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Quantile selection in non-linear GMM quantile models

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ABSTRACT

This note proposes a non-linear GMM quantile regression model to estimate the quantile as an additional parameter. The limiting distribution is studied. An empirical application to an intertemporal consumption model built on a structural dynamic quantile utility model illustrates the estimator. Using US data, it separately estimates the elasticity of intertemporal substitution and the risk attitude, which is captured by the estimated quantile.

1. Introduction

Since the seminal work of Koenker and Bassett (1978), quantile regression (QR) has attracted considerable interest in statistics and econometrics. QR estimates conditional quantile functions that provide insight into heterogeneous effects of variables of interest, indexed by the quantiles, $\tau \in (0, 1)$. Chernozhukov and Hansen (2005, 2006, 2008) extend QR methods and present results on identification, estimation, and inference for an instrumental variables QR (IVQR) model that allows for endogenous regressors; see Chernozhukov et al. (2017) for an overview of IVQR. Kaplan and Sun (2017) and de Castro et al. (2019) develop estimators for QR models with instruments in a generalized method of moments (GMM) framework. In general, investigating the entire quantile process is of interest because one may be interested in either testing global hypotheses about conditional distributions or making comparisons across different quantiles (for a discussion about inference in QR models see Koenker and Xiao, 2002). Nevertheless, in an attempt to provide the 'most representative quantile', Bera et al. (2016) consider a QR model within a quasi-maximum likelihood asymmetric Laplace framework and propose methods to estimate the quantile together with the parameters of interest.[1] The estimated quantile captures a measure of asymmetry of the distribution of innovations, and it does not necessarily lead to the mode, but to a point estimate that is most probable, in the sense it maximizes the entropy.

Recently, quantile preferences (QP) have attracted attention in modeling economic behavior in dynamic frameworks.[2] QP are an alternative to expected utility models with useful advantages, such as, in dynamic models, allowing the separation between risk aversion and elasticity of intertemporal substitution (EIS), the ability to capture heterogeneity by offering a family of preferences indexed by the quantile index, $\tau \in (0, 1)$, dynamic consistency and monotonicity. Giovannetti (2013) presents a two-period standard economy with one risky and one risk-free asset, where the agent has QP instead of the standard expected utility. de Castro and Galvao (2019a) develop a dynamic model of rational behavior under uncertainty, in which the agent maximizes the stream of the future quantile utilities, and derive the

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[1] See also Machado (1993) and Koenker and Machado (1999) where the quantile appears naturally as a location parameter and the pioneering work of Yu and Moyeed (2001) and Yu and Zhang (2005).

[2] Manski (1988) introduced QP, which have been subsequently axiomatized by Chambers (2009), Rostek (2010), and de Castro and Galvao (2019b).

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Euler equation from the maximization model. In this framework, the quantile τ has a structural interpretation since it captures the risk attitude, and as such, it may be seen as a parameter to be estimated.

de Castro et al. (2019) use the quantile Euler equation from a dynamic model as motivation to propose a quantile GMM method, and estimate the structural parameters providing estimates for the EIS for different levels of risk attitude, indexed by τ. Nevertheless, since the structural economic model could be interpreted as describing the behavior of a representative consumer, one may be interested in identifying both the EIS and the attitude towards risk (quantile) simultaneously. The empirical set-up developed in that paper, however, is not designed to estimate the quantile associated with the representative consumer. This paper proposes methods to bridge this gap.

In this paper we provide econometric methods for simultaneously estimating the parameters of interest together with the quantile by extending the quantile non-linear GMM model to consider an additional moment condition derived from Komunjer (2005, 2007) and Bera et al. (2016).[3] We derive the limiting distribution of the augmented quantile GMM estimator, and apply the methods to an empirical application using the dynamic quantile model.

2. A structural model for quantile identification

To motivate the econometric methods proposed in the next section, we consider an economic dynamic quantile model of intertemporal allocation of consumption as in de Castro and Galvao (2019a) to estimate the elasticity of intertemporal substitution (EIS).

The EIS is a parameter of central importance in several fields of economics. There is a large empirical literature that attempts to estimate the EIS; among others, Hansen and Singleton (1983), Hall (1988), Campbell and Mankiw (1989), Campbell and Viceira (1999), Campbell (2003), Yogo (2004), and Thimme (2017). The majority of the literature relies on the traditional expected utility framework.

Under certain assumptions, if an agent maximizes the τ-quantile of utility instead of expected utility, then the resulting Euler equation can be written as (see de Castro and Galvao, 2019a; de Castro et al., 2019 for details)

$$Q_\tau[\beta(1+r_{t+1})U'(C_{t+1})/U'(C_t) \mid \Omega_t] = 1,$$

where β is the discount factor, r_t is real interest rate, C_t is consumption, $U(\cdot)$ is the utility function, Ω_t is the information set, and $Q_\tau[W_t \mid \Omega_t]$ denotes the τth conditional quantile of W_t given Ω_t. With an isoelastic utility we obtain the following equation of interest:

$$Q_\tau[\beta(1+r_{t+1})(C_{t+1}/C_t)^{-\gamma} \mid \Omega_t] = 1, \tag{1}$$

where $1/\gamma$ captures the EIS.

The quantile Euler equation can be log-linearized with no approximation error, unlike the standard Euler equation. Since quantiles are invariant with respect to monotone transformations, by taking logarithms on both sides of (1) we have that

$$Q_\tau[-\ln(C_{t+1}/C_t) + \gamma^{-1}\ln(\beta) + \gamma^{-1}\ln(1+r_{t+1}) \mid \Omega_t] = 0. \tag{2}$$

de Castro et al. (2019) estimate model (2) for different quantiles using GMM as outlined below. Nevertheless, for the dynamic quantile model, the risk attitude is in fact measured by τ. As a result, the joint estimation of γ and τ allows for disentangling EIS from risk aversion, thus motivating estimation of τ as a structural parameter of interest.

[3] As discussed in Bera et al. (2016) this approach delivers estimates for the parameters together with a representative quantile, which represents a measure of skewness of the data that combines the information in the mean and the median to capture the asymmetry of the underlying innovations distribution.

3. The econometric model

We consider the following nonlinear conditional quantile model as in de Castro et al. (2019),

$$Q_\tau[\Lambda(Y_i, X_i; \alpha_{0\tau}) \mid Z_i] = 0, \tag{3}$$

where $Y_i \in \mathcal{Y} \subseteq \mathbb{R}^{d_Y}$ is the endogenous variable vector, $Z_i \in \mathcal{Z} \subseteq \mathbb{R}^{d_Z}$ is the full instrument vector that contains $X_i \in \mathcal{X} \subseteq \mathbb{R}^{d_X}$ as a subset, $\Lambda(\cdot)$ is the "residual function" that is known up to the finite-dimensional parameter of interest $\alpha_0 \in \mathcal{A} \subseteq \mathbb{R}^{p-1}$, and $\tau \in (0,1)$ is the quantile index. $\Lambda(\cdot)$ may be a nonlinear function of α_0. Unfortunately, traditional QR methods, that is, minimizing the check function as in Koenker and Bassett (1978), $\rho_\tau(\Lambda(Y_i, X_i; \alpha_{0\tau})) = \Lambda(Y_i, X_i; \alpha_{0\tau})\left[\tau - \mathbf{1}\{\Lambda(Y_i, X_i; \alpha_{0\tau}) < 0\}\right]$, where $\mathbf{1}\{\cdot\}$ is the indicator function, will not succeed in estimating the parameters of interest because of potential endogeneity.[4]

To estimate $\alpha_{0\tau}$, we use unconditional moments implied by (3):

$$\mathrm{E}\left[Z_i\left(\mathbf{1}\{\Lambda(Y_i, X_i; \alpha_{0\tau}) \le 0\} - \tau\right)\right] = 0. \tag{4}$$

If $d_Z = p-1$ then this corresponds to an exactly identified model; when $d_Z > p-1$ the model is over-identified.

We are interested, however, in a model where the parameters of interest are $\theta = (\alpha^\top, \tau)^\top$ (a vector of parameters of dimension p), for which (4) does not provide enough identification conditions. In fact, if α is indexed by τ there will be a continuum of solutions in (4), i.e. one for each τ.

Following Bera et al. (2016), we use the asymmetric Laplace distribution (ALPD) framework to motivate an additional moment condition to recover the quantile of interest. In the ALPD, the quantile τ measures the asymmetry of the underlying distribution and its mode. When evaluating $\Lambda(Y_i, X_i; \alpha, \tau)$ in terms of ALPD, then if $\mathrm{E}[\Lambda(Y_i, X_i; \alpha_0, \tau_0)] > 0$, the ALPD is skewed to the right, and this corresponds to $\tau_0 < 1/2$; on the contrary, when $\mathrm{E}[\Lambda(Y_i, X_i; \alpha_0, \tau_0)] < 0$, the ALPD is skewed to the left and $\tau_0 > 1/2$; moreover, when $\mathrm{E}[\Lambda(Y_i, X_i; \alpha_0, \tau_0)] = 0$, we have $\tau_0 = 1/2$. For the ALPD model, being an unimodal distribution, a positive (negative) mean is associated with right (left) skewness, for which the left (right) tail has a higher concentration of probability mass than the right (left) tail. Thus, τ captures a measure of asymmetry of the distribution of innovations, and it is also associated with the most probable quantile in the sense it maximizes the entropy.

Considering τ as a parameter to be estimated, an additional moment condition arises of the form (see Bera et al., 2016, p.84)[5]

$$\frac{1-2\tau_0}{\tau_0(1-\tau_0)}\mathrm{E}\left[\rho_{\tau_0}(\Lambda(Y_i, X_i; \alpha_0, \tau_0))\right] - \mathrm{E}[\Lambda(Y_i, X_i; \alpha_0, \tau_0)] = 0. \tag{5}$$

4. Estimator and limiting distribution

In order to estimate the parameters of interest, $\theta_0 = (\alpha_0^\top, \tau_0)^\top$, first define the following function

$$g(Y, X, Z; \theta) = \left[g_1(Y, X, Z; \theta)^\top \; g_2(Y, X, Z; \theta)\right]^\top, \tag{6}$$

[4] For nonlinear standard QR see Powell (1994, S2.2), Oberhofer and Haupt (2016), and references therein.

[5] Bera et al. (2016, p.84, eq. (19)) uses the ALPD maximum likelihood score function with respect to τ,

$$\frac{1-2\tau}{\tau(1-\tau)} = \frac{\mathrm{E}[\Lambda(Y_i, X_i; \alpha, \tau)]}{\sigma}.$$

As in the least squares case, the scale parameter σ can be interpreted as the expected value of the loss function, which in the QR case corresponds to the expectation of the $\rho_\tau(\cdot)$ function. Note that this is not necessarily equal to the standard deviation of the random variable. Thus, using the implied additional restriction (Bera et al., 2016, p.84, eq. (20)) $\sigma = \mathrm{E}\left[\rho_\tau(\Lambda(Y_i, X_i; \alpha, \tau))\right]$, we obtain a scalar moment function in (5).

where $g_1(Y, X, Z; \theta) = Z\left(\mathbf{1}\{\Lambda(Y, X; \alpha, \tau) \le 0\} - \tau\right)$, a $d_z \times 1$ function, and $g_2(Y, X, Z; \theta) = \frac{1-2\tau}{\tau(1-\tau)}\rho_\tau\left(\Lambda(Y, X; \alpha, \tau)\right) - \Lambda(Y, X; \alpha, \tau)$ a scalar function. Let

$$g_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} g(Y_i, X_i, Z_i; \theta),$$

be the empirical counterpart where sample average is used. The GMM-QR estimator is given by

$$\widehat{\theta}_n = \underset{\theta}{\operatorname{argmin}}\; g_n(\theta)^\top \widehat{W}_n\, g_n(\theta) \tag{7}$$

for a positive definite weighting matrix $\widehat{W}_n$.

Consider the following assumptions to establish the asymptotic properties of the estimator in (7).

Assumption 1. For each observation i among n in the sample, $Y_i \in \mathcal{Y} \subseteq \mathbb{R}^{d_Y}$, $Z_i \in \mathcal{Z} \subseteq \mathbb{R}^{d_Z}$; a subset of Z_i is $X_i \in \mathcal{X} \subseteq \mathbb{R}^{d_X}$, with $d_X \le d_Z$. The sequence $\{Y_i, Z_i\}$ is strictly stationary and weakly dependent.

Assumption 2. Let Θ be a compact set, with $\theta = (\alpha^\top, \tau)^\top \in \Theta$, where $\alpha \in \mathcal{A} \subset \mathbb{R}^{p-1}$ (with $p - 1 \le d_Z$) and $\tau \in \mathcal{T} \subset (0, 1)$, and θ_0 is an interior point of Θ and uniquely satisfies the moment condition $\mathrm{E}[g(Y_i, X_i, Z_i; \theta_0)] = 0$.

Assumption 3. Let $f_{\Lambda|Z}(\cdot|z)$ denote the conditional PDF of Λ_i given $Z_i = z$, and let $f_{\Lambda|Z,D}(\cdot|z, d)$ denote the conditional PDF of Λ_i given $Z_i = z$ and $D_i = d$. (i) For almost all z and d, $f_{\Lambda|Z}(\cdot|z)$ and $f_{\Lambda|Z,D}(\cdot|z, d)$ are at least r times continuously differentiable in a neighborhood of zero

Assumption 4. The function $\Lambda: \mathcal{Y}\times\mathcal{X}\times\mathcal{Z}\times\Theta \mapsto \mathbb{R}$ is known and has (at least) one continuous derivative in its Θ argument for all $y \in \mathcal{Y}, x \in \mathcal{X}$ and $z \in \mathcal{Z}$. The matrix $G \equiv \frac{\partial}{\partial\theta^\top}\mathrm{E}\left[g(Y_i, X_i, Z_i; \theta)\right]\Big|_{\theta=\theta_0}$ has rank p.

Assumption 5. CLT applies: $\sqrt{n}\{g_n(\theta_0) - \mathrm{E}[g_n(\theta_0)]\} \xrightarrow{d} Normal(0, \Sigma)$. For the weighting matrix, $\widehat{W}_n \xrightarrow{p} W$, and both are symmetric, positive definite matrices.

Then, the main result follows, the proof is in Appendix.

Theorem 1. *Under Assumptions 1–5, as $n \to \infty$, $\widehat{\theta}_n \xrightarrow{p} \theta_0$ and*

$$\sqrt{n}(\widehat{\theta}_n - \theta_0) \xrightarrow{d} Normal\left(0, (G^\top WG)^{-1}(G^\top W\Sigma WG)(G^\top WG)^{-1}\right), \tag{8}$$

where G and W are as defined in Assumptions 4 and 5.

5. Empirical estimation

In this section we estimate the dynamic quantile model described in Section 2 above. Data consist of aggregate level quarterly data for the United States (US) for 1947Q3–1998Q4. Consumption is measured at the beginning of the period, consisting of nondurables plus services for the US and total consumption for the other countries, in real, per capita terms. The real interest rate is the nominal short-term rate by the consumer price index. We consider 4 instrumental variables: the two quarter lagged dividend–price ratio for equities, two quarter lagged nominal interest rate, two quarter lagged inflation, and lagged consumption growth. For a complete description of the data, see Campbell (2003).

Our estimation follows Table 2 of Yogo (2004, p. 805). Yogo (2004) uses 2SLS to estimate the (structural) log-linearized model $\ln(C_{t+1}/C_t) = \delta_0 + \delta_1 \ln(1 + r_{t+1}) + u_{t+1}$, where r_{t+1} is the real interest rate, instrumenting for $\ln(1 + r_{t+1})$, where $\delta_1 = 1/\gamma$ is the EIS and $\delta_0 = \ln(\beta)/\gamma$. Yogo (2004) emphasizes that these are strong instruments that predict the real interest rate well, although formally characterizing "strong" for IVQR remains an open question. For practical estimation, the methods can be implemented sequentially. First, note that α can be obtained from Eq. (4) for every τ. Second, Eq. (5) can be used separately to find the appropriate level of τ. For our purposes, we use $\widehat{W}_n$ as block diagonal with components $(Z_n Z_n^\top)^{-1}$ for g_1 (where Z_n is the empirical matrix of instruments) and a large positive constant for g_2.

In order to illustrate the procedure the model is estimated for each τ. The results are reported in Fig. 1 where foc_4 reports the absolute value of the moment condition used in Eq. (5) together with the corresponding EIS estimate (eis_4). The solution requires that there should be a unique value of τ for which we can estimate $\alpha(\tau)$ that satisfies both (3) and (5). Table 1 presents the values of the estimates (EIS, τ, β) for the four lowest absolute values of the moment condition in (5), indexed by order. The table also reports the standard errors and 95% confidence interval calculated using 200 block bootstrap samples. The estimated EIS is 0.32 with associated τ of 0.74. The next 3 rows show the parameter estimates that follow in terms of the lowest values of foc_4, and they show that the EIS lies in between 0.26 and 0.35 and τ lies in between 0.64 and 0.74.

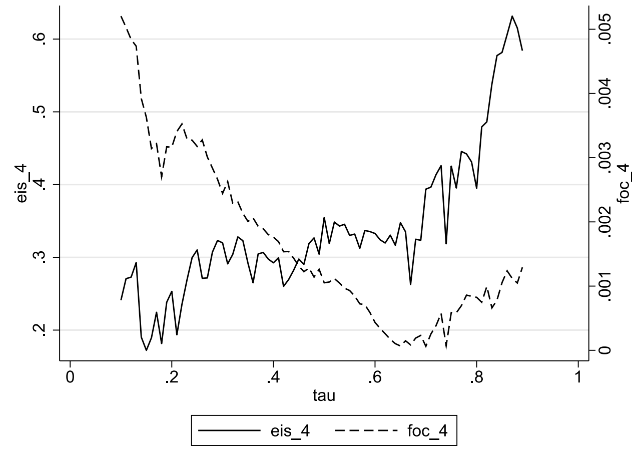

Fig. 1. EIS and τ estimates. United States 1947Q3–1998Q4.

Table 1
Results.

Order	EIS	τ	β
1st (GMM)	0.315	0.74	0.999
	(0.096)	(0.067)	(0.005)
	[0.206, 0.582]	[0.566, 0.833]	[0.991, 1.011]
2nd	0.348	0.65	1.004
3rd	0.263	0.67	1.005
4th	0.317	0.64	1.006

Notes: Standard errors in parenthesis and 95% confidence interval in brackets calculated using 200 block bootstrap replications for the GMM estimator.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Proof of Theorem 1. We show consistency of $\widehat{\theta}_{GMM}$ by establishing the conditions of Theorem 2.6 in Newey and McFadden (1994). The local identification condition (i) is satisfied by Assumptions 2 and 4. See also Chen et al. (2014, p. 787) when applying the full rank assumption of the partial derivative matrix with respect to θ. Both functions in g_1 and g_2 satisfy condition (iii). For g_1 note that $\Lambda(Y, X, \theta)$ is continuous in θ by Assumption 4 and continuously distributed by Assumption 3; for g_2 the same applies by Lemma A1 in Bera et al. (2016). They also satisfy condition (iv) by the boundedness of the indicator function, and the bounded moments of Z by Assumption 4. The other conditions follow from the assumptions. The asymptotic normality is shown by establishing the conditions of Theorem 7.2 in Newey and McFadden (1994). We first show that condition (v) is satisfied for g_1. By Assumption 4 and example 19.7 in van der Vaart (1998), the class of functions $\{\Lambda(Y, X, \theta) : \theta \in \Theta\}$ is P-Donsker. By Remark 4 in Chernozhukov and Hong (2003), the class of functions $\{1\{\Lambda(Y, X, \theta) \leq \tau\} : \theta \in \Theta\}$ is also P-Donsker. By Theorem 2.10.6 in van der Vaart and Wellner (1996), the class $\{1\{\Lambda(Y, X, \theta) \leq \tau\}Z : \theta \in \Theta\}$ is also P-Donsker, which implies condition (v) holds. The derivation for g_2 is in Lemma A1 in Bera et al. (2016). The other conditions hold directly from the assumptions. □

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