pi \in \cap_{i\in N}\mathcal{P}_i$. If $\sum_{i\in N} z_i = 0$, then $\sum_{i\in N} I_i(z_i) \leqslant \sum_{i\in N} I_\pi(z_i) = I_\pi\left(\sum_{i\in N} z_i\right) = 0$. Conversely, suppose that $\cap_{i\in N}\mathcal{P}_i = \varnothing$. Then, Theorem 2 of Billot et al. (2000) can be used to show that there are $(z_i)_{i\in N}$ such that $\sum_{i\in N} z_i = 0$ and $\sum_{i\in N} I_i(z_i) > 0$, completing the proof.
(ii) If $\upsilon_1 \leqslant \overline{\upsilon}_2$, then $I_1(1_A) = \upsilon_1(A) \leqslant 1 - \upsilon_2(A^c) = 1 - I_2(1_{A^c})$, which implies $I_1(1_A) + I_2(1_{A^c}) \leqslant 1$. Now, if $z = 1_A$, then $I_2(-z) = 1 - I_2(1_{A^c})$. Thus, $I_1(1_A) + I_2(-1_A) \leqslant 0$. We proceed now, as in the proof of Theorem 2.1, extending this to simple z and, by monotonicity, to $z \in B_\infty(\Sigma)$. □

Proof of Theorem 4.2. For a contradiction, assume that $(z_i)_{i\in N} \in T(\mathcal{E})$. Since u_i is concave by Assumption 7.1, Jensen's inequality gives[26]

$$U_i\left(e_i + z_i\right) = I_i\left(u_i\left(e_i + z_i\right)\right) \leqslant u_i\left(I_i\left(e_i + z_i\right)\right).$$

Since e_i is unambiguous, $I_i\left(e_i + z_i\right) = I_i\left(e_i\right) + I_i\left(z_i\right)$, by the Additivity Theorem (Theorem 2.1). This implies that: $U_i(e_i + z_i) \leqslant u_i\left(I_i\left(e_i\right) + I_i\left(z_i\right)\right)$. The assumption that $u_i(I_i(e_i)) = U_i(e_i)$ and the fact that $e_i + z_i \succcurlyeq_i e_i$ give:

$$u_i(I_i(e_i)) = U_i(e_i) \leqslant U_i(e_i + z_i) \leqslant u_i\left(I_i\left(e_i\right) + I_i\left(z_i\right)\right).$$

Since u_i is increasing, this implies that $I_i(z_i) \geqslant 0$, for all $i \in N$. Moreover, since $e_j + z_j \succ_j e_j$ for some $j \in N$, $I_j(z_j) > 0$. Therefore, $\sum_{i\in N} I_i(z_i) > 0$. However, this contradicts the conclusions of Lemma 7.8. □

The proof of Theorem 5.2 requires the following:

Lemma 7.9 *Given $P, P' \in \Delta(\Omega)$, suppose that $P' = fP$, for some $\mathcal{B}$-measurable function $f : \Omega \to \mathbb{R}_+$. Then, $P(\cdot|\mathcal{B}) = P'(\cdot|\mathcal{B})$.*

Proof Notice that f is (a version of) the Radon–Nikodym derivative $\frac{\mathrm{d}P'}{\mathrm{d}P}$. Fix $A \in \Sigma$ and define for each $B \in \mathcal{B}$, $\nu(B) = P(A \cap B)$ and $\nu'(B) = P'(A \cap B)$. It is easy to see that $\frac{\mathrm{d}\nu'}{\mathrm{d}\nu} = f$. Let Q and Q' denote the conditional probabilities of P and P', respectively. By condition (b) in the definition of conditional probability, it is clear that $\frac{\mathrm{d}\nu}{\mathrm{d}P} = Q$ and $\frac{\mathrm{d}\,\nu'}{\mathrm{d}P'} = Q'$. Now, by the chain rule for Radon–Nikodym derivatives (see Billingsley 1986, 32.6, p. 446), we have:[27]

$$\frac{\mathrm{d}\nu'}{\mathrm{d}P} = \frac{\mathrm{d}\nu'}{\mathrm{d}P'}\frac{\mathrm{d}P'}{\mathrm{d}P} = Q'f;$$

but also:

$$\frac{\mathrm{d}\nu'}{\mathrm{d}P} = \frac{\mathrm{d}\nu'}{\mathrm{d}\nu}\frac{\mathrm{d}\nu}{\mathrm{d}P} = fQ.$$

Therefore, $Q' = Q$, as we wanted to show. □

Proof of Theorem 5.2 $(i) \Rightarrow (iii)$: Suppose that $x = (x_i)_{i\in N}$ is a norm-interior unanimously unambiguous Pareto optimal allocation. By Theorem 5.1, we know that there exists $p \in \cap_{i\in N}\mathcal{P}_i(x_i)$, that is, $p = \frac{u_i'(x_i(\cdot))}{\int u_i'(x_i(\cdot))\,\mathrm{d}\pi_i}\pi_i$ for some $\pi_i \in \arg\min_{p\in\mathcal{P}_i}\int_\Omega u_i\left(x_i(\cdot)\right)dp$, for each $i \in N$. Since x_i is Σ^{uu}-measurable, the function

[26] Jensen's inequality can be easily extended for MEU and CEU paradigms.

[27] Of course, the equalities are in the "almost surely" sense. Since the measures are mutually absolutely continuous, there is no ambiguity on this.

$\lambda_i(\omega) \equiv \frac{u_i'(x_i(\cdot))}{\int u_i'(x_i(\cdot))\,\mathrm{d}\pi_i}$ is Σ^{uu}-measurable. By Lemma 7.9, $p(\cdot|\Sigma^{uu}) = \pi_i(\cdot|\Sigma^{uu})$, which proves that $p(\cdot|\Sigma^{uu}) \in \mathcal{P}_i^{uu}$ for all $i \in N$, that is, (iii) holds.

$(iii) \Rightarrow (ii)$: Suppose that $x = (x_i)_{i\in N}$ is a Pareto optimal allocation and that for some $i_0 \in N$, x_{i_0} is not Σ^{uu}-measurable. Let us first build a new (feasible) allocation y_i, which is unambiguous with respect to Σ^{uu}. For this purpose, take $\pi \in \cap_{i\in N}\mathcal{P}_i$ and define y_i by $y_i \equiv E_\pi\left[x_i|\Sigma^{uu}\right]$. Moreover, since e_N is Σ^{uu}-measurable by assumption, the feasibility of x, i.e., $\sum_i x_i = e_N$, gives: $E_\pi\left[\sum_i x_i|\Sigma^{uu}\right] = \sum_i y_i = e_N$, that is, y_i is feasible. If we prove that $y_i \succcurlyeq_i x_i$ for all $i \in N$ and $y_{i_0} \succ_i x_{i_0}$, this will contradict Pareto optimality of $(x_i)_{i\in N}$, hence proving the result. First, for any consumer i, $U_i(x_i) = \min_{\pi'\in\mathcal{P}_i} E_{\pi'}[u_i(x_i)] \leqslant E_\pi[u_i(x_i)]$. From Proposition 5.4, $E_\pi[u_i(x_i)] \leqslant E_\pi[u_i(y_i)]$. Since y_i is unambiguous for consumer i,

$$E_\pi[u_i(y_i)] = \min_{\pi'\in\mathcal{P}_i} E_{\pi'}[u_i(y_i)] = U_i(y_i).$$

Therefore, $U_i(x_i) \leqslant U_i(y_i)$ for all $i \in N$. Repeating this argument now with the strict inequality in Proposition 5.4, that is, $E_\pi[u_{i_0}(x_{i_0})] < E_\pi[u_{i_0}(y_{i_0})]$, we obtain $U_{i_0}(x_{i_0}) < U_{i_0}(y_{i_0})$. Thus, y Pareto improves x, which is a contradiction.

$(ii) \Rightarrow (i)$: Since $\mathcal{E}$ is well behaved, the assumptions of Proposition 12 of Rigotti et al. (2008) are satisfied. Therefore, there exist an individually rational Pareto optimal allocation x. Since e_i is norm-interior for each i and the economy is well behaved, x_i is also norm-interior. By (ii), x is a norm-interior unanimously unambiguous allocation. □

The proof of Theorem 5.3 requires some lemmas.

Lemma 7.10 *Assume that u_i is C^1, with $u_i' > 0$ for $i = 1, 2$. Let $x_1, x_2 \in \mathbb{R}_+$ be such that $0 < x_2 < x_1$. Then there exist $y_1 \in (0, x_1)$ and $y_2 \in (0, x_2)$ such that*

$$u_1'(y_1)u_2'(x_2 - y_2) = u_2'(x_1 - y_1)u_1'(y_2).$$

Proof Define $g(y_1, y_2) \equiv u_1'(y_1)u_2'(x_2 - y_2) - u_2'(x_1 - y_1)u_1'(y_2)$, for $(y_1, y_2) \in [0, x_1] \times [0, x_2]$. Since $x_2 < x_1$,

$$g(y, y) = u_1'(y)u_2'(x_2 - y) - u_2'(x_1 - y)u_1'(y) < 0$$

and

$$g(x_1, x_2) = u_1'(x_1)u_2'(x_2 - x_2) - u_2'(x_1 - x_1)u_1'(x_2) = u_2'(0)[u_1'(x_1) - u_1'(x_2)] > 0.$$

Since g is continuous, there exists $(y_1, y_2) \in (0, x_1) \times (0, x_2)$ such that $g(y_1, y_2) = 0$, which is the desired equality. □

Lemma 7.11 *Assume statement (i) in Theorem 5.3. Then, $\pi_i(B) = \pi_j(B)$ for any $B \in \mathcal{B}$, $\pi_i \in \mathcal{P}_i$, $\pi_j \in \mathcal{P}_j$ and $i, j \in N$.*

Proof Since Ω is finite, there is a partition of Ω associated with $\mathcal{B}$ in the sense that each element of $\mathcal{B}$ is a union of elements of this partition. Each element of this partition is called an atom. Fix an atom B of $\mathcal{B}$ and any two individuals, which without loss of generality, we can assume to be 1 and 2. Since e_N is $\mathcal{B}$-measurable, $e_N(\omega) = l_B$ for some constant $l_B \in \mathbb{R}_{++}$, for all $\omega \in B$. Define $l_{B^c} \equiv \inf_{\omega \in B^c} e_N(\omega) > 0$ and $l = \min\{l_B, l_{B^c}\}$. Choose $0 < \ell_{B^c} < \ell_B < l$. By Lemma 7.10, there exists $(x_1^B, x_1^{B^c}) \in (0, \ell_B) \times (0, \ell_{B^c})$ such that:

$$u_1'(x_1^B)u_2'(x_2^{B^c}) = u_2'(x_2^B)u_1'(x_2^{B^c}), \tag{10}$$

where $(x_2^B, x_2^{B^c}) = (\ell_B - x_1^B, \ell_{B^c} - x_1^{B^c})$. Now, define the following allocation:

$$\begin{aligned} x_1(\omega) &= x_1^B 1_B(\omega) + x_1^{B^c} 1_{B^c}(\omega); \\ x_2(\omega) &= x_2^B 1_B(\omega) + x_2^{B^c} 1_{B^c}(\omega); \\ x_i(\omega) &= \frac{l_B - \ell_B}{n-2} 1_B(\omega) + \frac{e_N(\omega) - \ell_{B^c}}{n-2} 1_{B^c}(\omega), \quad \text{for } i \geqslant 3. \end{aligned}$$

This allocation is clearly $\mathcal{B}$-unambiguous. Therefore, by (i), it is Pareto optimal. By Theorem 5.1, $\cap_{i \in N} \mathcal{P}_i(x_i) \neq \emptyset$. Since the allocation is unambiguous, $\mathcal{P}_i = \arg\min_{p \in \mathcal{P}_i} \int_\Omega u_i\,(x_i(\cdot))\,dp$ and there exist $\pi_i \in \mathcal{P}_i$, for $i = 1, 2$ such that:

$$\frac{u_1'(x_1(\cdot))}{\int u_1'(x_1)\,\mathrm{d}\pi}\pi_1(\cdot) = \frac{u_2'(x_2(\cdot))}{\int u_2'(x_2)\,\mathrm{d}\pi}\pi_2(\cdot),$$

which, restricted to the atom B, gives:

$$\frac{u_1'(x_1^B)}{u_1'(x_1^B)\pi_1(B) + u_1'(x_1^{B^c})\pi(B^c)}\pi_1(B) = \frac{u_2'(x_2^B)}{u_2'(x_2^B)\pi_2(B) + u_2'(x_2^{B^c})\pi_2(B^c)}\pi_2(B).$$

A simple algebraic manipulation simplifies this to:

$$\pi_1(B)\pi_2(B^c)u_1'(x_1^B)u_2'(x_2^{B^c}) = \pi_1(B^c)\pi_2(B)u_2'(x_2^B)u_1'(x_2^{B^c}).$$

By (10), we have:

$$\pi_1(B)\pi_2(B^c) = \pi_1(B^c)\pi_2(B),$$

which implies $\pi_1(B) = \pi_2(B)$. Since $\mathcal{B} \subset \Sigma_i$ for every $i \in N$, $\pi_1(B) = \pi(B)$ for every $\pi \in \mathcal{P}_1$, and the same holds for individual 2. Since the individuals and the atom were arbitrarily chosen, we have $\pi_i(B) = \pi_j(B)$ for any $B \in \mathcal{B}$, $\pi_i \in \mathcal{P}_i$, $\pi_j \in \mathcal{P}_j$ and $i, j \in N$, as we wanted to conclude. $\square$

Lemma 7.12 *Assume statement* (i) *in Theorem* 5.3*. Then, e is constant. Therefore,* $\mathcal{B} = \{\emptyset, \Omega\}$.

Proof Take $0 < \epsilon < \inf_{\omega \in \Omega} e_N(\omega)$, and define the allocation $x = (x_i)_{i \in N}$ by $x_i = \frac{\epsilon}{n-1}$ for $i \neq 1$ and $x_1(\omega) = e_N(\omega) - \epsilon$. By Theorem 5.1, for every atom B of $\mathcal{B}$, we have:

$$\frac{u_1'(x_1(\omega))}{\int u_1'(x_1(\cdot))\,\mathrm{d}\pi}\pi_1(B) = \pi_i(B), \quad \forall \omega \in B.$$

By the Lemma 7.11, $u_k'(x_k(\omega)) = \int u_k'(x_k(\cdot))\,\mathrm{d}\pi$, for all $\omega \in B$. Since B was arbitrary, this shows that x_1 and, therefore, e_N, is constant. □

Now, we can complete the proof of Theorem 5.3.

Proof of Theorem 5.3. $(i) \Rightarrow (ii)$: Since every full insurance allocation is a $\mathcal{B}$-unambiguous allocation, (i) implies that every full insurance allocation is Pareto optimal. By previous lemmas, e is constant. Thus, we are in the setting of Theorem 1 of Billot et al. (2000), which implies that $\cap_i \mathcal{P}_i \neq \emptyset$ (and all other implications in the theorem hold).
$(ii) \Rightarrow (i)$: Since e is constant, Theorem 1 of Billot et al. (2000) implies that every Pareto optimal allocation is a full insurance allocation and, therefore, $\mathcal{B}$-unambiguous. □

Proof of Proposition 5.4 Let us denote by $E^{\mathcal{B}}[Z]$ the conditional expectation $E\,[Z|\mathcal{B}]$. By Jensen's inequality, $E^{\mathcal{B}}[u(X)] \leqslant u(E^{\mathcal{B}}[X]) = u(Y)$, with strict inequality if u is strictly concave. Taking expectations in both sides, we obtain:

$$E[u(X)] = E\left[E^{\mathcal{B}}[u(X)]\right] \leqslant E[u(Y)],$$

with strict inequality if u is strictly concave. □

Proof of Theorem 5.5. Since the MEU economy is more ambiguous averse than the economy EU(π), the inclusion $O^u_{MEU} \subset O_{EU(\pi)} \cap A^u_{MEU}$ follows from Theorem 3.4, since all allocations are unambiguous in an economy with EU-DMs. Now, let $(x_i)_{i \in N} \in O_{EU(\pi)} \cap A^u_{MEU}$ for some $\pi \in \cap_{i \in N} \mathcal{P}_i$. Pareto optimality of the EU economy and Theorem 5.1 imply that

$$\frac{u_i'(x_i(\cdot))}{\int u_i'(x_i(\cdot))\,\mathrm{d}\pi}\pi = \frac{u_j'(x_j(\cdot))}{\int u_j'(x_j(\cdot))\,\mathrm{d}\pi}\pi = p.$$

Again by Theorem 5.1, it is sufficient to prove that $p \in \cap_{i \in N} \mathcal{P}_i(x_i)$, where:

$$\mathcal{P}_i\,(f) \equiv \left\{ \frac{u_i'\,(f(\cdot))}{\int_\Omega u_i'\,(f)\,\mathrm{d}\pi}\pi \,:\, \pi \in \arg\min_{p \in \mathcal{P}_i} \int_\Omega u_i\,(f(\cdot))\,\mathrm{d}p \right\}.$$

Since x_i is unambiguous for i, then $\pi \in \arg\min_{p \in \mathcal{P}_i} \int_\Omega u_i\,(x_i(\cdot))\,dp = \mathcal{P}_i$, which shows that $p \in \mathcal{P}_i(x_i)$ for every $i \in N$. This concludes the proof. □

Proof of Corollary 6.1. Recall that the individuals in each country are assumed to have the same set of priors $\mathcal{P}_C$, for $C = H, F$ and that the aggregate endowment in each country is unambiguous for the individuals in that country. Therefore, the result comes directly from Theorem 5.2. □

Proof of Corollary 6.2. This is a direct application of Theorem 5.2. □

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